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Hybrid NMPC of a Supermarket Refrigeration System using Sequential Optimization ⋆

Christian Sonntag [*,1] **Arvind Devanathan** [**] **Sebastian Engell** [*]

[*] *Process Dynamics and Operations Group, Dept. of Biochemical and Chemical Engineering, Technische Universität Dortmund, Germany*
[**] *Linde KCA GmbH, 01277 Dresden, Germany*

Abstract: Supermarket refrigeration systems are hybrid systems due to switching of the continuous dynamics and due to the presence of discretely switched actuators such as valves and compressors. The main control goal for these systems is to keep temperature and pressure levels within tight bounds while minimizing the wear of the compressors. In our previous work, a hierarchical model-predictive control scheme was proposed for supermarket refrigeration systems that overcomes the major drawback of traditional control schemes for these systems: excessive switching of the compressors due to synchronization of the local controllers of the display cases. In every NMPC iteration, low-level temperature controllers were employed, and the high-level optimization task was reduced to the determination of optimal parameters for these controllers. While this approach was successfully applied to systems of moderate size, it did not yield satisfactory results for larger systems due to the combinatorial growth of the search space. Furthermore, fast drastic changes of the external disturbances could not be handled well by this technique. This paper presents several extensions to overcome these problems.

Keywords: Predictive control, process control, refrigeration systems, switched systems, hybrid systems

1. INTRODUCTION

In supermarket refrigeration systems, a rack of compressors is used to feed liquid refrigerant to several open display cases that are used to cool edible goods. These systems exhibit both, discrete and continuous dynamics, and are thus hybrid systems: The control inputs (valves and compressors) can only be switched discretely, and the nonlinear continuous dynamics changes due to switching of the discrete inputs. Today, supermarket refrigeration systems are often controlled using decentralized schemes in which each display case is equipped with independent simple control loops [Larsen et al., 2005]. This approach often causes a severe reduction of the efficiency of the process and the lifespan of the equipment: Since the display cases are usually identical in design, the switching of the discrete expansion valves that control the flow of refrigerant to the display cases tends to synchronize. This synchronization causes the phenomenon that in some time periods, large amounts of refrigerant evaporate in the display cases while at other times, the amount of evaporated refrigerant is negligible. In consequence, the compressors have to be switched frequently to maintain the pressure requirements of the system. This frequent switching increases the wear of the compressors [Larsen et al., 2005].

To counteract these problems, the suitability of advanced model-predictive schemes for the control of supermarket refrigeration systems has been investigated in previous work [Larsen et al., 2005, Sarabia et al., 2007, Sonntag et al., 2007]. Intense research on the model-predictive control of hybrid systems in recent years has led to the development of a variety of techniques that were successfully applied to practical examples from different fields. MPC approaches for hybrid systems can be divided into two groups: techniques that operate on piecewise affine approximations of the (often nonlinear) system dynamics using (multi-parametric) MILP/MIQP optimization (see e.g. Bemporad and Morari [1999], Stursberg et al. [2002]), and approaches that directly apply nonlinear optimization techniques to nonlinear hybrid models (see e.g. de Prada et al. [2004]). In Larsen et al. [2005], the MPC approach from Bemporad and Morari [1999] was applied to a piecewise affine approximation of the nonlinear hybrid model of a supermarket refrigeration system that is also considered in this paper [2]. Although this approach succeeded in keeping most process variables within pre-specified bounds, the frequency of the compressor switchings was high due to the inaccuracy of the linear approximations of the nonlinear dynamics. In Sarabia et al. [2007], a nonlinear MPC scheme was proposed that operates directly on a nonlinear model. Since the direct solution of the resulting MINLP problem is computationally infeasible, the input signals are parameterized as discrete pulses, and only the occurrences

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[1] Corresponding author: c.sonntag@bci.tu-dortmund.de, +49-(0)231-7555124.

[2] This system is a benchmark problem of the European Network of Excellence on Hybrid Control [Larsen et al., 2007].

in time of these pulses are determined which leads to an optimization problem with only continuous decision variables. As in our scheme, the cost function is evaluated by simulation of the hybrid model. This approach was capable of keeping all process variables within the bounds. However, in each MPC iteration a complex NLP problem with many decision variables had to be solved within a short time horizon since the points in time at which the display case inlet valves switch were optimized directly.

