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FEEDBACK STABILIZATION OF THE TORA SYSTEM VIA INTERCONNECTION AND DAMPING ASSIGNMENT CONTROL

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Abstract: In this work, we consider the feedback stabilization problem of the so-called Translational Oscillator with a Rotational Actuator (TORA) system by applying the Interconnection and Damping Assignment (IDA) control methodology. To achieve this goal, the mechanical system is firstly transformed into the general port controlled Hamiltonian form and, then, the IDA design procedure is applied to synthesize the stabilizing control law. The Hamiltonian structure of the closed loop system is preserved and asymptotic stability of the mechanical position is achieved, which is verified by digital simulations.

Keywords: Design methods, Nonlinear system control, Asymptotic stabilization.

1. INTRODUCTION

Energy shaping methods pursue to preserve the physical structure of closed-loop systems. Total energy shaping in mechanical systems is guaranteed by firstly modifying the inertia matrix in the kinetic energy and, then, the potential energy is shaped. The Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) approach is used to achieve this goal in Ortega, *et al.* (2002a) for mechanical systems in the general Port-Controlled Hamiltonian (PCH) form (Ortega, *et al.* 2002b).

The stabilization problem of underactuated nonlinear systems has attracted attention of the control community in recent years (Olfati-Saber, 2001). The IDA-PBC design method deals with nonlinear systems with one degree of underactuation, achieving (asymptotic) stabilization of mechanical systems and endowing closed loop systems with a Hamiltonian structure and a desired energy function (Mahindrakar, *et al.,* 2006). In this paper we consider the problem of achieving asymptotic stability of the TORA system through a control law synthesized by applying the IDA-PBC methodology. This result is based on an alternative approach to the cascade and PBC control designs proposed by Escobar, Ortega and Sira-Ramirez, (1999); and by Pavlov, *et al.* (2005), which were employed for regulation and output feedback stabilization of the TORA system.

2. STABILIZATION OF UNDERACTUATED MECHANICAL SYSTEMS VIA IDA-PBC

2.1. Mechanical systems in PCH form

Consider underactuated mechanical systems with total energy function

$$H(q,p)=\frac{1}{2}p^T M^{-1}(q)p+V(q) \tag{1}$$

where $q\in\Re^n$, $p\in\Re^n$ represent the generalized positions and momenta, respectively, $M(q)=M^T(q)>0$ is the inertia matrix, and $V(q)$ denotes the potential energy of the system. By assuming that system (1) has not natural damping, it can be written as follows

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix}=\begin{bmatrix}0 & I_n\\ -I_n & 0\end{bmatrix}\begin{bmatrix}\nabla_q H\\ \nabla_p H\end{bmatrix}+\begin{bmatrix}0\\ G(q)\end{bmatrix}u \tag{2}$$

where I_n is the n-order identity matrix, $\nabla_q H,\nabla_p H$ are the gradient of H with respect to q and p, respectively, $u\in\Re^m$ is the control function and $G\in\Re^{nxm}$ with $rank(G)=m<n$.

In the application of the IDA-PBC approach two basic steps are followed (Ortega, *et al.,* 2002a; Mahindrakar, *et al.,* 2006): (1) *energy shaping*, on which the total energy is modified to assign a desired equilibrium point; and (2) *damping injection*, to achieve asymptotic stability and also to preserve the Hamiltonian form of the closed loop system.

