Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Constrained Control of Event-Driven Networked Systems ⋆

Leonidas Dritsas and Anthony Tzes *

* *University of Patras*
Electrical & Computer Engineering Department
Rio 26500, Greece (e-mail: tzes@ece.upatras.gr)

Abstract: This article focuses on the control of Networked Systems in which the packets are time-stamped and suffer from long (more than one sampling period) transmission delays. The inner sample arrival of the packet, coupled with other constraints posed by the system's characteristics such as control and/or state saturation impedes the system's performance. A constrained finite time optimal controller is designed for this system that is robust against the inner sample delays. The presented simulation studies investigate on the performance of the suggested controller.

Keywords: Networked Control Systems, Uncertain dynamic systems, Robust Stability, Control over networks.

1. INTRODUCTION

It is well known that in a Networked Control System (NCS) the inclusion of the delays between the sensor and the controller (delay $\tau_{sc}(k)$) and between the controller and the actuator (delay $\tau_{ca}(k)$) can destabilize the closed–loop system.

In a typical closed loop NCS along with network–induced delays, the state vector x is sampled, transmitted through the network, fed to the controller which computes the control effort and transmits it to the plant. The plant receives this command after a certain delay. The network–induced delays have in general different characteristics depending on the utilized network protocol and scheduling methods used in the NCS, while their presence can deteriorate the performance of the controlled system, sometimes even driving it to instability (Zhang et al. [2001], Walsh et al. [2001], Lian et al. [2001], F.L. Lian [2001]).

The network–induced delays in general can be categorized based on their characteristics as: a) Constant and Exactly known (e.g deterministic scheduling, intentional delay buffers), b) Constant and Unknown, with known bounds ($\tau_{\min} < \tau < \tau_{\max}$), c) Time–Varying and Exactly known ("Time–stamps" included in the data), d) Time–Varying (Uncertain) with known bounds ($\tau_{\min} < \tau(t) < \tau_{\max}$), and e) Time–Varying (stochastic process).

The usual approach for NCS design consists of the following steps: (i) design a controller ignoring the network, and (ii) analyze stability, performance and robustness with respect to the effects and the presented characteristics of network–delays and scheduling policies. The second step usually results in the selection of an appropriate scheduling protocol as well as setting bounds, so–called Maximum–Allowable–Transfer–Interval (or "M.A.T.I") on the transmission rate so that the desired properties of the network–free control system are preserved (G. Walsh and H. Ye and L. Bushnell [2002]).

If the delay τ^k is measured (assuming "time–stamps" in the packets arriving from the sensor to the controller) then appropriate compensation techniques can be applied (Nilsson et al. [1998]).

This research effort focuses on the case–(d), of bounded, varying but unknown time controller to actuator delays ($(d-1)h \leq \tau^k < dh$, where h is the sampling period, and d fixed and known integer) and instead of following the usual approach (design a controller ignoring the network and then analyze stability and performance), it takes into account the network delays in the controller design process.

In this work the case of SISO NCS is covered and the results of a "delay less than one sampling period delay" ($\tau^k < h$), are extended. This effort has been sparked by the recent work in the area of NCS concerning the M.A.T.I (Kim et al. [2003]), which has revived the interest even for this seemingly limiting case. This is due to the fact that for NCSs using Random (Ethernet–Type) Access Networks, M.A.T.I is actually the effective sampling–period and hence large values on M.A.T.I allow the employment of a slower (larger) sampling period which has also the beneficial effect of reducing network traffic.

In the best of authors' knowledge the issue of input constraints is not covered adequately in the NCS literature, although the incorporation of constraints would make the system modelling more realistic since it takes into account the inevitable limitations present in any actuator. Recently there are some relevant results from the research into constrained Time Delayed Systems (TDS). In reference (Fridman et al. [2003]) the case of time–delayed systems with saturating actuators is examined using the Lyapunov-Krasovskii functional and the descriptor

⋆ This work was partially funded by EU's FP6 Network of Excellence HYCON, contract number FP6-IST-511368

approach to the control of time–delay systems (Fridman and Shaked [2002]) is employed to design linear parameter varying controllers.