This paper proposes an extension of the hierarchical NMPC approach for supermarket refrigeration systems that was presented in Sonntag et al. [2007]. In this approach, the switching of the input valves of the display cases is not optimized directly, but simple low-level controllers are employed that regulate the temperatures in the display cases with a high sampling frequency. The parameters of these controllers are adjusted by a high-level NMPC optimizer that operates on a longer time horizon, thus leaving more computation time for the NLP step in every NMPC iteration. The high-level optimization also determines the switching sequences of the compressors. While this approach was successfully applied to a refrigeration system with two display cases and two compressors (see Sonntag et al. [2007]), it did not yield good results for larger systems for several reasons: (a) the enlargement of the discrete input space that is caused by an increase of the number of compressors could not be handled efficiently by this algorithm, (b) the algorithm considered only a single compressor switch in every NMPC iteration, thus severely limiting the available degrees of freedom, and (c) the computational effort for the simulation of the process model increased drastically with the number of display cases. Furthermore, sudden large changes of the external disturbances that e.g. occur when the system is switched from day-time to night-time operation could not be handled well. The extensions that are presented in this paper remedy the above-mentioned problems: (1) Only a subset of the possible compressor switches is considered over the immediate future, and a real value of the compressor capacity is assumed in the remote future, thus decoupling the number of alternative switchings from the dimension of the discrete input space. (2) The discrete search is performed by solving a sequence of continuous optimization problems with an increasing number of switches, and the search is stopped as soon as a policy is found that meets the specification. This reflects the main control goal, the minimization of the number of switches of the compressors. An additional advantage of this approach is that the number of compressor switchings is not included in the cost function, removing a discontinuous term in the cost function. (3) The simulation of the locally controlled system is performed using a fixed-step Runge-Kutta algorithm, leading to a linear growth of the numerical effort with the number of display cases. (4) An additional low-level controller is implemented that continuously monitors the process and switches the compressors directly if sudden large changes of the external disturbances occur.

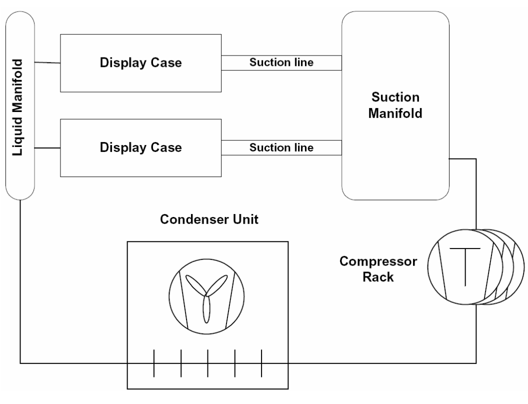


Fig. 1. A simplified scheme of a supermarket refrigeration system with two display cases.

2. SYSTEM DESCRIPTION

Edible goods that are sold in supermarkets are usually located in open refrigerated display cases to avoid deterioration and to enable easy access for the customers. Fig. 1 shows a schematic representation of a supermarket refrigeration system. It consists of four major parts: several display cases, a compressor rack, a suction manifold, and a condenser. Liquid refrigerant is supplied to the display cases through inlet valves. The refrigerant evaporates, thus removing heat from the air around the evaporator. The resulting vapor accumulates in the suction manifold and is fed to the condenser via the compressors which increase the pressure of the refrigerant vapor. Since the evaporation temperature of the refrigerant increases with the pressure, the thermal energy from the display cases can be removed in the condenser at room temperature. Finally, the liquefied refrigerant is fed back to the display cases. The cross-section of an open refrigerated display case is shown in Fig. 2. Cold air is circulated through the display case and forms an air curtain in front of the edible goods. Thermal energy is transferred from the goods to the air curtain ($\dot{Q}_{goods-air}$) and, since the temperature of the surrounding air is larger than that of the air curtain, the curtain also absorbs heat from the surroundings ($\dot{Q}_{airload}$). The absorbed thermal energy is transported to the evaporator ($\dot{Q}_{air-wall}$) in which the refrigerant evaporates and thus takes on the thermal energy ($\dot{Q}_e$).

