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Road Grade Estimation for Look-Ahead Vehicle Control

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Abstract: Look-ahead cruise controllers and other advanced driver assistance systems for heavy duty vehicles require high precision digital maps. This contribution presents a road grade estimation algorithm for creation of such maps based on Kalman filter fusion of vehicle sensor data and GPS positioning information. The algorithm uses data from multiple traversals of the same road to improve previously stored road grade estimates. Measurement data from three test vehicles and six road traversals have been used to evaluate the quality of the obtained road grade estimate compared to a known reference. The obtained final grade estimate compares favorably to one acquired from a specialized road grade measurement vehicle with a DGPS receiver and inertial measurement unit.

Keywords: Kalman filtering techniques in automotive control; Automotive system identification and modeling; General automobile/road-environment strategies

1. INTRODUCTION

Modern heavy duty vehicles (HDV) employ several electronic control systems which utilize information about the vehicle and its environment to increase efficiency, safety and comfort. The road grade is one key variable which heavily influences the longitudinal dynamics and energy flow in a heavy duty vehicle. Knowledge of the current and future road grade can be used in engine and gearbox control systems to help meet the instantaneous power demand while keeping fuel consumption and environmental impact as low as possible. If the road grade for the kilometer directly ahead of the vehicle is known, it is possible to automatically adjust the speed in advance of up- and downhill sections and thus conserve fuel without increasing trip time. The preview road grade information can also be utilized when determining if a gearshift should be performed or the state of some energy buffer changed.

Information about the current state of the vehicle is commonly acquired through various on-board sensors. Information about factors which will influence the vehicle in the future cannot generally be sensed directly. However, a map with stored information from previous trips can provide the required look-ahead information and enable new control algorithms to improve overall vehicle performance. In order to use the map the vehicle needs to be able to position itself, both when writing to and reading from the map. Satellite positioning receivers are already commonplace in vehicles, and they may be used for this task as well as other position-based services. Digital maps are widespread, but mostly used for navigation rather than direct vehicle control.

A sufficiently detailed road grade estimate is currently not generally available in navigation maps, and has to be obtained by other means. One method is to use on-board sensors to estimate the road grade and create a map as the vehicle drives down the road. If a road is driven frequently, many estimates of the road grade can be obtained. These can be used to increase confidence in the created map. This paper investigates properties of a proposed method for road grade estimation. The method combines road grade estimates based on standard mounted on-board sensors and information from a GPS receiver for many overlapping road traversals into a road grade map. Each time a known road is driven the map is updated. The method has been implemented and results from tests with the three types of HDVs shown in Figure 1 are presented.

Fig. 1. Vehicle types used for verification of the proposed road grade estimation method. Starting from the left a tractor-semitrailer combination (A), tractor only (B), and rigid truck (C) were used.

1.1 Related Work

The potential for improved energy efficiency through speed optimization based on future road grade has recently been treated, e.g., Terwen et al. [2004], Hellström et al. [2007], Fröberg and Nielsen [2007]. Knowledge of future energy needs combined with new auxiliary units which enable improved power consumption scheduling over time can improve total energy efficiency, as explored in Pettersson and Johansson [2006]. In this context the future road grade is assumed to be known, for example from a map. A multitude of methods for estimating the road grade can be found in the literature. One approach is to use a sensor to directly measure the grade. A direct road grade sensor for automotive use is described in a patent application filed as early as 1971 by Gaeke [1974]. A GPS receiver with 3D velocity output is used for example in Bae et al. [2001] where the grade is calculated from the ratio of the vertical and horizontal velocities. Such a method relies heavily on the existence of a high quality GPS signal, something which is not always available. The idea of using vehicle sensor information to find the road grade has been explored in Lingman and Schmidtbauer [2001] where a Kalman filter is used to process a measured or estimated propulsion force or estimated retardation force and a measured velocity. A similar method, where the grade is estimated using Recursive Least Squares based on a simple motion model has been suggested by Vahidi et al. [2005]. On-line road grade estimation based on accelerometers or a calculated driveline torque and a vehicle model is state-of-the-art in today's vehicles. These methods have the advantage of not needing any extra sensors, such as the GPS, but hence don't provide the extra bias compensation or easy inclusion of data from multiple road traversals. Earlier treatments of the proposed grade estimation method can be found in Sahlholm et al. [2007b,a].