The form of equation (1) suggests to propose the total energy function

$$H_d(q,p) = \frac{1}{2} p^T M_d^{-1}(q) p + V_d(q) \tag{3}$$

where $M_d = M_d^T > 0$ and V_d are the desired inertia matrix and the potential energy, respectively, to be defined. It is required that V_d has a local minimum at q^*, i.e.

$$q^* = \arg\min V_d(q) \tag{4}$$

In the PBC methodology the control input is usually decomposed into the following two terms (Ortega, *et al.,* 2002a)

$$u = u_{es}(q,p) + u_{di}(q,p) \tag{5}$$

with the first term used to achieve energy shaping and the second term is employed to inject damping into the system, in order to reach a closed loop system with the Hamiltonian form

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \left[J_d(q,p) + \quad R_d(q,p) \right] \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix} \tag{6}$$

where

$$J_d = \begin{bmatrix} 0 & M^{-1} M_d \\ -M_d M^{-1} & J_2(q,p) \end{bmatrix}, \quad R_d = \begin{bmatrix} 0 & 0 \\ 0 & G K_v G^T \end{bmatrix} \geq 0 \tag{7}$$

are the desired interconnection and damping structures. The skew-symmetric matrix J_2 (and some elements of M_d) are free parameters for design, whilst the term $M^{-1} M_d$ is considered to preserve the relationship $\dot{q} = M^{-1} p$. Furthermore, damping is provided by feeding back the new passive output $G^T \nabla_p H_d$. Thus, the term u_{di} of the equation (5) is chosen, with $K_v = K_v^T > 0$, as follows

$$u_{di} = -K_v G^T \nabla_p H_d \tag{8}$$

2.2. Stability

In order to guarantee stability of the closed loop system, the following proposition is used.

Proposition 1: *System (2) with* H_d *as in (3) and* q^* *the desired position holding (4) has a stable equilibrium point on* $(q^*, 0)$. *This equilibrium point is asymptotically stable if the system is locally detectable from the output* $G^T(q) \nabla H_d(q,p)$.

Proof. See Ortega, *et al.* (2002a) for a detailed proof.

2.3. Energy Shaping

To obtain the energy shaping term u_{es} in the controller, equations (5) and (7) are substituted in (2) and the result is equated to (6), i.e.

$$\begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u_{es} = \begin{bmatrix} 0 & M^{-1} M_d \\ -M_d M^{-1} & J_2(q,p) \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix} \tag{9}$$

where the term R_d of (6) has been cancelled by u_{di} of (7). The first row of (9) produces an identity, whilst the second row can be written as follows

$$G u_{es} = \nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 M_d^{-1} p \tag{10}$$

Since the system is underactuated, the control u_{es} has only effect on the image space of the operator G. Thus, for any choice of u_{es}, the following relationship should be satisfied

$$G^{\perp} \left\{ \nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 M_d^{-1} p \right\} = 0 \tag{11}$$

where $G^{\perp}$ is a full rank left annihilator of G (i.e., $G^{\perp} G = 0$). Equation (11) is a set of nonlinear partial differential equations (PDE) with unknown terms M_d *and* V_d, and J_2 being a free parameter, whilst p is an independent coordinate. If a solution is obtained for equation (11), the control term u_{es} is written in the form

$$u_{es} = \left(G^T G \right) \left(\nabla_q H - M_d M_{-1} \nabla_q H_d + J_2 M_d^{-1} p \right) \tag{12}$$

Equation (11) is equivalent to the pair of equations

$$G^{\perp} \left\{ \nabla_q \left(p^T M^{-1} p \right) - M_d M^{-1} \nabla_q \left(p^T M_d^{-1} p \right) + 2 J_2 M_d^{-1} p \right\} = 0 \tag{13}$$

$$G^{\perp} \left\{ \nabla_q V - M_d M_{-1} \nabla_q V_d \right\} = 0 \tag{14}$$

The partial differential equation (13) is nonlinear and should be solved with respect to unknown elements of the inertia matrix M_d. Then, using the already known matrix M_d, the equation (14) can be solved to obtain the desired potential energy V_d. When the inertia matrix in (1) is independent of the underactuated coordinate, it is common to assume a constant inertia matrix M_d, thus allows focusing on finding a solution to the potential energy V_d.

This work closely follows the results in Ortega, *et al.* (2002a) and Mahindrakar, *et al.* (2006), where to obtain a reduction of the PDE (13) and (14), the following assumptions hold:

<u>H1:</u> The system has underactuation degree one, i.e. $m = n - 1$.