In the rest of the paper the modelling of the network induced time–delays will be presented in Section 2, while the development of the CFTO–controller will be presented in Section 3. The simulation results that prove the efficacy of the proposed scheme will be presented in Section 4. and finally the conclusion remarks are provided in Section 5.

2. NCS MODELLING

The dynamics of the NCS under investigation is described by the combination of a continuous–time linear time–invariant plant with a discrete–time controller (Zhang et al. [2001], Dritsas and Tzes [2007]).

It should be noted that in the ensuing sections, the sampling period, h, is assumed to be constant and known, whereas both controller and actuator are event-driven, in the sense that their outputs are updated as soon as a new sample is received.

This effort is concerned with modeling and control issues for the case where the primary source of uncertainty is the transmission delay. Networks such as Controller Area Network (CAN) and Ethernet fall into this category. A different situation happens when computation and transmission delays are negligible and access delays serve as the main source of delays in NCS. It has been shown that for this case, the NCS can be modelled as a discrete-time switched system for which corresponding control synthesis techniques have recently appeared (H. Lin and P.Antsaklis [2004], H. Lin and G. Zhai and L. Fang and P. Antsaklis [2005], Lin et al. [2006]).

Inhere, the case of SISO systems with "long delays constrained within one sampling period", $((d-1)h \leq \tau^k < dh)$ is examined, where d is assumed to be constant and known a priori. It should be noted that this case corresponds to the "time-stamped" packet configuration where only one packet is recorded at each sampling period. In contrast to the generic case where $\tau^k < dh$ where control commands may arrive in a batch mode over a single period (Zhang et al. [2001], B. Lincoln and B. Bernhardsson [2000], Hu Shousong and Zhu Qixin [2003]), the presented examined case has significant simplicity. Its simplicity is dictated by the timeout periods of the networking hardware, which when tuned to expire over a single period it does not allow the fetching of more than one packet per period.

For NCS using random access MAC protocols (Ethernet, DeviceNet) the assumption of equidistant sampling and constant network delay may no longer be valid (see Naghshtabrizi and Hespanha [2006], Hespanha et al. [2007] for the variable sampling case). Hence a more cautious treatment of the modeling and discretization procedure is necessary, and even more so for the control synthesis.

The control architecture and the timing diagram for this case is shown in Figures 1 and 2, respectively, where a remote controller, non-collocated with the sensor and actuator is employed. This single-channel feedback NCS captures several important aspects of NCS dynamics and has been extensively used to investigate the effects of sampling and delay variations (uncertainties) in the stability of NCSs. The dynamics of the plant in this case can be

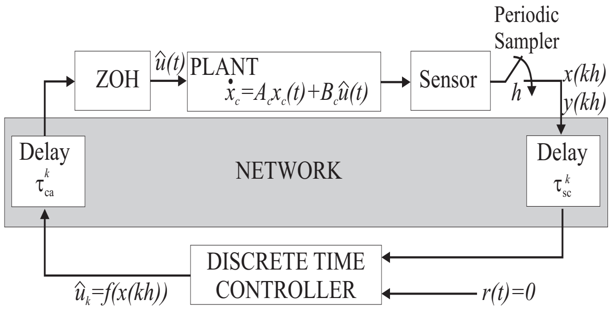

Fig. 1. NCS with uncertain time delays in the actuation and sensor path

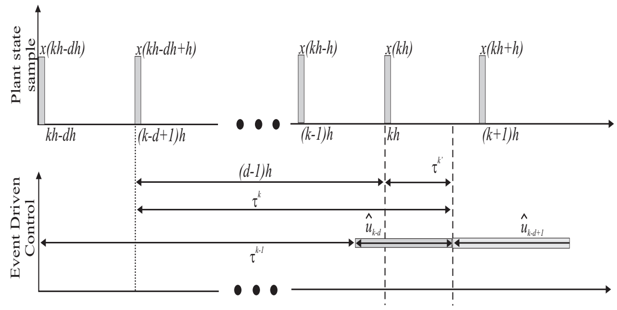

Fig. 2. NCS timing diagram $(d-1)h \leq \tau^k < dh$

cast in the following formulation:

$$\dot{x}(t) = A_c x(t) + B_c \hat{u}(t), \quad y(t) = C_c x(t)$$
$$t \in \left[kh + \tau^k, kh + h + \tau^{k+1}\right)$$
$$\hat{u}(t) = \begin{cases} \hat{u}_{k-d}, & t \in \left[kh - h + \tau^{k-1},\ kh + \tau^k\right) \\ \hat{u}_{k-d+1}, & t \in \left[kh + \tau^k,\ kh + h + \tau^{k+1}\right) \end{cases} \tag{1}$$
$$u_{\min} \leq \hat{u}(t) \leq u_{\max}, \quad x_{\min} \leq x(t) \leq x_{\max}, \tag{2}$$

where $x(t) \in R^n$ and the notation $x(t) \leq x_{\max}$ is an element-by-element inequality for the x-vector. For notation simplification reasons the notation $\{x_{k+1},\ x_k, \ldots\}$ will be used hereafter in order to denote the values $\{x(kh + h), x(kh), ...\}$ of the discrete–time signal coming out of the periodic sampler.

The state or output is sampled at time instance kh and presented to the event–driven remote controller for control computation purposes. The control–action is computed immediately after the reception of the sample $x(kh)$ and is transmitted via the network to the Zero–Order–Hold device and finally presented to the event–driven actuator, after a delay τ^k. Essentially, the control command computation delay and the network transmission delay are absorbed into τ^k. This inherent delay τ^k (apparent from the timing diagram in Figure 2), is in general a time–varying and uncertain quantity, reflecting the nature of the network involved, the network load, etc. (Zhang et al. [2001], Lian et al. [2001]).

In (1) $\hat{u}(t)$ is the "most recent" control action presented to the event–driven actuator at the time instance t within a sampling period (i.e. within the time interval

$[kh,\ kh+h))$, and can take either one of the two values $\hat{u}_{k-d}$ or $\hat{u}_{k-d+1}$.

It must be emphasized that the discrete–time piecewise constant control action $\hat{u}(t)$ experiences a "jump" at the uncertain time instance $kh+\tau^k$. At this time instance $(kh+\tau^k)$, the control action coming out of the event–driven ZOH device, is updated from value $\hat{u}_{k-d}$ into $\hat{u}_{k-d+1}$. Hence (unless τ^k is constant) it is not in general possible to treat the ensuing NCS in a standard "sampled-data" or "time–delayed" setting (Aström and Wittenmark [1997]). Instead a "hybrid" setup should rather be used (Branicky et al. [1998], Hassibi [2000], Naghshtabrizi et al. [2006, 2007]). Initial efforts towards this objective have been proposed and successfully used specifically for NCS in (Naghshtabrizi et al. [2006], Naghshtabrizi and Hespanha [2006], Hespanha et al. [2007], Dritsas et al. [2007b], Dritsas and Tzes [2007]).

Despite the "jump" nature of $\hat{u}(t)$, the discretization of (1) within a sampling period is straightforward, following (Aström and Wittenmark [1997]) for the discretization of input–delayed systems, and using the piecewise-constant control actions $\hat{u}_{k-d+1}$ and $\hat{u}_{k-d}$. The ensuing discretization is exact in the sense that it correctly describes the evolution of the state vector at the discrete time instances, and is given by Zhang et al. [2001], Dritsas and Tzes [2007]:

$$x(k+1)=\Phi x(k)+\Gamma_0(\tau^{k'})\hat{u}_{k-d+1}+\Gamma_1(\tau^{k'})\hat{u}_{k-d} \quad (3)$$

where

$$\Phi=\exp(A_c h),$$
$$\Gamma_0(\tau^{k'})=\int_0^{h-\tau^{k'}}\exp(A_c\lambda)B_c d\lambda,$$
$$\Gamma_1(\tau^{k'})=\int_{h-\tau^{k'}}^{h}\exp(A_c\lambda)B_c d\lambda. \quad (4)$$