The controlled variables of the system are the pressure inside the suction manifold and the temperatures of the

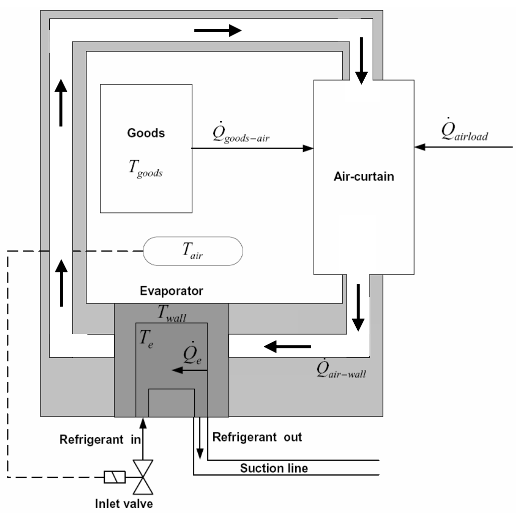


Fig. 2. The cross section of a display case.

air inside the display cases. As the system never reaches a steady state, the control goal is not to track setpoints, but to maintain these variables within specified bounds. This is achieved using two types of discrete control inputs: The expansion valves at the inlets of the display cases can be opened or closed, and the compressors can be switched on or off individually. The pressure in the suction manifold and the temperatures of the air and of the walls in the display cases can be measured directly, and an approximation of the temperature of the goods can be obtained from temperature sensors that are located close to the goods in the display cases. The external disturbances to the system are the energy transfer from the outside air to the display cases ($\dot{Q}_{airload}$) and an additional flow of refrigerant into the suction manifold ($\dot{m}_{ref-const}$) that originates from additional cooling facilities which are not modeled. These disturbances are measurable.

3. THE HYBRID MODEL

The hybrid model used in this work was proposed in Larsen et al. [2007]. It has been implemented as a *hybrid automaton* with continuous and discrete inputs [Lynch et al., 2003, Stursberg, 2004].

The hybrid model of the supermarket refrigeration system may contain an arbitrary number of display cases n_{dc}. The state of each display case is described by four differential state variables: the temperature of the goods ($T_{g,i}$), the temperature of the evaporator wall ($T_{w,i}$), the temperature of the air inside the case ($T_{air,i}$), and the mass of liquid refrigerant within the evaporator of the display case ($m_{ref,i}$). Thus, the vector of continuous state variables of the i-th display case is given by

$$\mathbf{x}_{dc,i} = [T_{g,i}, T_{w,i}, T_{air,i}, m_{ref,i}]^T. \quad (1)$$

The dynamics of the condenser unit is not modeled. Hence, the overall continuous state vector of the model can be written as

$$\mathbf{x} = [x_{dc,1}^T, \ldots, x_{dc,n_{dc}}^T, P_{suc}]^T, \quad (2)$$

where P_{suc} is the pressure in the suction manifold. Since each display case is equipped with an inlet valve for the refrigerant, the discrete input vector is given by

$$\mathbf{v} = [v_1, \ldots, v_{n_{dc}}, v_c]^T. \quad (3)$$

Here, $v_1, \ldots, v_{n_{dc}}$ are binary variables representing the state of the inlet valves (open/closed), and $v_c \in \Xi_c$ (given in %) determines the relative capacity [3] of the compressors that are currently running within the compressor rack. The set Ξ_c contains all discrete capacity levels that can be realized by switching one or several compressors on. In this paper, a system with three compressors of different capacities is investigated. For this system, Ξ_c is defined as:

$$\Xi_c := \{0\ \%, 30\ \%, 60\ \%, 100\ \%\}. \quad (4)$$

The continuous dynamics is modeled by a lumped-parameter ODE system [4] under the assumption that all display cases are of equal design. The amount of refrigerant in a display case and the current state of the corresponding inlet valve v_i influence the dynamics of the mass of refrigerant in a display case $m_{ref,i}$ according to

$$\frac{dm_{ref,i}}{dt} = \begin{cases} \dfrac{m_{ref,max} - m_{ref,i}}{\tau_{fill}} & \text{if } v_i = 1, \text{ (a)} \\ -\dfrac{\dot{Q}_{e,i}}{\Delta h_{lg}} & \text{if } v_i = 0. \text{ (b)} \end{cases} \quad (5)$$

Here, the maximum amount of refrigerant each display case can accommodate is represented by $m_{ref,max}$, $\dot{Q}_{e,i}$ is defined in Eq. 11, the specific enthaply of evaporation of the remaining liquefied refrigerant in the evaporator is given by Δh_{lg}, and τ_{fill} is a time constant. The display case is filled with refrigerant as long as the inlet valve is open (Eq. 5.a), and after the inlet valve has been closed, the remaining refrigerant evaporates according to Eq. 5.b. The temperature dynamics within the i-th display case is given by:

$$\frac{dT_{g,i}}{dt} = -\frac{\dot{Q}_{goods-air,i}}{m_{goods} \cdot cp_{goods}}, \quad (6)$$

