# A simple time-varying observer for speed estimation of UAV 

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#### Abstract

This note deals with the speed estimation of Unmanned Aerial Vehicle (UAV) using linear acceleration measurements. The estimator is a useful time-varying reduced-order Luemberger like observer such that the observation error is reduced to a time varying linear differential equation. Asymptotic stability of the estimation error is proved using the Lyapunov approach and the Barbalat lemma. Moreover, we generalize the proposed approach to systems with partial accelerations measurements. Conditions for the existence of the observer, which are less restrictive than those given in the literature, are given. A numerical simulation on a Quad-rotor UAV is performed to illustrate the effectiveness of the approach.


Keywords: Time-varying observer, Unmanned Aerial Vehicle, speed estimation, stability analysis, quadrotor

## 1. INTRODUCTION

UAV and more particularly mini-UAV must be as light as possible for technological reasons but also for low cost and low energy consumption. In fact, the weight of the drone can reduce its performances considerably. It is therefore rather natural to use a state estimator to replace possibly one or several sensors. Observers are usually used to estimate the non measured state components for output feedback control or to generate residual signals for fault diagnosis. During the last decade, tremendous research activities focused on structural, modelling and control design for UAV (see for example Castillo et al. [2005], Alabazares [2007], Lee et al. [2007]). However, very few results were established for the state estimation. Among the recent works on this subject, we mention the work of Lee et al. [2007], the authors propose a velocities estimator for a tracking control of an under-actuated Quad-rotor UAV using only linear and angular positions. In Madani and Benallegue [2007] and Benallegue et al. [2008], sliding-mode observers based control are proposed to estimate the effect of external perturbations using measurement of positions and yaw angle. Observer-based control for visual servo control of UAV has been proposed for example in LeBras et al. [2007] using image provided by a camera for the estimation of the velocity. For the use of observers in faults diagnosis, we can mention for exemple Sharma and Aldeen [2007], in which two cascaded sliding mode observers are proposed first to estimate the disturbances (effect of wind) and second to reconstruct actuators faults. In Bateman et al. [2007], a linear input decoupled functional observer is proposed to estimate actuators failures of UAV. The proposed method is based
on the linearization of the model around an operating point. A variable structure observer is performed in Slegers and Costello [2007] for actuator bias reconstruction of UAV without using a linearization. Extended Kalman Filter is used in Prevost et al. [2007] to estimate the state of a moving object detected by a UAV. The disadvantage of this method is its only local convergence. All the results cited above are obtained using linear and angular positions.
In Benzemrane et al. [2007], the authors propose a nice technique to estimate the speed of a Quad-rotor UAV from acceleration measurements, provided by Inertial Measurement Units. The approach is based on an adaptive observer technique using cascade nonlinear filters that lead, unfortunately, to a high order observer and therefore, the computational requirements increase considerably for on-line control purpose. More precisely, the observer is composed with two matrix differential equations in cascade with two nonlinear filters.
Motivated by this interesting work of Benzemrane et al. [2007], we propose here a useful and alternative approach for the speed estimation through a straightforward reduced-order time-varying observer. Indeed, one of the main contributions is that the state observers order is equal to the dimension of the state vector; this has the advantage to be implemented in real time applications. Furthermore, an extension to only partial acceleration measurements was established. In both cases we provide asymptotic stability conditions that are reduced to simple and checkable one. In the last section, numerical examples, describing a Quad-rotor, show the good performances of the proposed approach.

## 2. UAV MODELLING

Several works deal with the quadrotor modelling (see for example Alabazares [2007] and Castillo et al. [2005]). In this section we recall a sketch of modelling as well as the notations are introduced. In order to model the four-rotor rotorcraft dynamics two frames are defined i.e. $\mathcal{R}_{i}\left(O, \vec{E}_{1}, \vec{E}_{2}, \vec{E}_{3}\right)$ is an inertial frame attached to the earth and $\mathcal{R}_{b}\left(G, \vec{e}_{1}, \vec{e}_{2}, \vec{e}_{3}\right)$ is a body fixed frame attached to its center of mass.


