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Researches on Load Balancing Control Problem for the Systems with Multiple Parallel Entities Using Differences Control Technique

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Abstract: Load-balancing problem, which covers a large area in engineering fields where several parallel entities share the same load on the systems, is studied. A control technique for solving such problem, called differences control technique (DsCT), is investigated. Two types of systems whose parallel entities can be approximated respectively as an integral and a first-order-plus-dead-time (FOPDT) model are examined. The issue of system balanceability is elaborately addressed, and a sufficient condition for the systems to be balanceable is given. An application example is presented to demonstrate the effectiveness of the DsCT technique for solving the load-balancing problem.

1. INTRODUCTION

The issue of load balancing exists in a wide variety of engineering fields where a system consists of multiple parallel entities. For example, with the increasing popularity of multimedia streaming applications and some other Internet contents, traffic on Internet increases constantly, and hence a Web-server usually overloads in heavy-request period. As described in (Cardellini *et al*., 1999), Website administrators usually need to enlarge Web-server capacity. One of approaches is to replicate information across mirrored-server architecture. This traffic-distributing scheme lets users manually select alternative URLs for a Website. However, such architecture is not user-transparent, nor does it allow the Web-server system to control traffic distribution. Web-server clusters have been widely employed to provide high-efficiency Internet services (Cardellini *et al*., 1999, Liu & Yu, 2004, Shan *et al.*, 2002).

One commonly adopted structure of Web-server clusters is illustrated in Fig. 1. The N Web-servers are parallel, and based on some traffic-distributing policies, such as least connections, weighted percentage, round robin, etc. (Cisco Systems, Inc., Online), the total incoming client requests, f_U, are dispatched to the N Web-servers by the dispatcher. One fact about this topology is that the sum of the client requests dispatched to the N Web-servers is equal to the total incoming client requests to the Web-server cluster, that is, f_i, $i = 1, 2, \ldots, N$, satisfy

$$\sum_{i=1}^{N} f_i = f_U .$$

In petroleum refineries, in order to improve heat transfer efficiency and reduce the possibility of coke formation the fluid material is commonly divided into several parallel passes before entering a furnace, and these parallel passes join together again after leaving the furnace (Eitelberg, 1999, Garg, 1999, Wang & Zheng, 2005). Consider a furnace system as shown in Fig. 2, where the Oil inlet pipe is branched into N parallel passes labeled as Pass 1, Pass 2, …, and Pass N. Obviously, the inlet flowrates of the N parallel passes, $Flow_i$, $i = 1, 2, \ldots, N$ satisfy the following flowrate constraint:

$$\sum_{i=1}^{N} Flow_i = F_T ,$$

where F_T is the total flowrate of the oil inlet which is set by the management department of the factory and is a constant during a given period.

For product quality and other reasons, the outlet temperatures T_1 to T_N are required to be as identical as possible. However, the controlling of these temperatures is not an easy task due to the complexity of heating process and various disturbances (Abilov *et al.*, 2002, Eitelberg, 1999, Garg, 1999, Wang & Zheng, 2005). The goal of outlet temperatures uniformity usually cannot be obtained through static even distribution of F_T, and hence measures should be taken to dispatch the F_T among the parallel passes dynamically according to the N outlet temperatures.

In addition, the load-balancing problems also cover a large area in manufacturing fields, in computer parallel processing communities, and in data packet/cell switches, etc.

The load-balancing problem addressed in this paper can be formulated as follows: how to dispatch the total load (e.g. total user requests in Web-server cluster systems, or oil flowrate in multiple-passes furnace systems, etc.) among the

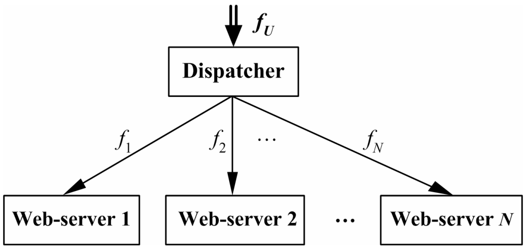

Fig. 1. An architecture of Web server clusters, where the total incoming client requests, f_U, are dynamically dispatched to the N Web-servers by the dispatcher.