1.2 Contribution

This paper introduces a method for HDVs to estimate the road grade using only standard mounted sensors and a GPS receiver. Two implementations are presented, one based on a non-linear vehicle model and extended Kalman filtering and one based on a piecewise linear model and a standard Kalman filter. The method includes a systematic way of improving the current grade estimate using new passes over a known road segment. Incremental improvements are made possible by the use of spatial sampling and storage of the estimated error covariance matrix for the current road grade estimate. The storage requirement for a particular road will not grow as new measurements are incorporated. A step by step illustration of the effects of adding new measurements is presented. The proposed method is evaluated using three test vehicles driven a total of six times over the same test road segment. The obtained final grade estimate compares favorably to one acquired from a specialized road grade measurement vehicle with a DGPS receiver and inertial measurement unit.

1.3 Outline

The paper is organized as follows. Section 2 describes the the road grade estimation method by introducing the vehicle model and the filtering, smoothing and data fusion steps. It also explains the experimental setup. Results are given in section 3, and the paper ends with conclusions and a discussion in section 4.

2. METHODOLOGY

A non-linear vehicle model and an extended Kalman filter (EKF) are used to estimate the road grade. A piecewise constant linear version of the vehicle model is also developed as a tool to evaluate the effect of the non-linearity. Road grade estimates based on six test runs on highway E4 south of Södertälje, Sweden have been calculated and merged using a the proposed method.

2.1 Vehicle Model and Measurements

The first step of the road grade estimation method is to combine a driving torque estimate from the engine control unit and vehicle speed measurements from the wheel sensors with GPS data. A longitudinal vehicle model is used to relate the sensor signals to the road grade. The road grade can be calculated from the model when the vehicle mass, engine torque, active gear and vehicle speed are all known. In this work the vehicle mass has been assumed known, which is reasonable in a lab setting but not in the real world. In a real system the mass will have to be estimated, which introduces an additional error source in the grade estimate. The engine torque estimate comes from the on-board engine management system and is based on fuel injector opening times. The current gear is continuously reported from the gearbox management system, and the vehicle speed is measured by standard mounted wheel speed sensors. The most important forces affecting the vehicle are shown in Figure 2. The forces

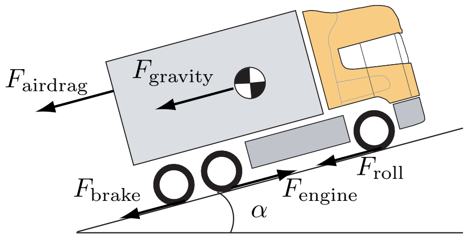


Fig. 2. Longitudinal forces acting on the vehicle.

are generally time varying, time has been left out of the equations for clarity. $F_{\text{engine}} = \frac{i_t i_f \eta_t \eta_f}{r_w} M$ is the net engine force. Knowledge of the current gear yields the gear ratio i_t and the efficiency η_t from tables. The final gear ratio i_f, efficiency η_f and wheel radius r_w are known vehicle constants. M denotes the engine torque. $F_{\text{airdrag}} = \frac{1}{2} c_w A_a \rho_a v^2$ is known through the measured vehicle speed v and the constants air drag coefficient c_w, vehicle frontal area A_a, and air density ρ_{air}. A very simple model $F_{\text{roll}} = mgc_r$ gives the rolling resistance from the vehicle mass m, gravity g, and coefficient of rolling resistance c_r. The road grade α enters the model through the gravity induced force $F_{\text{gravity}} = mg \sin \alpha$. The brake force F_{brake} is excluded from the model since it is generally unknown in a standard HDV, its influence is considered at a later stage. The total dynamic vehicle mass is expressed as $m_t = \frac{J_w}{r_w^2} + m + \frac{i_t^2 i_f^2 \eta_t \eta_f J_e}{r_w^2}$ where J_w and J_e represent the inertia of the engine and the wheels respectively. Newton's laws of motion are used to attain a differential equation describing velocity changes based on forces.