<u>H2:</u> The inertia matrix M depends only on the actuated coordinates.

<u>H3:</u> The system has two degree of freedom and, without loss of generality, take $G = [0 \quad 1]^T$.

Assumption *H3* is critical in this application. Assumptions *H1* and *H2* ensure that the term $G^{\perp}\nabla_q\left(p^T M^{-1} p\right)$ in the PDE (13) is zero. In this case (13) can be solved with a constant matrix M_d, by taking $J_2 = 0$. This allows us to focus on potential energy shaping and the PDE

$$G^{\perp}\left\{M_d M^{-1}\nabla V_d\right\} = 0. \qquad (15)$$

3. MATHEMATICAL MODEL OF THE TORA SYSTEM

The translational oscillator with rotational actuator (TORA) system (Pavlov, *et al.,* 2005) consists of a platform of mass m_1 which can oscillate without damping in the horizontal plane (see Fig. 1). On the platform a rotating eccentric mass m_2 is actuated by a DC motor. Its motion applies a force to the platform which can be used to damp the translational oscillations.

The inertia matrix of the system has the form

$$M = \begin{bmatrix} m_1 + m_2 & m_2 r\cos(q_2) \\ m_2 r\cos(q_2) & m_2 r^2 + I \end{bmatrix} \qquad (16)$$

where q_2 is the rotating angle of the mass m_2, r is the eccentricity ratio, and I is the moment of inertia. Denoting the generalized positions by $q = [q_1 \quad q_2]^T$, the gravitational constant by g, and the potential energy by $V(q_1, q_2)$, the system Lagrangian can be written as follows

$$L\left(q, \dot{q}\right) = \frac{1}{2}\begin{bmatrix} \dot{q}_1 & \dot{q}_2 \end{bmatrix} M(q_2)\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - V(q_1, q_2) \qquad (17)$$

where $V(q_1, q_2)$ represents the potential energy

$$V(q_1, q_2) = \frac{1}{2}Kq_1^{\,2} + m_2 gr\cos(q_2) \qquad (18)$$

and K stand by the constant stiffness of the spring.

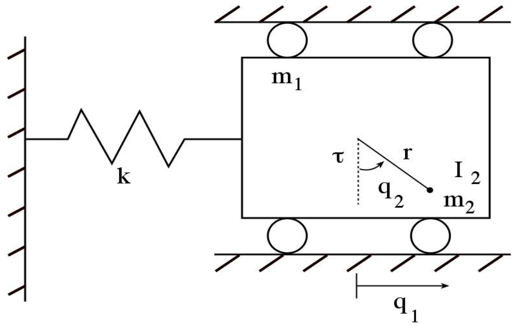

Fig. 1. The TORA system

Denoting the motor torque, acting at the rotating point of the mass m_2, by τ, the Euler-Lagrange equations of the TORA system has the form

$$\begin{cases} (m_1 + m_2)\ddot{q}_1 + m_2 r\cos(q2)\ddot{q}_2 - m_2 r\sin(q_2)\dot{q}_2^{\,2} + Kq_1 = 0 \\ m_2 r\cos(q_2)\ddot{q}_1 + \left(m_2 r^2 + I\right)\ddot{q}_2 + m_2 gr\sin(q_2) = \tau \end{cases} \qquad (19)$$

Thus, the TORA system has two degrees of freedom and underactuation degree one, with q_2 being the actuated coordinate and G being the matrix $G = [0 \quad 1]^T$. Hence, it can be easily verified that assumptions *H1, H2* and *H3* hold.