Fig. 1. Quadrotor model

The UAV model can be deduced from the rotation dynamic Newton-Euler law (1) and the translation dynamic Newton-Euler law (2)

$$
\begin{gather*}
\left(\frac{\mathrm{d} \mathbf{I} \vec{\omega}}{\mathrm{~d} t}\right)_{\mathcal{R}_{b}}+\vec{\omega} \wedge(\mathbf{I} \vec{\omega})=\overrightarrow{\mathcal{M}}  \tag{1}\\
m\left(\frac{\mathrm{~d} \vec{v}_{G}}{\mathrm{~d} t}\right)_{\mathcal{R}_{b}}+\vec{\omega} \wedge\left(m \vec{v}_{G}\right)=\vec{F} \tag{2}
\end{gather*}
$$

where $\mathbf{I}$ is the inertia matrix and $\vec{\omega}=(p, q, r)^{\top}$ is the angular velocity both expressed in the body fixed frame. $\overrightarrow{\mathcal{M}}$ represents the torque derived from the differential rotors thrusts, $m$ is the vehicle mass, $\vec{v}_{G}=(u, v, w)^{\top}$ is the center of mass velocity expressed in the body fixed frame and $\vec{F}$ is the sum of the four rotor thrust $\vec{T}=-T \vec{e}_{3}$ and the weight $\vec{P}=m g \vec{E}_{3}$. In order to express the weight with respect to the body fixed frame, the attitude matrix $R$ must be used. By means of Euler angles $\Phi$ (roll), $\theta$ (pitch), $\Psi$ (yaw), the attitude matrix can be written as

$$
R=\left(\begin{array}{ccc}
c_{\theta} c_{\Psi} & s_{\Phi} s_{\theta} c_{\Psi}-c_{\Phi} s_{\Psi} & c_{\Phi} s_{\theta} c_{\Psi}+s_{\Phi} s_{\Psi}  \tag{3}\\
c_{\theta} s_{\Psi} & s_{\Phi} s_{\theta} s_{\Psi}+c_{\Phi} c_{\Psi} & c_{\Phi} s_{\theta} s_{\Psi}-s_{\Phi} c_{\Psi} \\
-s_{\theta} & s_{\Phi} c_{\theta} & c_{\Phi} c_{\theta}
\end{array}\right)
$$

where $c .=\cos ($.$) and s .=\sin ($.$) .$
From the rotation dynamic Newton-Euler law (1) the dynamics of the angular velocity is given by

$$
\left\{\begin{array}{l}
\dot{p}=-\frac{I_{z z}-I_{y y}}{I_{x x}} q r+\frac{\tau_{\Phi}}{I_{x x}}  \tag{4}\\
\dot{q}=-\frac{I_{x x}-I_{z z}}{I_{y y}} p r+\frac{\tau_{\theta}}{I_{y y}} \\
\dot{r}=-\frac{I_{y y}-I_{x x}}{I_{z z}} p q+\frac{\tau_{\psi}}{I_{z z}}
\end{array}\right.
$$

where $I_{x x}, I_{y y}$ and $I_{z z}$ are the inertia matrix terms expressed in the principal inertia axis, $\tau_{\Phi}, \tau_{\theta}$ and $\tau_{\psi}$ represent the control torques due to the differential rotors thrusts. Using the center of mass dynamics equation (2) and the attitude matrix (3), the translation dynamics with respect to the body frame is

$$
\left\{\begin{array}{l}
\dot{u}=-q w+r v-g \sin \theta  \tag{5}\\
\dot{v}=-r u+p w+g \sin \Phi \cos \theta \\
\dot{w}=-p v+q u+g \cos \Phi \cos \theta-\frac{T}{m}
\end{array}\right.
$$

where $T$ is the total thrust which represents a control input. The time derivative of the Euler angles are related to the angular velocity $\vec{\omega}$ by the following relation

$$
\left\{\begin{array}{l}
\dot{\Phi}=p+q \tan \theta \sin \Phi+r \tan \theta \cos \Phi  \tag{6}\\
\dot{\theta}=q \cos \Phi-r \sin \Phi \\
\dot{\psi}=q \frac{\sin \Phi}{\cos \theta}+r \frac{\cos \Phi}{\cos \theta}
\end{array}\right.
$$

The dynamic model of the four-rotor rotorcraft is then given by equations (4), (5) and (6).

## 3. PROBLEM STATEMENT AND OBSERVERS DESIGN

Since the UAV is equiped only with an Inertial Measurement Unit it is assumed that the measured variables are the Euler angles $\eta=(\Phi, \theta, \psi)^{\top}$, the angular velocity $\vec{\omega}$ and the acceleration of the center of mass $(\dot{u}, \dot{v}, \dot{w})^{\top}$ given by the sensors embedded in the four-rotor rotorcraft. The states of equations (4) and (6) are then measured.