$$\frac{dT_{w,i}}{dt} = \frac{\dot{Q}_{air-wall,i} - \dot{Q}_{e,i}}{m_{wall} \cdot cp_{wall}}, \quad (7)$$

$$\frac{dT_{air,i}}{dt} = \frac{\dot{Q}_{goods-air,i} + \dot{Q}_{airload} - \dot{Q}_{air-wall,i}}{m_{air} \cdot cp_{air}}, \quad (8)$$

with

$$\dot{Q}_{goods-air,i} = UA_{goods-air} \cdot (T_{g,i} - T_{air,i}), \quad (9)$$

$$\dot{Q}_{air-wall,i} = UA_{air-wall} \cdot (T_{air,i} - T_{w,i}), \quad (10)$$

$$\dot{Q}_{e,i} = UA_{wall-ref}(m_{ref,i}) \cdot (T_{w,i} - T_e(P_{suc})), \quad (11)$$

$$UA_{wall-ref}(m_{ref,i}) = UA_{wall-refmax} \cdot \frac{m_{ref,i}}{m_{ref,max}}. \quad (12)$$

Here, m_{goods}, m_{wall}, m_{air}, cp_{goods}, cp_{wall}, cp_{air}, $UA_{goods-air}$, $UA_{air-wall}$, and $UA_{wall-refmax}$ are constant model parameters, and T_e is the evaporation temperature of the refrigerant which is a function of the suction pressure P_{suc}. The dynamics of the suction pressure is given by

$$\frac{dP_{suc}}{dt} = \frac{\dot{m}_{in-suc} + \dot{m}_{ref-const} - \dot{V}_c \cdot \rho_{suc}}{V_{suc} \cdot \frac{d\rho_{suc}}{dP_{suc}}}, \quad (13)$$

with

$$\dot{m}_{in-suc} = \sum_{i=1}^{n_{dc}} \frac{\dot{Q}_{e,i}}{\Delta h_{lg}}, \quad \dot{V}_c = \frac{v_c \cdot \eta_{vol} \cdot V_d}{100}. \quad (14)$$

The total mass flow of refrigerant from all display cases into the suction manifold is given by $\dot{m}_{in-suc}$, and $\dot{m}_{ref-const}$ is an external disturbance that represents an additional flow of refrigerant from other unmodeled cooling facilities into the suction manifold. Using the relative capacity of the running compressors v_c (see Eq. 3), the volume flow $\dot{V}_c$ from the suction manifold can be computed. ρ_{suc} is a nonlinear refrigerant-dependent function modeling the density of the vapor in the suction manifold, and η_{vol} and V_d are constant model parameters.

4. THE HIERARCHICAL CONTROL SCHEME

A scheme of the control strategy developed in this work is shown in Fig. 3. At every sampling instant, low-level controllers provide discrete control inputs to the plant while the high-level NMPC scheme sets the parameters of the low-level controllers and determines a suitable strategy for the compressor switches on a larger time horizon.

[3] Thus, the values 0% (100%) always indicate that all compressors are off (on), independently of the number of compressors.

[4] See Larsen et al. [2007] for a more detailed description of the model.

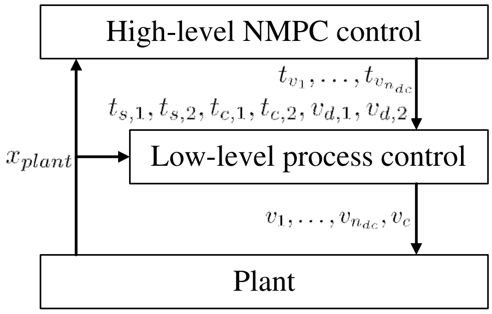


Fig. 3. Scheme of the control strategy.

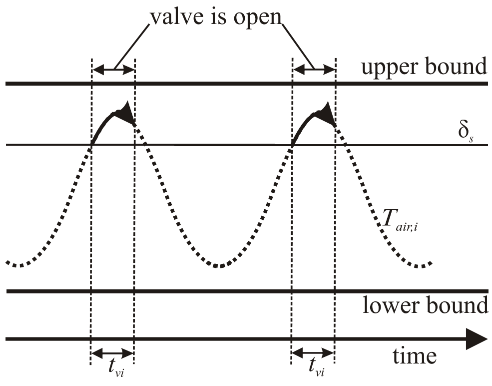


Fig. 4. Switching strategy for the expansion valves for an exemplary evolution of the air temperature of display case i.