A GPS receiver provides a three dimensional position (latitude, longitude, and altitude) together with a signal indicating the number of satellites used for the position fix. The vehicle speed and the road grade are used to calculate the time derivative of the altitude and thus provide a link between the GPS and the vehicle model. Changes in the road grade are assumed to be random on the time scale of the filter, and are thus not modeled. The engine torque is regarded as an input signal $u(t) = M(t)$. Put together with the state vector $x = [v\ z\ \alpha]^T$ this gives the continuous time vehicle and road model $\dot{x}(t) = f(x)$ with

$$\begin{aligned}\dot{v}(t) =& \frac{1}{m_\mathrm{t}}(F_\mathrm{engine} \\ &- F_\mathrm{airdrag} - F_\mathrm{roll} - F_\mathrm{gravity}) \\ \dot{z}(t) =& v(t)\sin\alpha(t) \\ \dot{\alpha}(t) =& 0\end{aligned} \tag{1}$$

See Kiencke and Nielsen [2003]. In order to easily obtain estimates at specific spatial locations rather than time instants a spatially sampled version of the model is derived. The continuous model is then discretized with the distance step Δs. The discretized model is

$$\underbrace{\begin{bmatrix} v_k \\ z_k \\ \alpha_k \end{bmatrix}}_{x_k} = \underbrace{\begin{bmatrix} v_{k-1} + \Delta s \dfrac{dv_{k-1}}{ds} \\ z_{k-1} + \Delta s \sin\alpha_{k-1} \\ \alpha_{k-1} \end{bmatrix}}_{f_k(x_{k-1},u_{k-1})} + \underbrace{\begin{bmatrix} w_{k-1}^v \\ w_{k-1}^h \\ w_{k-1}^\alpha \end{bmatrix}}_{w_{k-1}} \tag{2}$$

The rate of change in velocity is given

$$\frac{dv_{k-1}}{ds} = c_1 \frac{M_{k-1}}{v_{k-1}} - c_2 v_{k-1} - c_3 \frac{1}{v_{k-1}} (c_\mathrm{r} + \sin\alpha_{k-1}) \tag{3}$$

$$c_1 = \frac{r_\mathrm{w} i_\mathrm{t} i_\mathrm{f} \eta_\mathrm{t} \eta_\mathrm{f}}{J_\mathrm{w} + m r_\mathrm{w}^2 + i_\mathrm{t}^2 i_\mathrm{f}^2 \eta_\mathrm{t} \eta_\mathrm{f} J_\mathrm{e}}$$

$$c_2 = \frac{\frac{1}{2} r_\mathrm{w}^2 c_\mathrm{w} A_\mathrm{a} \rho_\mathrm{a}}{J_\mathrm{w} + m r_\mathrm{w}^2 + i_\mathrm{t}^2 i_\mathrm{f}^2 \eta_\mathrm{t} \eta_\mathrm{f} J_\mathrm{e}}$$

$$c_3 = \frac{r_\mathrm{w}^2 m g}{J_\mathrm{w} + m r_\mathrm{w}^2 + i_\mathrm{t}^2 i_\mathrm{f}^2 \eta_\mathrm{t} \eta_\mathrm{f} J_\mathrm{e}}$$

It can be noted that the values of c_1,c_2, and c_3 depend on the vehicle parameters as well as the selected gear. The presence of the efficiencies η_t and η_f also make the expression (3) dependent on whether the net engine torque is positive or negative.

To evaluate the influence of the nonlinearity in the vehicle model a piecewise constant linear version is derived. The linear model is changed at gear changes and depends on the direction of power flow in the drive line. Each gear and power flow direction will lead to a different mode, denoted by m, with a specific required torque to maintain a constant speed, and equilibrium in the model. The linear discretized model around the equilibrium x_m is given by the system transition matrix F_m and the input model G according to

$$\tilde{x}_k = F_m \tilde{x}_{k-1} + G\tilde{u}_{k-1} \tag{4}$$

where $\tilde{x} = x - x_m$ is the state relative to the linearization point and $\tilde{u} = M - M_m$ is the relative engine torque. The transition matrix is given by $F_m = I + \frac{\partial f}{\partial x}\Big|_{x_m,u_m} \Delta s$.

Two states and the input torque M are available for the state estimation. The measured states are the vehicle velocity v and the altitude z. This leads to a linear measurement equation

$$y_k = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{H_k} \begin{bmatrix} v_k \\ z_k \\ \alpha_k \end{bmatrix} + \underbrace{\begin{bmatrix} e_k^\mathrm{v} \\ e_k^\mathrm{z} \end{bmatrix}}_{e_k} \tag{5}$$

which is be used with both the linear and non-linear vehicle models.