$\Gamma_0(\tau^{k'})$ can be computed from (4) via the following identity for integrals of matrix exponentials (C. van Loan [1978])

$$\Gamma_0(\tau^{k'})=\left[\, I_n\ \bar{0}^T \,\right]\exp\left(\begin{bmatrix} A_c & B_c\\ \bar{0} & 0\end{bmatrix}(h-\tau^{k'})\right)\begin{bmatrix}\bar{0}^T\\ 1\end{bmatrix} \quad (5)$$

where $\bar{0}\triangleq 0_{1\times n}$ is an n-column zero row-vector and I_n is the $n\times n$ identity matrix (n being the dimension of the state vector). $\Gamma_1(\tau^{k'})$ can be computed by noticing that

$$\Gamma_1(\tau^{k'})=-\Gamma_0(\tau^{k'})+\int_0^h\exp(A_c\lambda)B_c d\lambda. \quad (6)$$

2.1 Inner Sample Uncertain Term Description

Motivated by the arbitrarily varying (but bounded) and uncertain nature of the delay τ^k, and following procedures and arguments similar to the ones described in (Tipsuwan and Chow [2003], Wang et al. [1994]), the system's nominal model and the corresponding control synthesis is intentionally based on the choice of the average delay $\tau^\circ=(\tau_{\min}+\tau_{\max})/2$ as the nominal value of the uncertain delay. The actual uncertain delay can be modelled (decomposed) as

$$\tau^{k'}=\tau^\circ+(\frac{\tau_{\max}-\tau_{\min}}{2})\delta\ , \quad (7)$$

where $\mid\delta\mid\leq 1$. Based on the above decomposition (7) of the uncertain–delay into a constant–known part and an uncertain one, the matrices $\Gamma_0(\tau^{k'})$, $\Gamma_1(\tau^{k'})$ can be accordingly decomposed into constant and known nominal parts $\Gamma_0(\tau^\circ)$, $\Gamma_1(\tau^\circ)$ and uncertain (though bounded) parts $\Delta\Gamma_0(\tau^{k'})$, $\Delta\Gamma_0(\tau^{k'})$. $\Gamma_0(\tau^{k'})$ in (4) can be decomposed as:

$$\Gamma_0(\tau^{k'})=\int_0^{h-\tau^\circ}\exp(A_c\lambda)B_c d\lambda+\int_{h-\tau^\circ}^{h-\tau^{k'}}\exp(A_c\lambda)B_c d\lambda$$
$$\triangleq\Gamma_0(\tau^\circ)+\Delta\Gamma_0(\tau^{k'})\ . \quad (8)$$

Note that the expression for $\Gamma_0(\tau^\circ)$ is amenable to computation via identity (5). Alternative expressions for $\Delta\Gamma_0(\tau^{k'})$, amenable to computation via (5), can be taken by changing again the integration variable into $\lambda_2\triangleq\lambda-h+\tau^\circ$ in order to alternatively get (Dritsas et al. [2006a,b, 2007a])

$$\Delta\Gamma_0(\tau^{k'})=\exp(A_c(h-\tau^\circ))\int_0^{\tau^\circ-\tau^{k'}}\exp(A_c\lambda_2)B_c d\lambda_2. \quad (9)$$

Similarly $\Gamma_1(\tau^{k'})$ in (4) can be decomposed as

$$\Gamma_1(\tau^{k'})=\int_{h-\tau^{k'}}^{h-\tau^\circ}\exp(A_c\lambda)B_c d\lambda+\left[\int_0^h\exp(A_c\lambda)B_c d\lambda-\Gamma_0(\tau^\circ)\right]$$
$$\triangleq\Delta\Gamma_1(\tau^{k'})+\Gamma_1(\tau^\circ)=-\Delta\Gamma_0(\tau^{k'})+\Gamma_1(\tau^\circ)\ . \quad (10)$$