### 3.1 Observer design with three measured accelerations

To estimate the angular velocity $x=(u, v, w)^{T}$ we consider the model of system (5). Taking into account the measured variables, the system (5) can be rewritten as:

$$
\begin{align*}
& \dot{x}(t)=A(t) x(t)+b(t)  \tag{7}\\
& y(t)=\dot{x}(t)
\end{align*}
$$

with

$$
A(t)=\left(\begin{array}{ccc}
0 & r & -q \\
-r & 0 & p \\
q & -p & 0
\end{array}\right)
$$

and

$$
b(t)=\left(\begin{array}{c}
-g \sin \theta \\
g \sin \Phi \cos \theta \\
g \cos \Phi \cos \theta-\frac{T}{m}
\end{array}\right)
$$

We consider the standard state observer form

$$
\begin{equation*}
\dot{\hat{x}}(t)=N(t) \hat{x}(t)+M(t) b(t)+K(t) y(t) \tag{8}
\end{equation*}
$$

where the time varying matrices $N(t), M(t), K(t)$ will be defined later.

Lemma 1. If the time varying matrices $N(t), M(t), K(t)$ are chosen as

$$
\begin{align*}
N(t) & =-\gamma A^{T}(t) A(t)  \tag{9a}\\
M(t) & =-\gamma A^{T}(t)  \tag{9b}\\
K(t) & =I+\gamma A^{T}(t) \tag{9c}
\end{align*}
$$

where $\gamma$ is a strictly positive tuning parameter and if $\vec{\omega}(t)$, $\dot{\vec{\omega}}(t)$ and $\ddot{\vec{\omega}}(t)$ are bounded and if at least one component of the vector $\vec{\omega} \wedge \dot{\vec{\omega}}$ does not go to zero at infinity then the observation error is asymptotically stable.

Proof. Using equations (7) and (8) the observation error $\epsilon=x-\hat{x}$ dynamics can be written as

$$
\begin{equation*}
\dot{\epsilon}=N \epsilon+(A-N-K A) x+(I-M-K) b \tag{10}
\end{equation*}
$$

It is easy to see that the unbiasedness conditions

$$
\begin{array}{r}
A-N-K A=0 \\
I-M-K=0
\end{array}
$$

are satisfied if matrices $N(t), M(t), K(t)$ are chosen as in (9) then the observation error becomes

$$
\begin{equation*}
\dot{\epsilon}(t)=-\gamma A^{\top}(t) A(t) \epsilon(t) \tag{11}
\end{equation*}
$$

The stability proof follows Lyapunov techniques and the use of Barbalat's lemma (see Khalil [1992]). Consider the following Lyapunov function candidate $V(\epsilon)=\epsilon^{\top} \epsilon$. The time derivative of $V$ along the observation error dynamics (11) leads to

$$
\begin{equation*}
\dot{V}=-2 \gamma\|A \epsilon\|^{2} \leq 0 \tag{12}
\end{equation*}
$$

Since $V(\epsilon(t))$ is monotonicaly non increasing and bounded below by zero, $V(\epsilon(t)) \rightarrow l$ as $t \rightarrow \infty$ where $l$ is finite. Then $\epsilon(t)$ is bounded.
Now, we use Barbalat's lemma in order to prove $\dot{V}(t) \rightarrow 0$. The time derivative of $\dot{V}$ along equation (11) gives

$$
\ddot{V}(\epsilon(t))=4 \gamma^{2} \epsilon^{\top} A^{\top} A A^{\top} A \epsilon-4 \gamma \epsilon^{\top} \dot{A}^{\top} A \epsilon
$$

Since $\epsilon(t), A(t), \dot{A}(t)$ are bounded then $\ddot{V}$ is bounded, which implies that $\dot{V}(\epsilon(t))$ is uniformly continuous. Now using Barbalat's lemma it follows that $\dot{V}(\epsilon(t)) \rightarrow 0$ as $t \rightarrow \infty$. Then, from relation (12) $A(t) \epsilon(t) \rightarrow 0$ as $t \rightarrow \infty$.
In order to use again Barbalat's lemma, we compute the time derivatives of the function $\varphi(t)=A(t) \epsilon(t)$. A simple calculation gives

$$
\begin{aligned}
\dot{\varphi}(t) & =\left(\dot{A}-\gamma A A^{\top} A\right) \epsilon \\
\ddot{\varphi}(t) & =\left(\ddot{A}-2 \gamma \dot{A} A^{\top} A-\gamma A \dot{A}^{\top} A-\gamma A A^{\top} \dot{A}\right) \epsilon
\end{aligned}
$$