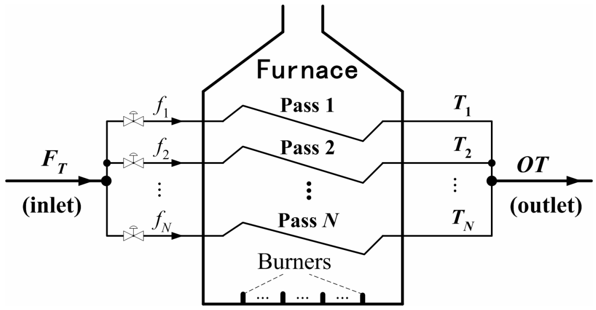

Fig. 2. Schematic diagram of a multi-passes furnace system.

N parallel entities such that they have identical states under the following load constraint:

$$\sum_{i=1}^{N} l_i = L_T \, , \tag{1}$$

where L_T is the total load on the system, and l_i is the load dispatched to the ith entity, i = 1, 2, …, N.

The approach of the generalized switched server (GSS) has been proposed to solve the above load-balancing problem (Wang & Zheng, 2006). With the GSS scheme, the system would be a discretely controlled continuous system, and hence the GSS architecture would be a hybrid one, requiring a high-level expertise for system analysis and control design. The reiterative difference control technique (rDCT) has been proposed to control the parallel streams temperatures to realize the pass balancing for a multi-passes furnace system (Wang & Zheng, 2005). As pointed out in (Wang & Zheng, 2007), with the rDCT technique, there needs certain times of reiterative employment of the DCT technique, and the control system is not scalable. Later, Wang & Zheng (2007) have further generalized the rDCT technique and proposed the differences control technique (DsCT) to solve the pass-balancing problem for the multi-passes furnace system. In this paper, the DsCT technique is first investigated, and then two types of systems whose parallel entities can be approximated respectively as an integral and a first-order-plus-dead-time (FOPDT) model are examined. For the two types of systems, the issue of the system balanceability is addressed, and a sufficient condition for such systems to be balanceable is given respectively. An application example is presented to demonstrate the effectiveness of the DsCT technique for solving the load-balancing problem.

The rest of the paper is organized as follows. Section 2 briefly reviews some of the related works, investigates the differences control technique, and introduces the definition of the system balanceability. The framework of an Integral model controlled with the DsCT scheme, which can be modeled as an N-tanks system is given in Section 3. This section also discusses the balanceablility problem of the control system. Section 4 presents the differences control model for some industrial cases, where the parallel entities can be approximated as a first-order-plus-dead-time (FOPDT) model. An application example of the DsCT to a real-life furnace system is given in Section 5. Finally, Section 6 concludes this paper.

2. DIFFERENCES CONTROL TECHNIQUE

This section first briefly reviews some related works, and then investigates the differences control technique (DsCT).

2.1 *Related Works*

2.1.1 *Generalized Switched Server Systems*

The tank-pair is the basic switching unit of the GSS system (Wang & Zheng, 2006). It consists of two tanks and a controller. Each tank has an inflow f_i and an outflow g_i, i = 1, 2, and the x_1 and x_2 are the levels of tank 1 and tank 2 respectively. The controller's input is level difference $x := x_1 - x_2$, and its output is flowrate variation φ.

The main idea of the GSS is that, at any given instant, two worst-needing-control (WNC) tanks are chosen to form a tank-pair while the other tanks being kept in no control state. So, the sum of the inflows f_1 to f_N can also be kept as a constant. When the system evolves, some other tank or tanks may become the WNC tanks. Based on certain switching policy, the two WNC tanks are dynamically chosen to form the tank-pair, and then all the N tanks can be controlled in a time-sharing manner.

2.1.2 *Reiterative Difference Control Technique*

The difference control technique (DCT) has been proposed to dynamically distribute the fluid among the passes to maintain the uniformity of the stream temperatures and applied to a furnace with four passes successfully (Wang & Zheng, 2005). The principle of the DCT is that the temperature difference of the two passes, *TD*, is controlled to be zero by a controller C_{12}. The output of controller C_{12} is the flowrate variation. This variation is added to the pass whose outlet temperature is high and at the same time subtracted from the one whose outlet temperature is low. Thus, the sum of the two flowrates is always a constant, and the two passes can be controlled using one controller.

Based on the two-passes temperatures control, the difference control technique can be reiteratively employed to control more passes. Take a system with four passes as an example (Wang & Zheng, 2005). The outlet temperature difference of Pass 1 and Pass 2 can be controlled using a controller C_{12}. Similarly for Pass 3 and Pass 4 using another controller C_{34}. Thus, the difference between the two outlet temperatures of Pass 1 and Pass 2 is reduced to zero, and similarly for Pass 3 and Pass 4. However, the outlet temperature of Pass 1 (or Pass 2) may differ from that of Pass 3 (or Pass 4) as subsystems S_{12} and S_{34}, which consist of Pass 1 and Pass 2, Pass 3 and Pass 4, respectively, are independent. So, a third controller is needed to control the subsystems S_{12} and S_{34}.