By defining the generalized inertia momenta and the system parameters, as $p = M\dot{q}$ and $c_1 = m_1 + m_2$, $c_2 = m_2 r$, $c_3 = m_2 r^2 + I$, the inertia matrix can be written as follows

$$M(q_2) = \begin{bmatrix} c_1 & c_2\cos(q_2) \\ c_2\cos(q_2) & c_3 \end{bmatrix} \qquad (20)$$

where it is assumed that the condition

$$c_1 c_3 - c_2^{\,2} > 0 \qquad (21)$$

hold. The total system energy is expressed as

$$H(p, q) = \frac{1}{2}p^T M^{-1}(q_2)p + V(q_1, q_2). \qquad (22)$$

Thus, the Hamiltonian form of the TORA system, with $G = \begin{bmatrix} 0 & 1 \end{bmatrix}^T \in \Re^{2x1}$, can be written as follows

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}\begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix}u. \qquad (23)$$

Substituting (20) and (22) in (23), the open loop model of the TORA system is obtained

$$\begin{cases} \dot{q}_1 = \dfrac{1}{\delta}\left[c_3 p_1 - c_2 p_2 \cos(q_2)\right] \\ \dot{q}_2 = \dfrac{1}{\delta}\left[c_1 p_2 - c_2 p_1 \cos(q_2)\right] \\ \dot{p}_1 = -K q_1 \\ \dot{p}_2 = \dfrac{c_2^{\,2} \sin(q_1)\cos(q_2)}{\delta^2} \\ \times\left[c_3 p_1^{\,2} - 2 c_2 p_1 p_2 \cos(q_2) + c_1 p_2^{\,2}\right] - \dfrac{c_2 p_1 p_2 \sin(q_2)}{\delta} + u \end{cases} \tag{24}$$

where $\delta = c_1 c_3 - c_2^{\,2} \cos^2(q_2)$ is the determinant of the matix M. Finally, the equilibrium points belong to the set

$$E = \left\{ \begin{array}{l} (q_1, q_2, p_1, p_2, u) \; / \; q_1 = 0, \\ p_1 = 0, \quad p_2 = 0, \quad u = 0 \;\; , q_2 \;\; arbitrary \end{array} \right\} \tag{25}$$

4. STABILIZATION OF THE TORA SYSTEM

4.1. *Energy Shaping*

By focusing on the solution of the PDE (14), the desired constant inertia matrix M_d can be chosen

$$M_d = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_3 \end{bmatrix}, \qquad a_1 > 0 \qquad a_1 a_3 - a_2^{\,2} > 0 \tag{26}$$

Thus, the equation (14) yields

$$\begin{bmatrix} \dfrac{a_1 c_3 - a_2 c_2 \cos(q_2)}{a_2 c_1 - a_1 c_2 \cos(q2)} \end{bmatrix} \nabla q_1 V_d + \nabla q_2 V_d = \begin{bmatrix} \dfrac{\delta}{a_2 c_1 - a_1 c_2 \cos(q2)} \end{bmatrix} K q_1 \tag{27}$$

Denoting

$$\frac{\gamma_1}{\gamma_2} = \frac{a_1 c_3 - a_2 c_2 \cos(q_2)}{a_2 c_1 - a_1 c_2 \cos(q2)} = \frac{b_3 + b_4 \cos(q2)}{b_1 + b_2 \cos(q_2)} \tag{28}$$

with

$$b_1 = a_2 c_1, \quad b_2 = -a_1 c_2, \quad b_3 = a_1 c_3, \qquad b_4 = -a_2 c_2. \tag{29}$$

It can be obtained

$$\frac{\gamma_1}{\gamma_2} = \frac{b_4}{b_2} \Leftrightarrow b_3 b_2 - b_1 b_4 = 0 \tag{30}$$

and, equivalently,

$$\frac{\gamma_1}{\gamma_2} = \frac{a_2}{a_1} \Leftrightarrow a_2 = \pm\sqrt{\frac{c_3}{c_1}} a_1 = \alpha \; a_1 \tag{31}$$

which is defined on the subset $(a_1, a_2, a_3) \in \Re^3$ satisfying (26), with $a_2 = \alpha \; a_1$. By taking into account that $\sqrt{c_1 c_3} - c_2 \cos(q_2) \neq 0$, it is finally obtained

$$\frac{\alpha}{q_q} \nabla q_1 V_d + \frac{1}{q_1} \nabla q_2 V_d = \frac{K}{a_1}\left[\sqrt{c_1 c_3} + c_2 \cos(q_2)\right]. \tag{32}$$