2.2 System State Augmentation

The state vector has to be further augmented with d extra variables to include all the delayed input terms (the dimension of the input signal u being 1 for the SISO case studied). The system's state space description in terms of the augmented state vector is now given by:

$$\begin{bmatrix} x_{k+1}\\ \hat{u}_{k-d+1}\\ \vdots\\ \hat{u}_{k-1}\\ \hat{u}_k\end{bmatrix}=\begin{bmatrix}\Phi & \Gamma_1(\tau^{k'}) & \Gamma_0(\tau^{k'}) & \cdots & 0\\ \bar{0} & 0 & 1 & \cdots & 0\\ \vdots & \vdots & & \ddots & \\ \bar{0} & 0 & 0 & \cdots & 1\\ \bar{0} & 0 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix} x_k\\ \hat{u}_{k-d}\\ \vdots\\ \hat{u}_{k-2}\\ \hat{u}_{k-1}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \vdots\\ 0\\ 1\end{bmatrix}\hat{u}_k$$
$$\triangleq A_d z_k+B_d\hat{u}_k \quad (11)$$

where $z_k=[x_k,\hat{u}_{k-d},\ldots,\hat{u}_{k-1}]^T$. The matrix A_d can be decomposed as

$$A_d=\begin{bmatrix}\Phi & \Gamma_1(\tau^\circ) & \Gamma_0(\tau^\circ) & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & & \ddots & \\ 0 & 0 & 0 & \cdots & 1\\ 0 & 0 & 0 & \cdots & 0\end{bmatrix}$$
$$+\begin{bmatrix}0_n & \Delta\Gamma_1(\tau^{k'}) & \Delta\Gamma_0(\tau^{k'}) & \cdots & 0\\ 0 & 0 & 0 & \cdots & 0\\ \vdots & \vdots & & \ddots & \\ 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & \cdots & 0\end{bmatrix}$$
$$=A_d^\circ+\Delta A_d(\tau^{k'})$$

The matrix $\Delta A_d(\tau^{k'})$ can then be expressed as follows

$$\Delta A_d(\tau^{k'}) = \left[\frac{I_n}{0_{(n+d)\times n}}\right] \Delta\Gamma_1(\tau^{k'}) \left[\bar{0}|1|-1|0\ \dots\ 0\right] \quad (12)$$

Let the ith element ($\Delta\Gamma_{1,i},\ i = 1, \dots, n$) of the $\Delta\Gamma_1(\tau^{k'})$ vector be bounded as

$$\Delta\Gamma^-_{1,i} \leq \Delta\Gamma_{1,i}(\tau^{k'}) \leq \Delta\Gamma^+_{1,i},\ \forall\tau^{k'} \in [0, h)\ . \quad (13)$$

Let the set $\widetilde{\Delta\Gamma_1}$ with its 2^n members defined by the binary enumeration of the aforementioned extremum values of the $\Delta\Gamma_{1,i}$ elements, or

$$\widetilde{\Delta\Gamma_1} \in \left\{ \begin{bmatrix} \Delta\Gamma^-_{1,1} \\ \Delta\Gamma^-_{1,2} \\ \vdots \\ \Delta\Gamma^-_{1,n} \end{bmatrix}, \begin{bmatrix} \Delta\Gamma^+_{1,1} \\ \Delta\Gamma^-_{1,2} \\ \vdots \\ \Delta\Gamma^-_{1,n} \end{bmatrix}, \begin{bmatrix} \Delta\Gamma^-_{1,1} \\ \Delta\Gamma^+_{1,2} \\ \vdots \\ \Delta\Gamma^-_{1,n} \end{bmatrix}, \dots, \begin{bmatrix} \Delta\Gamma^+_{1,1} \\ \Delta\Gamma^+_{1,2} \\ \vdots \\ \Delta\Gamma^+_{1,n} \end{bmatrix} \right\},$$

then the $\Delta\Gamma_1(\tau^{k'})$ vector belongs to the convex hull of the orthotopic set defined by the vertices of $\widetilde{\Delta\Gamma_1}$, or

$$\Delta\Gamma_1(\tau^{k'}) \in \text{con}\left\{\widetilde{\Delta\Gamma_1}\right\} \quad (14)$$