Thus, for a system with N the parallel passes, there requires reiterative employment of the DCT N–1 times. Too many times of iteration makes the DCT technique inconvenient to apply.

2.2 Differences Control Technique

Different from the GSS, the differences control technique (DsCT) is based on the traditional control approaches; and comparing to the rDCT technique, the DsCT technique processes all the parallel entities as a whole, not needing the reiterative employment of the DCT technique.

The principle of the DsCT is illustrated in Fig. 3, where x_1, x_2, …, and x_N represent the states of the N parallel entities. They are averaged to x_{avrg}, and the difference of the x_{avrg} and x_i is called the *token* of entity i, that is,

$$x_{avrg} = \frac{1}{N}\sum_{i=1}^{N} x_i \text{ , and}$$

$$token_i = x_i - x_{avrg} \text{ , } i = 1, 2, \ldots, N.$$

Thus, there are N tokens, labeled as $token_1$, $token_2$, …, and $token_N$ respectively, in the system, and they satisfy

$$\sum_{i=1}^{N} token_i = \sum_{i=1}^{N} x_i - N \cdot x_{avrg} = 0 . \tag{2}$$

In Fig. 3, the φ_i represents the load deviation from l_{si}, the base value of the load on the ith entity. So, the load dispatched to the ith entity, l_i, is given by

$$l_i = l_{si} + \varphi_i , \qquad i = 1, 2, \ldots, N. \tag{3}$$

It is through controlling of the load deviation, φ_i, that the load is balanced. Each entity has its own independent controller, the input of which is its token, and the output of which is the load deviation φ_i from the base value.

Considering that $l_i = l_{si}$, $i = 1, 2, \ldots, N$, when the outputs of the N controllers are all the zero, i.e., φ_i=0, $i = 1, 2, \ldots, N$, we can conclude from (1) that the l_{s1} to l_{sN} should satisfy

$$\sum_{i=1}^{N} l_{si} = L_T . \tag{4}$$

In order to make (1), (3), and (4) hold simultaneously, a condition that the sum of the N load deviations is equal to zero must be satisfied. The following theorem indicates that such condition can be conveniently satisfied.

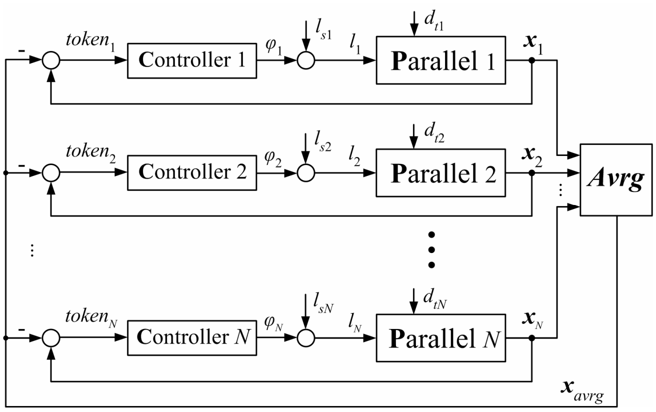

Fig. 3. The principle diagram of the DsCT technique, where the load dispatching is implemented through regulating the load deviation φ_i from the base value l_{si}.

Theorem 2.2.1: The sum of the N load deviations is always equal to zero, if all the N controllers are identical.

Proof: Please see (Wang & Zheng, 2007). ■

With the DsCT scheme, the load-balancing problem of the system with N parallel entities is transformed to N independent single-loop control problem, which successfully solves the load-sharing problem (Eitelberg, 1999) and gives great convenience to the system analysis and controller design.

The next issue needed addressing is the system balanceability, which answers the question of, with differences control scheme, whether there must exist some PID controller that could drive the N states of the concerned system to be identical.

Definition 2.2.1: A differences control system is said to be balanceable, if, for a given system with multiple parallel entities, there must exist some PID controller such that

$$\lim_{t\to\infty} \max_{i,j\in\{1,\ldots,N\}} \left| x_i(t) - x_j(t) \right| = 0 , \tag{5}$$

where $x_k(t)$ is the state of the kth entity at time t, $k = 1, 2, \ldots, N$, and $|\bullet|$ denotes the absolute value of $\bullet$.