2.2 State Estimation

Two different Kalman filters are used to estimate the road grade and other model states. The non-linear model is used together with an EKF, and the piecewise linear model with a standard Kalman filter (KF).

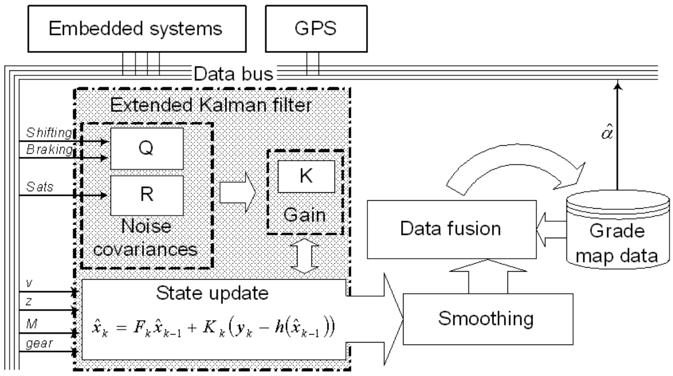


Fig. 3. Overview of the data filtering, smoothing and fusion of the proposed road grade estimation method.

Using the notation of the previous section the estimation model for the nonlinear EKF with a linear measurement equation is given by

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}) + w_{k-1} \\ y_k &= Hx_k + e_k \end{aligned} \tag{6}$$

The process and measurement noise covariances are updated depending on the characteristics of the driving situation and GPS positioning conditions. In the EKF the non-linear model is linearized around the current state at every time step. The obtained transition matrix F_k is then used to complete the steps of the standard Kalman filter recursions. These recursions are described by two update steps: a time update and a measurement update. In the time update the system model is used to predict the future state of the system. Using the notation $\hat{x}_{k|k-1}$ to denote the quantity $\hat{x}$ at time k based on information available up to time $k-1$ the time update is done according to

$$\begin{aligned} \hat{x}_{k|k-1} &= f(x_{k-1}, u_{k-1}) \\ P_{k|k-1} &= F_k P_{k-1|k-1} F_k^T + Q_k \end{aligned} \tag{7}$$

Similarly to F_m in the piecewise linear model the transition matrix F_k is defined to be the Jacobian $F_k = \frac{\partial f}{\partial x}\Big|_{\hat{x}_{k-1|k-1},u_{k-1}}$. $P_{k|k-1}$ is the estimated error covariance, and $Q_k = E[w_k^2]$ is the process noise covariance. After the time update the measurement at time k is used in a measurement update to improve the estimate. The measurement update is described by

$$\begin{aligned} K_k &= P_{k|k-1} H^T (H P_{k|k-1} H^T + R_k)^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k (y_k - H\hat{x}_{k|k-1}) \\ P_{k|k} &= (I - K_k H) P_{k|k-1} \end{aligned} \tag{8}$$

Here K_k is the Kalman gain, and $R_k = E[e_k^2]$ is the measurement noise covariance.

The piecewise constant linear model is used with a regular Kalman filter. At each mode change between different linearizations the final state of the old filter is used to initialize the new one. The linear system model in each mode is

$$\begin{aligned}\tilde{x}_k &= F_m\tilde{x}_{k-1} + G\tilde{u}_{k-1} + w_{k-1} \\ \tilde{y}_k &= H\tilde{x}_k + e_k\end{aligned} \tag{9}$$

where $\tilde{y}_k = y_k - Hx_m$. This leads to the KF time update equations

$$\begin{aligned}\hat{x}_{k|k-1} &= F_m\hat{x}_{k-1|k-1} + Gu_{k-1} \\ P_{k|k-1} &= F_kP_{k-1|k-1}F_k^T + Q_k\end{aligned} \tag{10}$$

The measurement equations are identical to the EKF case.

For this method the true process and noise covariances R_k and Q_k are not known from the start. Instead they are used as time varying design parameters to tune the filter to different driving situations. To simplify the design the noise covariance matrices were chosen to be diagonal. The diagonal elements are directly associated to the three model states and two measured quantities. For normal driving at a fixed gear Q_k was tuned to give a filter with a time constant similar to the one used to produce our reference road grade estimate. R_k was adjusted depending on the number of GPS satellites available. While other factors also affect the GPS position accuracy the number of satellites was the only relevant signal available from the satellite receiver used. When satellite coverage was lost a very high variance for was set for the altitude measurement, causing the grade estimate only to depend on vehicle signals. Driving events such as gearshifts and braking affect the vehicle in ways that are not covered by the relatively simple vehicle model given in (1). To account for this the process variance for the velocity state was increased during those events.