As $A(t), \dot{A}(t), \ddot{A}(t), \epsilon(t)$ are bounded, then $\ddot{\varphi}(t)$ is bounded which implies that $\dot{\varphi}(t)$ is uniformly continuous. Since $\varphi(t) \rightarrow 0$ and $\dot{\varphi}(t)$ is uniformly continuous it follows from Barabalat's lemma that $\dot{\varphi}(t)=\left(\dot{A}-\gamma A A^{\top} A\right) \epsilon \rightarrow 0$. Now since $\dot{\varphi}(t) \rightarrow 0$ and $A \epsilon \rightarrow 0$ then $\dot{A} \epsilon \rightarrow 0$.
Next, the two conditions $A \epsilon \rightarrow 0$ and $\dot{A} \epsilon \rightarrow 0$ can be written as

$$
\begin{cases}r(t) \epsilon_{2}(t)-q(t) \epsilon_{3}(t) & \rightarrow 0  \tag{13}\\ -r(t) \epsilon_{1}(t)+p(t) \epsilon_{3}(t) & \rightarrow 0 \\ q(t) \epsilon_{1}(t)-p(t) \epsilon_{2}(t) & \rightarrow 0\end{cases}
$$

and

$$
\begin{cases}\dot{r}(t) \epsilon_{2}(t)-\dot{q}(t) \epsilon_{3}(t) & \rightarrow 0  \tag{14}\\ -\dot{r}(t) \epsilon_{1}(t)+\dot{p}(t) \epsilon_{3}(t) & \rightarrow 0 \\ \dot{q}(t) \epsilon_{1}(t)-\dot{p}(t) \epsilon_{2}(t) & \rightarrow 0\end{cases}
$$

Using the first relations from (13) and (14) it is easy to see that $(q(t) \dot{r}(t)-\dot{q}(t) r(t)) \epsilon_{3}(t) \rightarrow 0$. Suppose that the function $q(t) \dot{r}(t)-\dot{q}(t) r(t)$ (the first component of vector $\vec{\omega} \wedge \dot{\vec{\omega}})$ does not go to zero at the inifinity, then $\epsilon_{3} \rightarrow 0$. If $r(t) \rightarrow 0$, then $\dot{r}(t) \rightarrow 0$ as $\dot{r}(t)$ is uniformly continuous which is impossible while $q(t) \dot{r}(t)-\dot{q}(t) r(t)$ does not go to zero at the infinity. Notice that $q$ and $\dot{q}$ are bounded. Using $\epsilon_{3}(t) \rightarrow 0$, from relation (13), it is obvious that $\epsilon_{1}(t) \rightarrow 0$ and $\epsilon_{2}(t) \rightarrow 0$. The same reasoning can be used with the other components of the vector $\vec{\omega} \wedge \dot{\vec{\omega}}$ which proves the asymptotical stability of the observation error. This ends the proof.

### 3.2 Observer design with two measured accelerations

In this section, only the two first components of the acceleration are measured. To estimate the angular velocity $x=(u, v, w)^{T}$ we consider the model of system (5). Taking into account the measured variables, the system (5) can be rewritten as

$$
\begin{align*}
& \dot{x}(t)=A(t) x(t)+b(t)  \tag{15}\\
& y(t)=C \dot{x}(t)
\end{align*}
$$

with

$$
C=\left(\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0
\end{array}\right)
$$

We consider again the state observer given in (8).
Lemma 2. Assume that the time-varying matrices $N(t)$, $M(t), K(t)$ are chosen as

$$
\begin{align*}
L(t) & =\left(\begin{array}{cc}
1 & 0 \\
0 & 1 \\
-p / r & -q / r
\end{array}\right)  \tag{16a}\\
K(t) & =L+\gamma A^{T}(t) C^{T}  \tag{16b}\\
N(t) & =A(t)-K(t) C A(t)  \tag{16c}\\
M(t) & =I-K(t) C \tag{16d}
\end{align*}
$$

where $\gamma$ is a strictly positive tuning parameter and if $\vec{\omega}(t)$, $\dot{\vec{\omega}}(t)$ and $\ddot{\vec{\omega}}(t)$ are bounded and if at least one of the two first components of the vector $\vec{\omega} \wedge \dot{\vec{\omega}}$ does not go to zero at infinity then the observation error is asymptotically stable.