A solution of (32) has the form

$$\begin{aligned} V_d(q_1, q_2) = F(s) + \frac{K}{a_1}\sqrt{c_1 c_3}\,\frac{1}{2\alpha}\left[q_1^{\,2} - (q_1 - \alpha \; q_2)^2\right] \\ + \frac{K}{a_1} c_2 \alpha \cos(q_2) + \frac{K}{a_1} c_2 q_1 \sin(q_2) - \frac{K}{a_1} c_2 \alpha \end{aligned} \tag{33}$$

where $F(s)$ is an arbitrary function of the variable $s = q_1 - \alpha q_2$. To assign the equilibrium point at the origin of V_d, $F(s)$ can be defined as $F(s) = \frac{1}{2} R (q_1 - \alpha q_2)^2$, with R a design parameter.

The obtained function $V_d(q_1, q_2)$ satisfies $V_d(0,0) = 0$, and also

$$\nabla q V_d(q_1, q_2) = 0 \Leftrightarrow \quad q_1 = 0, \qquad q_2 = 0 \tag{34}$$

where

$$\nabla_q V_d = \begin{bmatrix} R(q_1 - \alpha \; q2) + \dfrac{K\sqrt{c_1 c_3}\, q_2}{a_1} \\ \left(\dfrac{K}{a_1} - R\right)\alpha(q_1 - \alpha \; q_2) + \dfrac{K c_2 \cos(q_2)}{a_1 \alpha} \end{bmatrix} \tag{35}$$

In order to have a minimum of $V_d(q_1, q_2)$ at the origin, it is necessary that the Hessian matrix of V_d at $(0,0)$, denoted by $Hess(V_d(0,0))$, be positive definite. Notice that $Hess(V_d(0,0))_{11} = R$, and

$$\begin{aligned} \det Hess(V_d(0,0)) = \frac{K}{a_1^{\,2}} \{ R a_1 \sqrt{c_1 c_3}\, \alpha + R a_1 c_2 \alpha \\ - K c_1 c_3 - 2K\sqrt{c_1 c_3}\, c_2 - K c_2^{\,2} \} \end{aligned} \tag{36}$$

therefore, $Hess(V_d(0,0))$ will be positive definite if the parameter R is chosen such that it satisfies the two conditions:

$$(i) \;\; R > 0, \qquad (ii) \;\; R > \frac{K}{a_1}\left[\sqrt{c_1 c_3} + c_2\right] \tag{37}$$

Finally, it should be noticed that the desired inertia matrix has the form

$$M_d = \begin{bmatrix} a_1 & \alpha\, a_1 \\ \alpha\, a_1 & a_3 \end{bmatrix}, \qquad a_1 > 0, \quad a_1 a_3 - \alpha^2 a_1{}^2 > 0 \quad (38)$$

Thus, by using (38) together with the potential energy (34), the desired total energy can be defined as

$$H_d(q,p) = \frac{1}{2} p^T M_d{}^{-1}(q) p + V_d(q) \quad (39)$$

B. Stability Analysis

The stability of the equilibrium for the TORA system can be established by applying the invariance LaSalle principle. To this end, let us define the residual set

$$\Omega = \left\{ (q_1, q_2, p_1, p_2) \in \Re^4 \,/\, \dot{H}_d(q_1(t), q_2(t), p_1(t), p_2(t)) = 0 \right\} \quad (40)$$