Subsequently,

$$\Delta A_d(\tau^{k'}) \in \left[\frac{I_n}{0_{(n+d)\times n}}\right] \text{con}\left\{\widetilde{\Delta\Gamma_1}\right\} \left[\bar{0}|1|-1|0\ \dots\ 0\right]$$

and

$$A_d(\tau^{k'}) \in A^\circ_d + \left[\frac{I_n}{0_{(n+d)\times n}}\right] \text{con}\left\{\widetilde{\Delta\Gamma_1}\right\} \left[\bar{0}|1|-1|0\ \dots\ 0\right]\ . \quad (15)$$

Essentially the uncertainty about the $\tau^{k'}$ transforms the NCS–description to a polytopic (orhtotopic) uncertainty, where $A_d(\tau^{k'})$ in the orthotope defined in (15). The case of $d = 1$ has been examined in (Dritsas et al. [2006a,b], Tzes et al. [2005]), while the case where $\tau^{k'} = 0$ and d switches between the members of the set $d \in \{1, \dots, D\}$ where D is known a priori has been examined in (Dritsas et al. [2007a]).

3. CFTO CONTROLLER SYNTHESIS

Consider the aforementioned NCS described in equation (11) written in a compact form as

$$z_{k+1} = A_d z_k + B_d \hat{u}_k\ , \quad (16)$$

where A_d is in the aforementioned orthotope. Rather than focusing on the simple problem of state and input saturation constraints defined in (2) the more generic case of the so–called “guard functions” is examined, where

$$\begin{bmatrix} z_k \\ \hat{u}_k \end{bmatrix} \in \mathcal{P} = \{H_i z_k + J_i \hat{u}_k \leq K_i, i = 1, \dots, C\}\ . \quad (17)$$

The controller synthesis procedure for the constrained uncertain polytopic system, can be handled using the Constrained Finite Time Optimal Control–CFTOC machinery (Bemporad et al. [2003], Kvasnica et al. [2004]). The CFTOC (“multi–parametric”) approach consists in computing the optimizer vector $U_N = \left\{\hat{u}'_0, \dots, \hat{u}'_{N-1}\right\}'$ with N the prediction horizon, which minimizes the following cost function:

$$J_N(z_0) = \min_{U_N}\left[||Pz_N||_l + \sum_{i=0}^{N-1}(\|R\hat{u}_k\|_l + \|Qz_k\|_l)\right] \quad (18)$$

subject to the linear system dynamics and state/input constraints defined before. The cost in (18) may be linear (e.g., $l \in \{1, \infty\}$) or quadratic (e.g., $l = 2$) depending on the vector norm employed. The initial condition z_0 is the currently available sample of the state vector, while z_k with $k = 0, \dots, N-1$, are the predicted values of the state vector through equation (16) starting from $z_0 = z(0)$ and applying the input sequence U_N. Moreover N is the prediction horizon, Q, R and P are the full column rank weighting matrices on the corresponding optimization variables i.e predicted states, control effort and the desired final state, respectively. The predicted final state z_N is usually supposed to belong to a predefined set Z_{set} a choice typically dictated by stability and feasibility requirements especially when CFTOC is implemented in a Receding Horizon fashion.

For a given initial state z_0, the problem described in equations (16, 17, 18) can be solved as an Linear (LP) or Quadratic (QP) Program for linear or quadratic cost objectives respectively. It is well known (Borrelli [2002], Bemporad et al. [2003], Grieder et al. [2004], Kvasnica et al. [2004]) that the “multi-parametric” CFTOC optimizer is a continuous piecewise affine state feedback of the following form:

$$\hat{u}_k = F_j z_k + G_j,\ \text{ if }\ z_k \in R_j \quad (19)$$

defined over convex polyhedra R_j–henceforth referred to as “regions”– which are also generated by the CFTOC–algorithm. The algorithm additionally provides the feasibility set $Z_f \subseteq \Re^{n+d}$ which is the set of all initial states z_0 for which the CFTOC problem is feasible, i.e. $Z_f = \{z_0 \in \Re^{n+d} | \exists(\hat{u}_0, ..., \hat{u}_{N-1}) \in \Re^N, z_k \in Z, u_{k-1} \in U, \forall k \in \{1, ..., N\}$.