4.1 The Low-Level Control System

The main variables of the supermarket refrigeration system exhibit periodic behavior and never reach a steady state. This fact is used to design low-level controllers for the display cases that keep the controlled variables within the prescribed bounds. The switching strategy for the valves of display case i is shown in Fig. 4. The valve of the corresponding display case remains closed as long as the air temperature remains below the switching threshold δ_s. After the refrigerant has evaporated, the air temperature starts to rise. Once the air temperature crosses δ_s from below, v_i is opened for a constant period of time t_{v_i}, and the air temperature will decrease again. The time period t_{v_i} is a continuous parameter that is assigned by the high-level optimizer for each display case ($t_{v_1}, \ldots, t_{v_{n_{dc}}} \in \mathbb{R}^{\geq 0}$ in Fig. 3). The value of δ_s is determined from simulation studies assuming that $T_{air,i}$ will always decrease shortly after the valve v_i is opened (which can be deduced from the continuous model dynamics and parameters). The controller parameters $t_{s,1}$, $t_{s,2}$, $t_{c,1}$, $t_{c,2}$, $v_{d,1}$, and $v_{d,2}$ are computed by the high-level optimization scheme to initiate one or several switchings of the compressors, as explained in the following section.

To compensate for fast changes of the external disturbances, an additional low-level controller is employed. This controller monitors the stationary continuous compressor capacity v_{cs} that is needed to keep all process variables within the admissible region over long time periods. From the ODE system, v_{cs} can be computed as:

$$v_{cs} = 100\ \% \cdot \left(\frac{\dot{m}_{in-suc} + \dot{m}_{ref-const}}{\rho_{suc} \cdot \eta_{vol} \cdot V_d} \right). \qquad (15)$$

If v_{cs} changes by more than $\delta_c = \frac{1}{2} \cdot \frac{100\ \%}{n_c}$ (n_c is the number of compressors) over a time period of 30 seconds, the current NMPC iteration is interrupted. Subsequently, the controller switches the compressors to the discrete capacity level that is closest to v_{cs} and restarts the high-level optimizer.

4.2 The High-Level NMPC Controller

Since the temperature control of the display cases is ensured by the low-level controllers, the time interval t_{opt} that is available in every iteration of the high-level optimizer can be chosen to be significantly larger than the sampling time of the process control system.

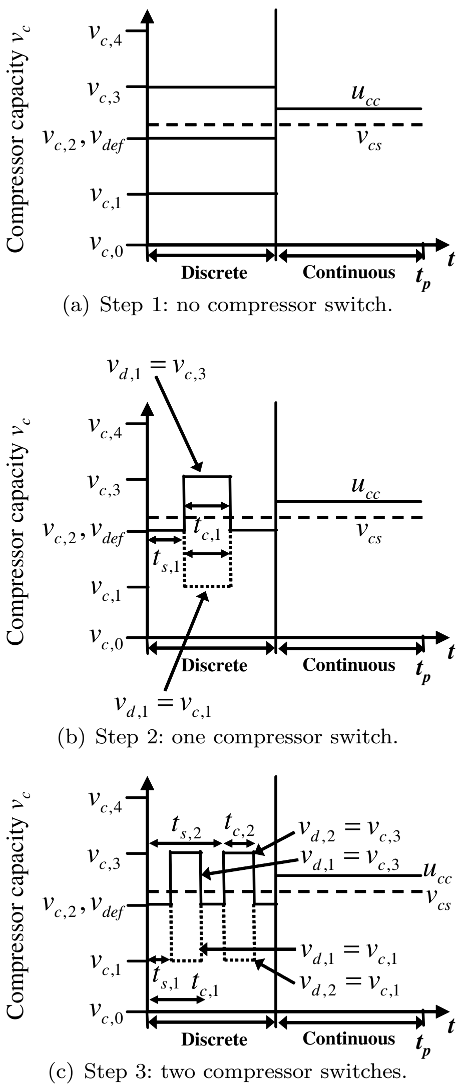
In each iteration, the algorithm solves a sequence of continuous optimization problems of increasing complexity to avoid discontinuities in the cost function that are introduced by an explicit consideration of the number of compressor switchings in the cost function. The first optimization problems do not include any switching of the compressors which represents the optimal case if all process variables can be kept within the predefined bounds. If needed, problems with one or two switchings are solved subsequently. The optimization problems that are solved in each step are of the form:

$$\min_{v_{d,1} \in \Xi_{d,1}, v_{d,2} \in \Xi_{d,2}} \left(\min_{\mathbf{u}_{ct}} \Omega(t_p, \sigma_j, \mathbf{u}_{ct}, v_{d,1}, v_{d,2}, v_d) \right), \qquad (16)$$

with $\Xi_{d,1}, \Xi_{d,2} \subset \Xi_c$. σ_j is the hybrid state[5] of the plant at the beginning of the NMPC iteration j, t_p is the prediction horizon, Ω is the cost function, and v_d is the initial compressor capacity. Solutions of these problems are obtained by solving the continuous minimization problems for all possible combinations of $v_{d,1}$ and $v_{d,2}$. The cost function is evaluated by the simulation of a hybrid model of the supermarket system over the prediction horizon t_p. This model is obtained by including the low-level control system into the hybrid model presented in Sec. 3. The formulations of the optimization problems are different for the three sequential steps of the algorithm as shown exemplarily in Fig. 5 for a system with 4 compressors of equal capacity ($\Xi_c = \{v_{c,0}, v_{c,1}, v_{c,2}, v_{c,3}, v_{c,4}\}$ with $v_{c,0} = 0\ \%$, $v_{c,4} = 100\ \%$). The prediction horizon is separated into two parts: an initial period in which only discrete capacity levels are considered, and a second, usually longer period in which the discrete compressor capacity is relaxed into a continuous value $u_{cc} \in [0\ \%, 100\ \%]$. This relaxation is based on the fact that a continuous capacity value u_{cc} can be approximated accurately by a suitable switching strategy of the compressors over a sufficiently long horizon.

Instead of considering all discrete compressor capacity levels and all possible switching sequences as e.g. in Sonntag et al. [2008] which is infeasible for many compressors, a subset of fixed cardinality that contains only promising candidate sequences is investigated. A reduction of the set of capacity levels can be achieved by considering only three

[5] σ_j comprises the continuous state of the plant and the current discrete location z. The discrete locations are defined in accordance with the switching dynamics. The definition is not given here due to space limitations.



(a) Step 1: no compressor switch.

(b) Step 2: one compressor switch.

(c) Step 3: two compressor switches.

Fig. 5. The sequential optimization scheme.

levels that are closest to v_{cs} (see Eq. 15). The default capacity level v_{def} is defined that corresponds to the capacity level that is closest to v_{cs} ($v_{c,2}$ in Fig. 5), and the levels adjacent to v_{def} are from here on denoted by $v_{def,-}$ and $v_{def,+}$ ($v_{c,1}$ and $v_{c,3}$ in Fig. 5).

The cost function is defined as[6]:

$$\Omega = \sigma\left(\{P_{suc,s}(k)\}_{0\le k\le t_p}\right) + 100\cdot\sum_{i=0}^{t_p}\left(\underset{\underline{P}_s,\overline{P}_s}{\lambda}(P_{suc,s}(i)) + \sum_{j=1}^{n_{dc}}\underset{\underline{T}_s,\overline{T}_s}{\lambda}(T_{air,j,s}(i))\right). \quad (17)$$

Here, the sequences $\{P_{suc,s}(l)\}$ and $\{T_{air,j,s}(l)\}$, $0 \le l \le t_p$, $l \in \mathbb{N}$, are computed by uniformly sampling the scaled simulated time evolutions of the state variables. $\underline{P}_s$, $\underline{T}_s$, $\overline{P}_s$, and $\overline{T}_s$ are the lower and upper bounds of the admissible region, the operator $\sigma(\bullet)$ computes the standard deviation, and λ is defined as:

$$\underset{\underline{y},\overline{y}}{\lambda}(y) = \begin{cases} 0 & \text{if } \underline{y} \le y \le \overline{y}, \\ \underline{y} - y & \text{if } y < \underline{y}, \\ y - \overline{y} & \text{if } y > \overline{y}, \end{cases} \quad (18)$$

with $\underline{y}, \overline{y}, y \in \mathbb{R}$. The first term of the cost function aims at the minimization of the variance of the suction pressure and, thus, at the desynchronization of the air temperatures in the display cases, and the second term adds a penalty for violations of the admissible region.