By differentiating H_d, it is obtained

$$\dot{H}_d(t) = -(\nabla p H_d)^T G K_v G^T \nabla p H_d = -K_v \left\| G^T \nabla p H_d \right\|^2 \quad (41)$$

with $\| \,.\, \|$ denoting the usual Euclidean norm. Then,

$$\begin{aligned} \Omega &= \left\{ (q_1, q_2, p_1, p_2) \in \Re^4 \,/\; G^T \nabla p H_d = 0 \right\} \\ &= \left\{ (q_1, q_2, p_1, p_2) \in \Re^4 \,/\; G^T M_d{}^{-1} p = 0 \right\}. \end{aligned} \quad (42)$$

From the time derivative of $G^T M_d{}^{-1} p = 0$, and the value of $\dot{p}$ in equation (6) it is obtained

$$G^T M_d{}^{-1} \left(-M_d M^{-1} \nabla q H_d \right) = -G^T M^{-1} \nabla q V_d = 0 . \quad (43)$$

Now, pre-multiplying by G^T the expression

$$M^{-1} \nabla q V_d = \frac{1}{\delta} \begin{bmatrix} c_3 \nabla q_1 V_d - c_2 \cos(q_2) \nabla q_2 V_d \\ -c_2 \cos(q_2) \nabla q_1 V_d + c_1 \nabla q_2 V_d \end{bmatrix} \quad (44)$$

it is obtained

$$\frac{1}{\delta} \left[-c_2 \cos(q_2) \nabla q_1 V_d + c_1 \nabla q_2 V_d \right] = 0 . \quad (45)$$

This equation has a numerable set of solutions given by

$$\left\{ \left(0, N\frac{\pi}{2} \right) \right\} \cup \left\{ \left(q_1{}^*, N\frac{\pi}{2} \right) \right\}, N = 0, \pm 1, \pm 3, \ldots, \; q_1{}^* \; const.$$

For all $N \neq 0$, $\cos\left(N\frac{\pi}{2} \right) = 0$ holds, and $q_1(t)$, $q_2(t)$ are constants, thus the equations in (6) are reduced to

$$\begin{cases} \dot{q}_1 = \dfrac{1}{\delta} \left[c_3 p_1 \right] = 0 \\ \dot{q}_2 = \dfrac{1}{\delta} \left[c_1 p_2 \right] = 0 \end{cases} \quad (46)$$

and

$$\begin{cases} \dot{p}_1 = \dfrac{1}{\delta} [-a_1 c_3] \left(\nabla q_1 V_d \right) + \dfrac{1}{\delta} [-a_2 c_1] \left(\nabla q_2 V_d \right) - K_v \dot{q}_1 = 0 \\ \dot{p}_2 = \dfrac{1}{\delta} [-a_2 c_3] \left(\nabla q_1 V_d \right) + \dfrac{1}{\delta} [-a_3 c_1] \left(\nabla q_2 V_d \right) - K_v \dot{q}_2 = 0 \end{cases} \quad (47)$$

It means that

$$p_1(t) \equiv 0, \;\; p_2(t) \equiv 0 \;\; \text{and} \;\; \dot{p}_1(t) \equiv 0, \;\; \dot{p}_2(t) \equiv 0 \quad (48)$$

The determinant of the associated matrix of the linear system (47) is

$$\Delta = \det \begin{bmatrix} -a_1 c_3 & -a_2 c_1 \\ -a_2 c_3 & -a_3 c_1 \end{bmatrix} = -(c_1 c_3)\left(a_1 a_3 - a_2{}^2 \right) \neq 0 \quad (49)$$

Thus, $\nabla q_1 V_d(q_1, q_2) = 0$, $\nabla q_2 V_d(q_1, q_2) = 0$ and, recalling (34), this implies that $q_1(t) = 0$, $q_2(t) = 0$. Considering this result together with (49), it is concluded that $\Omega = \{(0,0,0,0)\}$ and, by virtue of the LaSalle invariance principle, the origin is an asymptotically stable equilibrium point.