Notice that even though the computations of the multi–parametric (CFTOC) control law are carried out off–line, they quickly become prohibitive for larger problems. This is not only due to the high complexity of the multi–parametric programs involved, but mainly because of the exponential number of transitions between regions which can occur when a controller is computed in a dynamic programming fashion (Borrelli et al. [2003]). Thus the number of the controller's regions is not only a measure of the controller's complexity but also affects directly its on–line implementation in the form of a look–up table.

4. SIMULATION STUDIES

Consider the following open loop unstable continuous–time system $G_s = \frac{1}{s-1}$, where the following constraints on the input and state are $u_{\min} = -1000 \leq u(t) \leq +1000 = u_{\max}$ and $x_{\min} = -10 \leq x(t) \leq +10 = x_{\max}$ respectively. The sampling period is $h = 0.05$, while the uncertain delay was allowed to vary between h and $2h$, or ($d = 2$). The nominal system for $\tau^\circ = \frac{h}{2}$ is

$$\begin{bmatrix} x_{k+1} \\ \hat{u}_{k-1} \\ \hat{u}_k \end{bmatrix} = \begin{bmatrix} 1.0513 & 0.026 & 0.0253 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \hat{u}_{k-2} \\ \hat{u}_{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \hat{u}_k$$

The variation of the scalar quantity $\Delta\Gamma_1(\tau^{k'})$ w.r.t. $\tau^{k'} \in [0, h)$ was numerically investigated, providing the extremum values $\Delta\Gamma^-_1 = -0.026$ and $\Delta\Gamma^+_1 = 0.0253$.

Based on the aforementioned values

$$\Delta A_d(\tau^{k'}) \in \text{con}\left\{ \begin{bmatrix} 0 & 0.0253 & 0.026 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -0.026 & 0.0253 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

The tuning parameters of the CFTOC for the regulation problem were chosen as $Q = 10^3 I_3,\ P = 10^6 I_3, R = 1$ while the initial (augmented) state was $z_0 = [3, 0, 0]^T$. The control signal was applied to the continuous time system in equations (1) via a clocked–driven ZOH.

The resulting controller partitioning for a prediction horizon $N = 3$ and 2–vector–norm is presented in Figure 3. The output response of the continuous system, when this regulation control law is applied, along with the control effort, is displayed in Figure 4. Similar results can be