Remark 1: Equation (5) is equivalent to that the system has N identical states;

Remark 2: In fact, Equation (5) gives a strict definition for the system to be balanceable. In practice, the zero in the right side of (5) can be replaced by a given positive number, say, ε.

Some systems (e.g. Web-server cluster systems) can be approximated as an Integral process, and hence they can be modeled as a multiple-tanks system, whereas for some industrial processes (e.g. oil heating process), they can be modeled as an FOPDT system. The differences control for the multiple-tanks and FOPDT systems are to be discussed respectively in next two sections.

3. DIFFERENCES CONTROL OF N-TANKS SYSTEMS

This section first induces the *N*-tanks systems, then presents their differences control models, and finally analyzes the system balanceability and discusses controller design.

3.1 N-Tanks Systems

An *N*-tanks system is shown in Fig. 4, which can be used to model the Web server clusters (as shown in Fig. 1,) or similar systems. In Fig. 4, the F_T is dispatched to the N servers, and it can be used to represent the total user requests to a Website; The inflows f_1 to f_N represent user requests dispatched to servers 1 to N respectively; x_i, the level of tank i, is viewed as the overstock of the user requests of the server i; and the outflows g_1 to g_N are used to represent the processing rates of the N servers, which can be treated as disturbances. Thus, the levels uniformity of the N tanks can be used to describe the traffic (load) balancing on the N Web servers (Wang & Zheng, 2006).

3.2 Differences Control Model

The schematic diagram of differences control of *N*-tanks system is given in Fig 4, where the levels x_1 to x_N are averaged to x_{avrg}. The difference of x_i and x_{avrg} is defined as the ith tank's *token*, i = 1, 2, …, N. Thus, every tank has its own independent controller, the input and output of which are the tank's token and the flowrate deviation φ respectively.

Let us examine the dynamics of the control system. Denote $Q_i = f_{si} + \varphi_i$ as net inflow of tank i (please note that the outflow g_i is treated as a disturbance). Assume for simplicity that the cross-section areas of the N tanks are all one, then the level of tank i, $x_i(t)$, is given by

$$x_i(t) = x_i(0) + \int_0^t (f_{si} + \varphi_i)dt\,, \quad i = 1, 2, \ldots, N. \tag{6}$$

Without loss of generality, the controller is assumed to be with PID structure, and the differences control model of the system can be obtained as follows.

$$\mathbf{X}'' + p\mathbf{X}' + q\mathbf{X} = \mathbf{R}\,, \tag{7}$$

where $\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$; $p = K_1/(1+K_3)$; $q = K_2/(1+K_3)$; and

$$\mathbf{R} = \{\tfrac{K_2}{1+K_3}[x_{avrg0} + \tfrac{1}{N}\int_0^t F_T dt] + \tfrac{K_1}{1+K_3}\tfrac{1}{N}F_T + \tfrac{1}{N}F_T{}'\}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}.$$

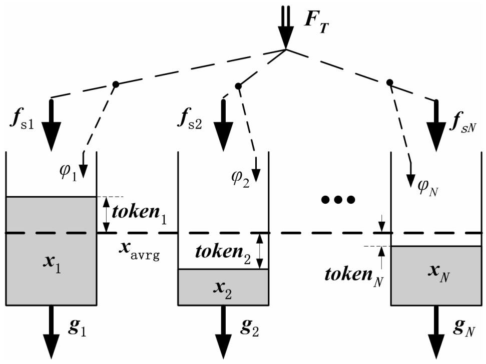

Fig. 4. Schematic diagram of differences control of *N*-tanks system. Based on each tank's token, the total inflow F_T is dynamically dispatched to the N tanks.

3.3 Balanceability Analysis and Controller Design

It is known according to (5) that the differences control system is balanceable if the difference of any two levels tends to be zero when time t approaches to infinity.

Let us consider the difference of x_i and x_j, i, j = 1, 2, …, N. Denote $x_{ij}(t) := x_i(t) – x_j(t)$. From (7) we have

$$\ddot{x}_{ij}(t) + p\dot{x}_{ij}(t) + qx_{ij}(t) = 0\,, \tag{8}$$

where i, j = 1, 2, …, N; p and q are as in (7).