By carrying out the estimation off-line when complete road sections have already been recorded it is possible to use smoothing to compensate for the filtering delay and include later measurements in the estimate for each data point. Rauch-Tung-Striebel fixed point smoothing algorithm, introduced in Rauch et al. [1965], was used in this work.

2.3 Data Fusion

In order to merge data from many passes over the same road segment a distributed data fusion method is used. The distributed approach has the important advantage that the data which has to be stored does not increase as additional measurements of known road segments are incorporated into the map. For each road segment, the map consists of the road related states (altitude z and slope α) and the associated estimated error covariance estimates for those states. Based on the estimated error covariances stored in the map and the estimated error covariances of a new smoothed estimate an updated map is created each time a new measurement of a road segment becomes available. The new map becomes a weighted average of the two sources

$$\begin{aligned}P_k^f &= ((P_k^1)^{-1} + (P_k^2)^{-1})^{-1} \\ \hat{x}_k^f &= P_k^f((P_k^1)^{-1}\hat{x}_k^1 + (P_k^2)^{-1}\hat{x}_k^2)\end{aligned} \tag{11}$$

where P_k^f is the resulting error covariance, $\hat{x}_k^f$ is the new state estimate for the map. The quantities P_k^1, P_k^2, $\hat{x}_k^1$, and $\hat{x}_k^2$ are the source estimates and estimated error covariances. Details on the data fusion algorithm (11) can be found in Gustafsson [2000].

2.4 Experiment setup

The proposed road grade estimation algorithm has been tested on highway E4 south of Södertälje in Sweden. Three test vehicles, representing the different types shown in Figure 1 were used. Important properties for the test vehicles are listed in Table 1. A total of six round-trip measurements were conducted. The different vehicles were driven on different days under varying weather conditions. Most of the signals needed for the road grade estimation are available on the CAN bus of stock production trucks. These are the vehicle speed, engine torque (calculate based on fuel injection times), current gear, gearshift status, and brake utilization. The CAN bus signals were recorded using a laptop. There was no GPS data available on the vehicle bus, instead an external VBOX GPS receiver with a CAN interface was used. The GPS data was logged using the same computer as the vehicle data.

Table 1. Key properties of the test vehicles used to collect experiment data.

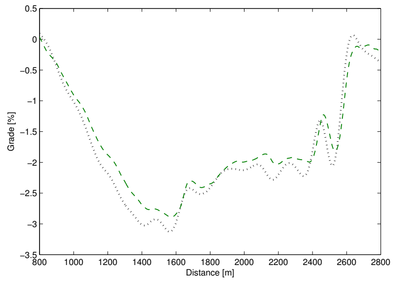
Vehicle	Configuration	Weight	Axles	Meas.
A	Tractor and semi-trailer	39 t	5	1,2,3
B	Tractor	13 t	2	4,5
C	Rigid truck	21 t	3	6

The absolute position obtained from the GPS was used to synchronize data from the different measurements. A reference point was chosen in one of the measurements, the closest points in the other measurements were then used as their respective starting points. From the starting point the traveled distance information in each measurement was used to resample all signals to a common distance vector. With common distance indexing it was then possible to complete the road grade estimation and data fusion steps.

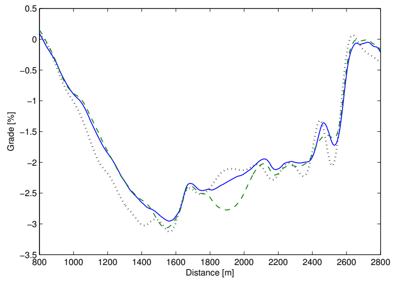
3. RESULTS

Road grade estimates obtained from regular highway driving at the normal cruising speed are very good. Using more than one road traversal and more than one vehicle improves the final grade estimate. All result figures presented share the same distance scale for easy cross-referencing. A reference grade profile obtained from a specialized measurement vehicle is used to evaluate the estimates. Figure 4 shows the agreement of the final grade estimate with the reference for a part of the test road. The part of the test road shown in Figure 4 contains a downhill section, from $s = 1000$ m to $s = 2600$ m. Around $s = 2000$ m vehicle A needs to apply the brakes in order to avoid over speeding. During braking the torque affecting the vehicle is unknown. The process noise term in Q_k corresponding to the velocity state is increased in order to decrease the reliance on the model and increase the estimated slope error covariance.