Based on the concepts introduced above, the execution of the sequential optimization scheme in every NMPC iteration can be written as follows (see also Fig. 5):

(1) Compute v_{cs} and determine $v_{def,-}, v_{def}, v_{def,+}$.
(2) **First step (no compressor switch):**
  - In Eq. 16, define $\mathbf{u}_{ct} = [t_{v_1}, \ldots, t_{v_{n_{dc}}}, u_{cc}]$.
  - Define $\Xi_{d,1} = \Xi_{d,2} = v_d = v_{def,-}$ and solve (16).
  - Define $\Xi_{d,1} = \Xi_{d,2} = v_d = v_{def}$ and solve (16).
  - Define $\Xi_{d,1} = \Xi_{d,2} = v_d = v_{def,+}$ and solve (16).
  - Extract the best found solution of all solved problems $\mathbf{u}^*_{ct}, v^*_{d,1}, v^*_{d,2}$. If the penalty term in Eq. 17 is zero for this solution, go to step (5).
(3) **Second step (one compressor switch):**
  - Define $\mathbf{u}_{ct} = [t_{v_1}, \ldots, t_{v_{n_{dc}}}, u_{cc}, t_{s,1}, t_{c,1}]$.
  - Define $\Xi_{d,1} = \{v_{def,-}, v_{def,+}\}$ and $\Xi_{d,2} = v_d = v_{def}$, and solve (16).
  - Extract the best found solution $\mathbf{u}^*_{ct}, v^*_{d,1}, v^*_{d,2}$. If the penalty term in Eq. 17 is zero for this solution, go to step (5).
(4) **Third step (two compressor switches):**
  - Define $\mathbf{u}_{ct} = [t_{v_1}, \ldots, t_{v_{n_{dc}}}, u_{cc}, t_{s,1}, t_{c,1}, t_{s,2}, t_{c,2}]$.
  - Define $\Xi_{d,1} = \Xi_{d,1} = \{v_{def,-}, v_{def,+}\}$ and $v_d = v_{def}$, and solve (16).
(5) Assign the best found solution $\mathbf{u}^*_{ct}, v^*_{d,1}, v^*_{d,2}$ to the low-level control system.

Evidently, the complexity of the continuous optimization problems increases from the first step to the third step. In simulation studies, it was found that the third step is only rarely executed which indicates that this scheme offers sufficient freedom to efficiently control supermarket refrigeration systems.

During the embedded simulation of the hybrid model, the time points at which the air temperatures in the display cases cross the threshold δ_s from below must be computed. The event detection mechanism of the *Matlab*-based ODE solver *ODE45* that was used in Sonntag et al. [2007] causes a large increase of the simulation time with the number of display cases. To overcome this problem, the fact is exploited that the sampling time of the process control system is known to be 1 second. Keeping this in mind, a fixed-step 4^{th}-order Runge-Kutta algorithm without event detection is used to simulate the system in steps of 1 second, and after every simulation step, it is determined if any of the air temperatures caused an event. Since accurate event detection is not necessary in this approach, the computational effort for simulation increases linearly with the number of display cases.

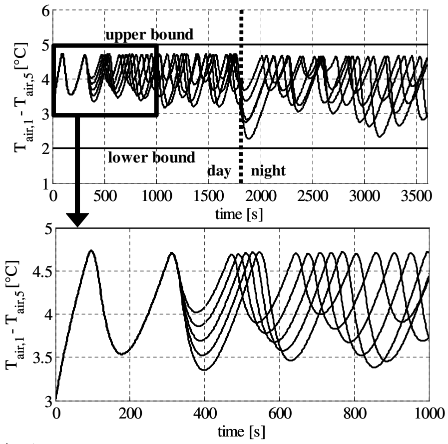
5. SIMULATION RESULTS

This section presents some results of the application of the control scheme to a supermarket refrigeration system

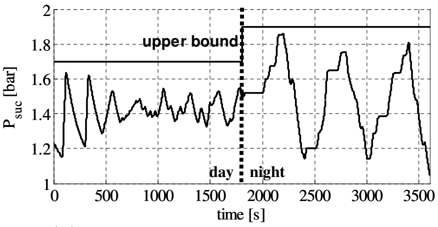
[6] The subscript $_s$ denotes that the corresponding values that are determined by simulation are scaled to the range $[0, 1]$ before the cost function evaluation.