Proof. Using equations (15) and (8) the observation error $\epsilon=x-\hat{x}$ dynamics can be written as

$$
\begin{equation*}
\dot{\epsilon}=N \epsilon+(A-N-K C A) x+(I-M-K C) b \tag{17}
\end{equation*}
$$

It is easy to see that the unbiasedness conditions

$$
\begin{array}{r}
A-N-K C A=0 \\
I-M-K C=0
\end{array}
$$

are satisfied if matrices $N(t), M(t)$ are chosen as in (16). Then the observation error becomes

$$
\begin{equation*}
\dot{\epsilon}(t)=N \epsilon(t) \tag{18}
\end{equation*}
$$

Using the expression of $K(t)$, one obtains

$$
\begin{equation*}
N(t)=A(t)-L(t) C A(t)-\gamma A^{T}(t) C^{T} C A(t) \tag{19}
\end{equation*}
$$

Now, choosing $L$ as in (16) yields $A(t)-L(t) C A(t)=0$ and finally the observation error becomes

$$
\begin{equation*}
\dot{\epsilon}(t)=-\gamma A^{T}(t) C^{T} C A(t) \epsilon(t) \tag{20}
\end{equation*}
$$

Consider the following Lyapunov function candidate $V(\epsilon)=$ $\epsilon^{\top} \epsilon$. The time derivative of $V$ along the observation error dynamics (20) leads to

$$
\begin{equation*}
\dot{V}=-2 \gamma\|C A \epsilon\|^{2} \leq 0 \tag{21}
\end{equation*}
$$

Since $V(\epsilon(t))$ is monotonicaly non increasing and bounded below by zero, $V(\epsilon(t)) \rightarrow l$ as $t \rightarrow \infty$ where $l$ is finite. Then $\epsilon(t)$ is bounded.
Now, we use Barbalat's lemma in order to prove $\dot{V}(t) \rightarrow 0$. The time derivative of $\dot{V}$ along equation (20) gives

$$
\ddot{V}(\epsilon(t))=4 \gamma^{2} \epsilon^{\top} A^{T} C^{T} C A A^{T} C^{T} C A \epsilon-4 \gamma \epsilon^{T} \dot{A}^{T} C^{T} C A \epsilon
$$

Since $\epsilon(t), A(t), \dot{A}(t)$ are bounded then $\ddot{V}$ is bounded, which implies that $\dot{V}(\epsilon(t))$ is uniformly continuous. Now using Barbalat's lemma it follows that $\dot{V}(\epsilon(t)) \rightarrow 0$ as $t \rightarrow \infty$. Then, from relation (21) $C A(t) \epsilon(t) \rightarrow 0$ as $t \rightarrow \infty$.
In order to use again Barbalat's lemma, we compute the time derivatives of the function $\varphi(t)=C A(t) \epsilon(t)$. A simple calculation gives

$$
\dot{\varphi}(t)=\left(C \dot{A}-\gamma C A A^{T} C^{T} C A\right) \epsilon
$$

It is easy to show that if $A(t), \dot{A}(t), \ddot{A}(t), \epsilon(t)$ are bounded, then $\ddot{\varphi}(t)$ is bounded which implies that $\dot{\varphi}(t)$ is uniformly continuous. Since $\varphi(t) \rightarrow 0$ and $\dot{\varphi}(t)$ is uniformly continuous it follows from Barabalat's lemma that $\dot{\varphi}(t)=\left(C \dot{A}-\gamma C A A^{T} C^{T} C A\right) \epsilon \rightarrow 0$ as $t \rightarrow \infty$. Now since $\dot{\varphi}(t) \rightarrow 0$ and $C A \epsilon \rightarrow 0$ then $C \dot{A} \epsilon \rightarrow 0$.
Next, the two conditions $A \epsilon \rightarrow 0$ and $\dot{A} \epsilon \rightarrow 0$ can be written as

$$
\left\{\begin{align*}
r(t) \epsilon_{2}(t)-q(t) \epsilon_{3}(t) & \rightarrow 0  \tag{22}\\
-r(t) \epsilon_{1}(t)+p(t) \epsilon_{3}(t) & \rightarrow 0
\end{align*}\right.
$$

and

$$
\begin{cases}\dot{r}(t) \epsilon_{2}(t)-\dot{q}(t) \epsilon_{3}(t) & \rightarrow 0  \tag{23}\\ -\dot{r}(t) \epsilon_{1}(t)+\dot{p}(t) \epsilon_{3}(t) & \rightarrow 0\end{cases}
$$