Theorem 3.1: An *N*-tanks system with differences control scheme is balanceable; and the control system may have N identical levels if the parameters of a PID controller satisfy that $K_1 > 0$, $K_2 > 0$, and $K_3 \geq 0$.

Proof: Due to page limit, the proof is omitted here. ■

4. DIFFERENCES CONTROL OF FOPDT SYSTEMS

Section 3 has discussed the *N*-tanks model, which can be used to resolve the load-balancing problems of some systems whose parallel entities can be approximated as an Integral process. However, in some industrial cases where the parallel entities can be approximated as an FOPDT model (Lee et al., 2002, Moon & Lee, 2003, Saffer II *et al.*, 2005). This section analyzes the differences control of FOPDT system in frequency domain.

A block diagram of a system with multiple parallel entities controlled with the differences control scheme is given in Fig. 5, where $G_i(s)$ represents the transfer function of the ith parallel entity which can be approximated as an FOPDT model and $C_i(s)$ is Laplace transform of the ith entity's controller, i = 1, 2, …, N. Denote $\Phi_i(s)$, $L_{si}(s)$, $D_i(s)$, $X_i(s)$, and $X_{avrg}(s)$ as the Laplace transform of the $\varphi_i(t)$, $l_{si}(t)$, $d_{ti}(t)$, $x_i(t)$, and $x_{avrg}(t)$ respectively, i = 1, 2, …, N.

It is known from Fig. 5 that the Laplace transform of $x_i(t)$ can be given by

$$X_i(s)=\frac{G_i(s)}{1+C_i(s)G_i(s)}L_{si}(s)+\frac{C_i(s)G_i(s)}{1+C_i(s)G_i(s)}X_{avrg}(s) + \frac{G_i(s)}{1+C_i(s)G_i(s)}D_i(s)\ , \tag{9}$$

where $i = 1, 2, \ldots, N$.

As in Section 3, let us consider the difference between x_i and x_j, $i, j = 1, 2, \ldots, N$. If any of them tends to be zero, the states of the N parallel entities would be identical. Due to structure symmetry of the system, all the $G_i(s)$ should be the same. Denote $G_1(s)$ to $G_N(s)$ as $G(s)$. In addition, the differences control technique requires identical $C_i(s)$ such that the sum of the N load deviations is equal to zero (see Theorem 2.2.1). Denote $C_i(s)$, $i = 1, 2, \ldots, N$, as $C(s)$. Thus, it can be found from (19) that, for any $i, j = 1, 2, \ldots, N$, the Laplace transform of $x_{ij}(t) = x_i(t) - x_j(t)$ is given by

$$X_{ij}(s)=\frac{G(s)}{1+C(s)G(s)}[D_i(s)-D_j(s)]\,. \tag{10}$$

For a step disturbance $d_{ti}(t)$, and according to final value theorem, if the closed-loop system is stable, i.e., the roots of equation $1+C(s)\cdot G(s) = 0$ are all in left-hand plane, the limit of $x_{ij}(t) = x_i(t) - x_j(t)$ when time t tends to infinity can be given by

$$\lim_{t\to\infty} x_{ij}(t)=(d_i-d_j)\cdot\lim_{s\to 0}\frac{G(s)}{1+C(s)G(s)}\,. \tag{11}$$

where d_i is the magnitude of the disturbance $d_{ti}(t)$.

For the FOPDT model, the limit of $G(s)$ when s tends to be zero is given by

$$\lim_{s\to 0} G(s)=\lim_{s\to 0}\frac{K}{(Ts+1)}e^{-\tau s}=K\,. \tag{12}$$

where $G(s)$ is the transfer function of parallel entities.

It can be found from (11) and (12) that, under the condition that the closed-loop system is stable, the difference of any two entities' states tends to be zero as time t approaches to infinity, if and only if the limit of $C(s)$ when s tends to be zero is infinity. From the above analysis, we have the following conclusion.

Theorem 4.1: For a system with multiple parallel entities whose transfer functions can be approximated as (12), the differences control system is balanceable if there exists some controller such that

- the roots of equation $1+C(s)\cdot G(s) = 0$ are all in left-hand plane; and
- it includes (at least) an Integral term.