C. Damping Injection

According to (12) and considering that $J_2 = 0$, the control term u_{es} is given by

$$
\begin{aligned}
u_{es} = & -\frac{c_2^{\,2}\sin(q_2)\cos(q_2)}{\delta^2}\left[c_3 p_1 - 2c_2 p_1 p_2 \cos(q_2 + c_1 p_2^{\,2}\right] \\
& + \frac{c_2 p_1 p_2 \sin(q_2)}{\delta} - \frac{\left[a_2 c_3 - a_3 c_2 \cos(q_2)\right]}{\delta} \\
& \times \left[R\left(q_1 - \alpha q_2\right) + \frac{k\sqrt{c_1 c_3}\, q_2}{a_1} + \frac{k c_2 \sin(q_2)}{a_1} \right] \\
& - \frac{\left[a_3 c_1 - a_2 c_2 \cos(q_2)\right]}{\delta} \times \left[-R\left(q_1 - \alpha q_2\right)\alpha + \frac{k\sqrt{c_1 c_3}}{a_1}\left(q_1 - \alpha q_2\right) \right. \\
& \left. + \frac{k c_2 \alpha \sin q_2}{a_1} \right]
\end{aligned} \tag{50}
$$

Furthermore, the control term u_{di} is given by

$$
\begin{aligned}
u_{di} &= -K_v \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \nabla p_1 H_d \\ \nabla p_2 H_d \end{bmatrix} = -K_v \begin{bmatrix} 0 & 1 \end{bmatrix} M_d^{-1} p \\
&= -K_v \frac{1}{a_1 a_3 - a_2^{\,2}} \left[a_1 p_2 - a_2 p_1 \right]
\end{aligned} \tag{51}
$$

These results can be summarized in the following proposition.

Proposition 2: *The TORA system (24), with the parameters* c_1, c_2, c_3 *verifying the inequality (21), in closed loop with the IDA-PBC control law* $u = u_{es} + u_{di}$, *with* u_{es} *and* u_{di} *given by the expressions (50) and (51), has an asymptotically stable equilibrium point at the origin.*

5. DIGITAL SIMULATIONS

Digital simulations were carried out for the TORA system with parameters taken from Olfati-Saber, (2001), $c_1 = 12$, $c_2 = 1$, $c_3 = 11$, $K = 5$, and the design parameters where $R = 66$ and $K_v = 20$. A response of the controlled system is shown in Fig. 2.

6. CONCLUSIONS

In this work the stabilization problem for the TORA system has been solved by applying the IDA-PBC method. The design method take advantage of two properties of the TORA system: it has underactuation degree one and the inertia matrix depends only on the actuated coordinates. These conditions allowed us to choose a constant desired inertia matrix and pay attention on the potential energy shaping, and finding a solution for the arising PDE.

Digital simulations were carried out to evaluate the performance of the designed control law. The controlled system exhibited a good behavior with an acceptable settling time.

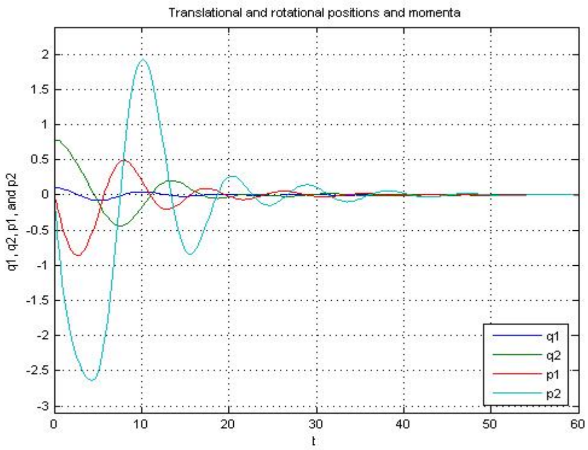

Fig. 2. Controlled response of the TORA system

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