Using the first relations from (22) and (23) it is easy to see that $(q(t) \dot{r}(t)-\dot{q}(t) r(t)) \epsilon_{3}(t) \rightarrow 0$. Suppose that the function $q(t) \dot{r}(t)-\dot{q}(t) r(t)$ (the first component of vector $\vec{\omega} \wedge \dot{\vec{\omega}})$ does not go to zero at the inifinity, then $\epsilon_{3} \rightarrow 0$. If $r(t) \rightarrow 0$, then $\dot{r}(t) \rightarrow 0$ as $\dot{r}(t)$ is uniformly continuous which is impossible while $q(t) \dot{r}(t)-\dot{q}(t) r(t)$ does not go to zero at the infinity. Notice that $q$ and $\dot{q}$ are bounded. Using $\epsilon_{3}(t) \rightarrow 0$, from relation (22), it is obvious that $\epsilon_{1}(t) \rightarrow 0$ and $\epsilon_{2}(t) \rightarrow 0$. The same reasoning can be used with the second component of the vector $\vec{\omega} \wedge \dot{\vec{\omega}}$ which proves the asymptotical stability of the observation error. This ends the proof.

## 4. NUMERICAL SIMULATIONS

In this section, we applied our approach to design an observer for the linear velocity of an UAV. In order to show the influence of the design parameter $\gamma$, simulations are carried out with two values of this parameter i.e. $\gamma=50$ and $\gamma=100$ (fastest dynamics). The simutions are performed with the following parameters (Benzemrane et al. [2007]) $m=2.5 \mathrm{~kg}, I_{x x}=22493110^{-7} \mathrm{~kg} \cdot \mathrm{~m}^{2}$, $I_{y y}=22261110^{-7} \mathrm{~kg} . \mathrm{m}^{2}$ and $I_{z z}=32513010^{-7} \mathrm{~kg} . \mathrm{m}^{2}$.
The following figures represent the observation error $\epsilon_{i}\left(\epsilon_{i}\right.$ is the ith component of $\epsilon$ ) for different cases

- case 1 : Observer given by lemma 1 without measurement noise (figures 2 to 4)
- case 2: Observer given by lemma 1 with Gaussian (standard deviation : 1, mean value : 0 ) measurement noise (figures 5 to 7 )
- case 3 : Observer given by lemma 2 with Gaussian (standard deviation : 1 , mean value : 0 ) measurement noise (figures 8 to 10)


Fig. 2. Observation errors $\varepsilon_{1}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 1


Fig. 3. Observation errors $\varepsilon_{2}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 1 .


Fig. 4. Observation errors $\varepsilon_{3}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 1 .


Fig. 5. Observation errors $\varepsilon_{1}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 2 .


Fig. 6. Observation errors $\varepsilon_{2}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 2 .


Fig. 7. Observation errors $\varepsilon_{3}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 2 .


Fig. 8. Observation errors $\varepsilon_{1}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 3 .


Fig. 9. Observation errors $\varepsilon_{2}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 3 .


Fig. 10. Observation errors $\varepsilon_{3}(t)$ with $\gamma=100$ (fastest dynamics) and $\gamma=50$ : case 3 .

One can see that all the observation errors are asymptotically stable. The tuning parameter $\gamma$ allows to adjust the observation error convergence rate without modifying the stability conditions. Notice that the behaviour of the observation error is weakly sensitive to the noise measurement.

## 5. CONCLUSION

In this work a simple time-varying reduced-order observer has been presented to estimate the linear velocity of an UAV using full or partial acceleration measurements. Sufficient conditions for the asymptotic stability of the observation error are given. Simulation results are promising and seems to lead to an exponential stability. Future research concerning a proof of exponential stability and observer robustness will be done.

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