with 5 display cases and 3 compressors with capacities as defined in Eq. 4. The controller and the simulation model were implemented in *Matlab*, and the *TOMLAB*-based NLP solver *NPSOL* was employed for the solution of the continuous optimization problems [7]. Fig. 6 shows the trajectories of the air temperatures and of the suction pressure, and the switching behavior of the compressors. Initially, the system is operated in a day-time scenario ($\dot{Q}_{airload} = 3000\ W$, $\dot{m}_{ref-const} = 0.2\ \frac{kg}{s}$, and $\overline{P} = 1.7\ bar$). After 1800 seconds, the system is switched into night-time operation, i.e. the display cases are covered with blankets which leads to a reduction of $\dot{Q}_{airload}$ to 1800 W, and the unmodeled cooling facilities are switched off ($\dot{m}_{ref-const} = 0\ \frac{kg}{s}$). Furthermore, $\overline{P}$ is increased to 1.9 bar. The controller is capable of keeping all process variables within the bounds and desynchronizes the air temperatures very quickly. Furthermore, the low-level compressor controller detects the drastic change in the external disturbances and switches off all compressors. Although there is no direct comparison available for this system, a comparison to the simulation results of a smaller system under traditional control given in Larsen et al. [2007] shows that the presented control scheme adheres better to the constraints and significantly reduces the frequency of the controller switching.

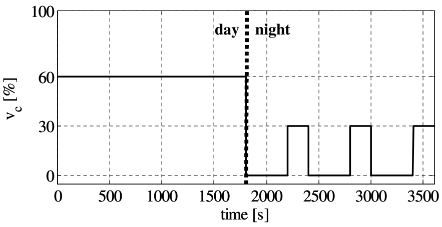
[7] The following parameter values were used: $t_p = 900\ s$, $t_{opt} = 200\ s$, $\underline{T} = 2\ \%$, $\overline{T} = 5\ \%$, and $\underline{P} = 1\ bar$.



(a) Air temperatures in the display cases. The lower figure shows an enlargement of a small part of the upper figure.



(b) Pressure in the suction manifold.



(c) Compressor capacity.

Fig. 6. Trajectories of the main process variables.

6. CONCLUSIONS AND FUTURE WORK

In this paper, several extensions of the NMPC-based control strategy for supermarket refrigeration systems that was introduced in Sonntag et al. [2007] were presented. The extensions enable the application of the scheme to large systems that are subject to drastic disturbances by (1) restricting the complexity of the optimization task, (2) introducing a sequential optimization scheme to increase the available degrees of freedom and to avoid discontinuities in the cost function, (3) implementing an efficient simulation strategy for large-scale systems, and (4) introducing a low-level pressure controller. Simulation studies show that the new control scheme is capable of keeping the process variables within tight bounds while reducing the switching frequency of the compressors. Our current research aims at a stronger exploitation of the model structure to allow for a more efficient optimization.

REFERENCES

A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35: 407–427, 1999.

C. de Prada, S. Cristea, D. Sarabia, and W. Colmenares. Hybrid control of a mixed continuous-batch process. In *Proc. European Symp. on Computer-Aided Process Engineering*, pages 473–484, 2004.

L. F. S. Larsen, T. Geyer, and M. Morari. Hybrid MPC in supermarket refrigeration systems. In *Proc. 16th IFAC World Congress*, 2005. Th-E12-TO/5.

L. F. S. Larsen, R. I. Zamanabadi, R. Wisniewski, and C. Sonntag. Supermarket refrigeration systems - a benchmark for the optimal control of hybrid systems. Technical report for the HYCON NoE, 2007. http://tinyurl.com/23nrkc.

N. Lynch, R. Segala, and F. Vaandrager. Hybrid I/O automata. *Information and Computation*, 185(1):105–157, 2003.

D. Sarabia, F. Capraro, L. F. S. Larsen, and C. de Prada. Hybrid control of a supermarket refrigeration system. In *Proc. ACC*, pages 4178–4185, 2007.

C. Sonntag, A. Devanathan, S. Engell, and O. Stursberg. Hybrid nonlinear model-predictive control of a supermarket refrigeration system. In *Proc. IEEE Multi-Conf. on Systems and Control*, pages 1432–1437, 2007.

C. Sonntag, W. Su, O. Stursberg, and S. Engell. Optimized start-up control of an industrial-scale evaporation system with hybrid dynamics. *To appear in: Control Engineering Practice.*, 2008. doi:10.1016/j.conengprac.2007.09.003.

O. Stursberg. A graph search algorithm for optimal control of hybrid systems. In *Proc. 43rd IEEE Conf. on Decision and Control*, pages 1412–1417, 2004.

O. Stursberg, S. Panek, J. Till, and S. Engell. Generation of optimal control policies for systems with switched hybrid dynamics. In *Modelling, Analysis, and Design of Hybrid Systems*, volume 279 of LNCIS, pages 337–352. 2002.