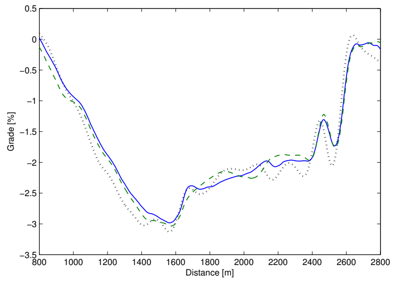
Figure 5 shows a comparison of the smoothed estimates from all six traversals with the final grade estimate and the reference grade profile. The downhill section from



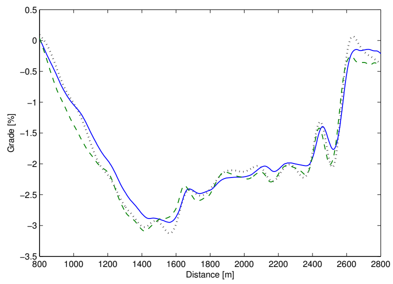
(a) The first measurement forms a road grade map by itself. Estimation errors cause it to differ from the reference road grade.



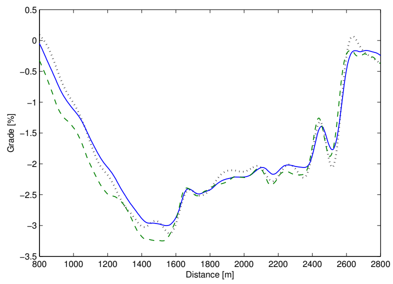
(b) When a second measurement is added to the one in (a) a new road grade map is obtained. The large disturbance in measurement two at $s = 1900$ m has high uncertainty and thus a low weight in the data fusion.



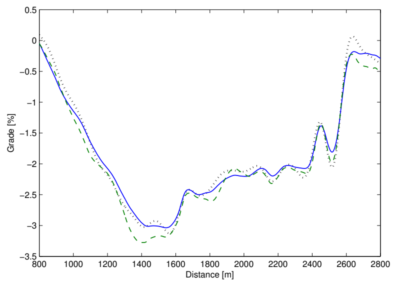
(c) The third estimate from vehicle A does not differ much from the map based on the previous two road traversals.



(d) The larger difference in the fourth estimate is probably due to different model parameter errors in relation to vehicle B.



(e) Estimate five is based on vehicle B, just like the one in (d).



(f) When the sixth estimate, recorded with vehicle C, has been added the map is complete.

Fig. 6. As more measurements are added the road grade map is improved. The sub-figures (a)-(f) show the progression as six measurements are combined into one road grade map. Each figure shows the latest measurement (dashed), the road grade map based on all measurements added so far (solid) and the reference road grade (dotted).

$s = 1300$ m to $s = 2300$ m is the hardest part of the test road to estimate accurately. The mean value at each sample point is included to illustrate the effect of the data fusion step. The grade maps resulting from the progressive inclusion of the six recorded road traversals can be seen in Figure 6.

The results from using the piecewise constant linear model instead of the time-varying non-linear model indicated only marginal changes in the estimated slope. A comparison of road grade estimates obtained with the two methods is shown in Figure 7. The main non-linearity in the vehicle model, for the magnitude of slopes considered, is in the velocity. The linear model is only valid for velocities close to the linearization point of 80 km/h. During most of the test road measurements the velocity of the measuring vehicle was close to this value. The proposed method is primarily suited for highway estimation, and it would probably be wise to reject any data sets with large velocity deviations.

4. CONCLUSIONS AND DISCUSSION

For the investigated test cases the piecewise linear model performs in a similarly to the time-varying non-linear model for the task of estimating highway grades. This opens up possibilities both to lower the computational requirements and to gain more insight into how the filter can be improved. One such planned extension is the estimation of the true process and measurement noise covariances Q and R. Better synchronization of the different measurement runs by the use of more reference positions is likely to improve the performance gain from using multiple traversals.