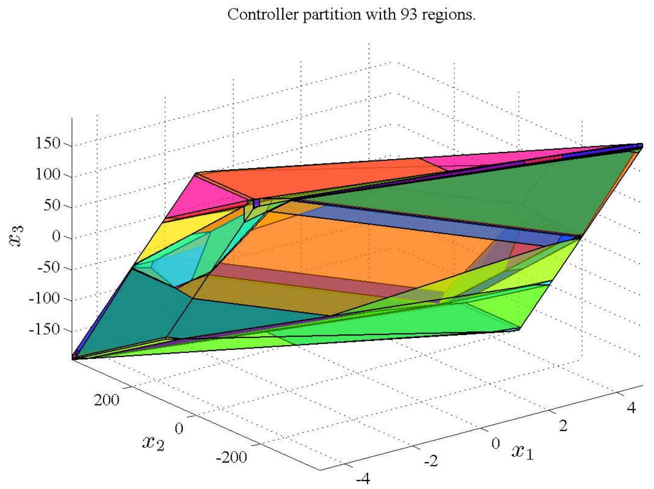

Fig. 3. Controller Partition with 2-norm and N=3

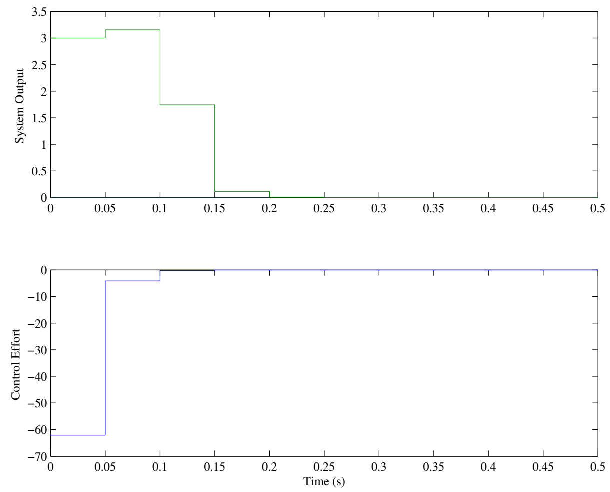

Fig. 4. Evolution of the sampled–system's output & control effort (2–norm, $N = 3$)

traced in Figures 5 and 6 for the case of a larger prediction horizon $N = 4$. The polytopic controller partitioning for a prediction horizon $N = 3$ (4) resulted in 93 (191) regions. Despite the smaller extremum control requirements in the case with a larger horizon, there is no clear improvement on the system's performance despite the increase in the controller'c complexity.

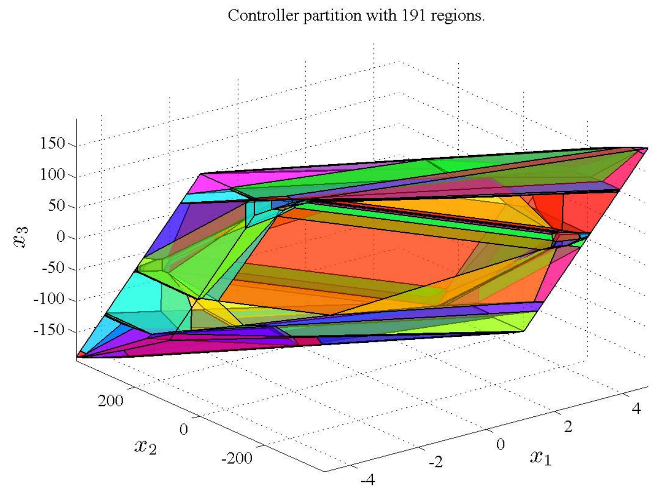

Fig. 5. Controller Partition with 2-norm and N=4

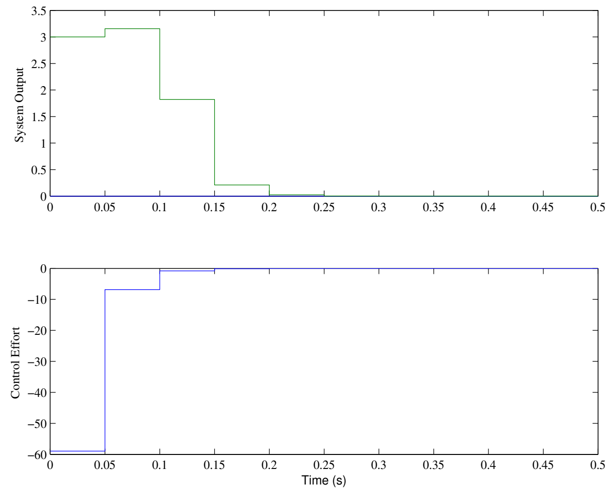

Fig. 6. Evolution of the sampled–system's output & control effort (2–norm, $N = 4$)

5. CONCLUSIONS

In this article a constrained optimal controller for networked systems with varying latency times that are restricted within one known sampling period has been presented. This approach allows the embedding of both the delay variation and the input/state constraints in the controller synthesis procedure for this class of NCS. It is proven that the discretized version of the system under study is a polytopic system. The proposed CFTO–controller has been applied to a linear NCS and multiple simulation results have been presented proving the efficacy of the proposed control scheme.

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