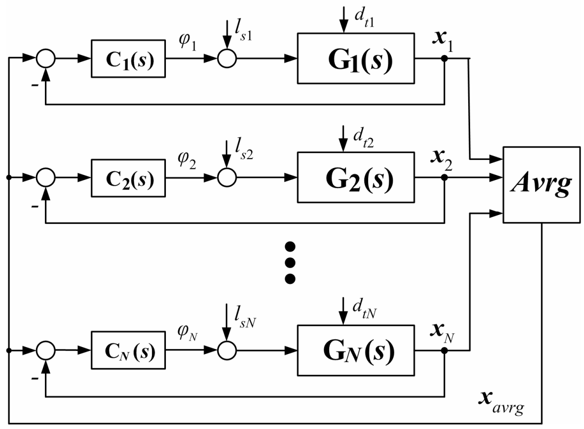

Fig. 5. Block diagram of a system with multiple parallel entities controlled using the differences control scheme.

5. AN APPLICATION EXAMPLE

This section considers a four-passes furnace system, which has been discussed in (Wang & Zheng, 2005), as an example and reports the application results of the DsCT to such example.

5.1 System Balanceability and Controller Design

Based on the data sampled from the furnace system, the transfer function of the four parallel passes can be for simplicity approximated as

$$G(s)=\frac{5}{120s+1}e^{-60s}\,. \tag{13}$$

It can be seen that the limit of (13) when s tends to zero is 5 (a finite constant). Assume for simplicity that the controller is with Proportional-Integral (PI) structure, that is,

$$C(s)=K_1+\frac{K_2}{s}=\frac{K_1 s+K_2}{s}\,, \tag{14}$$

where K_1 and K_2 is Proportional and Integral gain respectively.

For a PI controller, the differences control system is balanceable if there exist some K_1 and K_2 such that the roots of equation $1+C(s)\cdot G(s) = 0$ are all in left-hand plane, or equivalently, the Nyquist plot of $C(s)\cdot G(s)$ does not encircle the point (-1, 0).

The Nyquist plot of $C(s)\cdot G(s)$ with K_1=0.04 and K_2=0.003 does not encircle the point (-1, 0).

5.2 Control Effect

Under the guidance of the balanceability analysis in previous subsection the controller parameters K_1 and K_2 are tuned.

After the whole commissioning job, the following control results are obtained:

- The maximum difference of the four outlet temperatures can be maintained within 2 °C almost all the time, provided that the skewness of the firing status of some burners is not too serious; or else,

- The system would give warning to alert the operator to go to furnace to adjust the fuel valves of relevant burners manually.

Fig. 6 comparatively shows the trends of temperatures and flowrates of the four passes before and after being controlled with differences control scheme. Before the system is switched to the automatic mode the outlet temperature difference is commonly above 10 °C. Once the system is switched to differences control mode, the temperature difference among the four passes decreases gradually with about 1 minute delay.

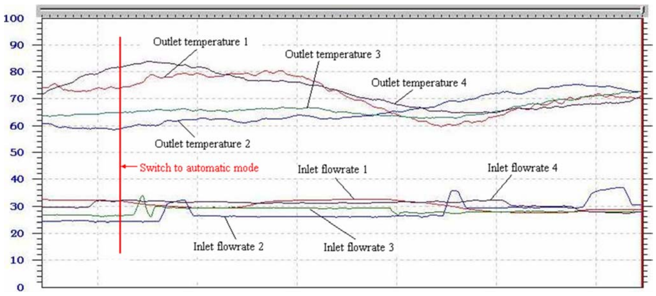

Fig. 6. Control effect of the differences control scheme of a preheat furnace system.

6. CONCLUSIONS AND DISCUSSIONS

In this paper, a class of load-balancing problems has been studied. A differences control technique (DsCT) has been investigated. The load-balancing problems of the systems with the N parallel entities has been transformed to N independent single-loop control problems by using the differences control scheme, which successfully solves the load-sharing problem and gives great convenience to the system analysis and controller design.

The issue of system balanceability has been addressed. It has been shown that, with the DsCT technique, an N-tanks system is balanceable.

The differences control of the FOPDT system has been analyzed in frequency domain. It has been proven that, for an FOPDT system, the differences control system is balanceable if the controller meets some specified conditions.

Finally, an application example of a furnace system has been reported, which indicates that the DsCT technique is effective for solving the load-balancing problems.

It should be pointed out that, throughout the paper, the disturbances in the system are assumed to be step ones. For most cases, these assumptions are reasonable. For example, the processing rate of the user requests of a server in a given Web server clusters should be a nonzero constant or zero (server fails). For some cases where for somewhat reasons the disturbances cannot be treated as step ones, the Theorems 3.1 and 4.1 should be modified accordingly.

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