Measurements from more vehicles and more road passes will make it possible to deduce more precisely what grade estimation errors are random and reduced with additional data, and which are systematic and more crucial to deal with in the method. Further analysis of the linear vehicle models can help identify areas for improvement. Already at this stage the proposed method is feasible for collecting road grade data of sufficient quality for model predictive control based energy optimization of the vehicle longitudinal motion.

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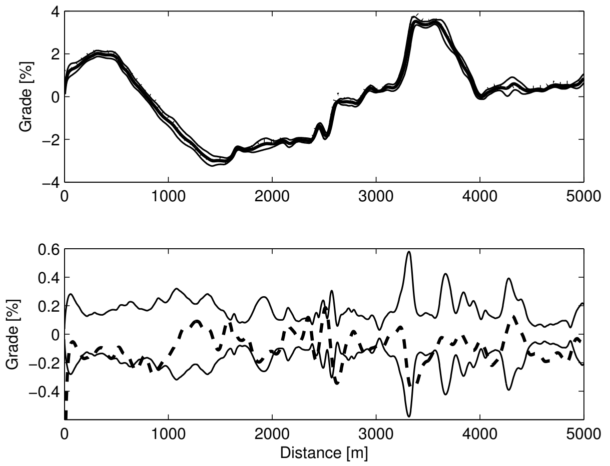

Fig. 4. The top figure shows the final grade estimate calculated through data fusion based on six road traversals (solid). It agrees well with the reference grade profile from a specialized measurement vehicle (dashed). The numerical one standard deviation confidence interval (assuming normal distribution) around the final grade estimate at each sample point, based on the six experiments, is also shown (thin lines). The bottom figure shows the difference between the fused grade estimate and the reference profile (dashed), as well as the standard deviation from the top figure (solid).

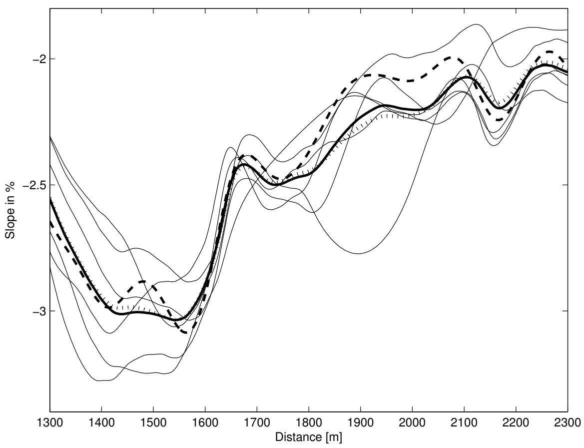

Fig. 5. The final merged road grade estimate (solid) is shown with the reference grade profile (dashed) and the mean value of all smoothed estimates (dotted). The estimates from the individual traversals are also included (thin lines). This is a magnification of the most challenging part of the test road. Measurement two is particularly at odds with the rest at 1900 m. This is due to a combination of poor GPS coverage and the effect of the braking. The detrimental effect on the fused estimate is smaller than on the mean.

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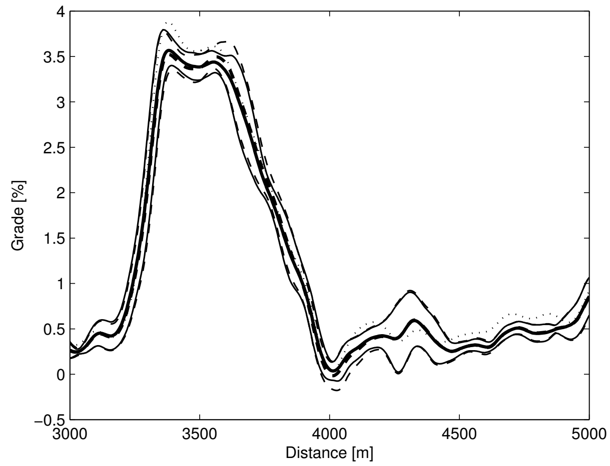

Fig. 7. The final grade estimate based on the non-linear model (solid) with confidence interval (thin lines) is shown together with the one based on the piecewise constant linear model (dashed). The reference grade is also shown (dotted thin line). The differences between the two methods are slight, and significantly smaller than the deviation from the reference grade.

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