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Friction Compensation in Flexible Joints Robot with GMS Model: Identification, Control and Experimental Results

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Abstract: In this paper the position control of robot manipulators considering joint flexibilities and friction compensation is presented. For the control purposes a cascade control strategy is presented and the friction compensation is described using the Generalized Maxwell-Slip (GMS) model. The GMS parameters are identified and a friction observer based on this model is proposed and incorporated to the cascade strategy so that the stability and performance can be improved. An experimental setup was constructed to validate the proposed control and friction compensation strategy: a planar two degrees of freedom robot with joint flexibilities prototype. The behavior of the cascade control with GMS model was tested in simulation and it was validated in the experimental setup.

1. INTRODUCTION

In the last years new applications in various fields have motivated many developments in research and specifications of the areas of robotics. To attend the existent demand the robot's structures have been proposed to meet high relationships between the load and robot weights. With these requirements, undesirable effects are introduced, among them the inherent flexibility [Albu-Schäffer and Hirzinger, 2000]. One usual example of these effects is the flexibilities due to the transmission of the harmonic drives. Besides the flexibility, the search for precision in the movements demands a good knowledge of the physical phenomena involved in the system dynamics. Such an important phenomenon in robotic systems is the friction, which has significant influence in manipulator's performance.

Related to the friction compensation, several models have been used to describe the phenomenon dynamics. In Gandhi et al. [2002] an extension to the LuGre model for friction compensation in a harmonic drive is presented. Also, in flexible robots, Jeon and Tomizuka [2005] analyze the consequences of the limit cycles. Recently, the Generalized Maxwell-Slip (GMS) [Lampaert et al., 2003] has been proposed to describe the friction dynamics appropriate for control purposes.

This work proposes a control law to solve the tracking problem, considering the joints flexibilities and the GMS model to describe the friction dynamics. For this end, the Cascade control strategy is adapted to receive the information of the friction model. To apply this approach on the 2 DOF prototype, the parameters are identified. The stability analysis is proved based on the Lyapunov theory. To consolidate the theoretical with practical aspects two experimental cases are performed: steady-state and inversions of velocities.

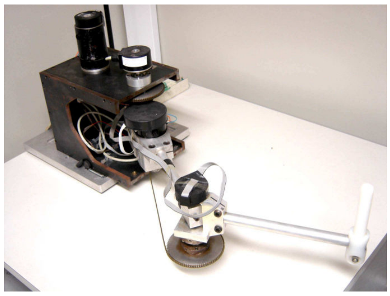

Fig. 1. Prototype robot with flexible joints

The paper is organized as follows. In the Section 2,the experimental setup is presented. The GMS friction model is described in Section 3. The identification methods used to obtain the friction parameters are presented in Section 4. The Cascade controller, the friction observer and the stability proof are stated in Section 5. In Section 6 the results are presented. Finally the main conclusions and perspectives to further works are outlined in Section 7.

2. EXPERIMENTAL SETUP

In order to validate the theoretical results involved in this paper, we used a planar robot prototype with two degrees of freedom (Fig. 1). Between each motor inertia and link inertia are placed transmission gears with ratios and torsional springs. The gravity terms are disregarded in the equations, because the motion is restricted to the horizontal plan. The model used in this work is that proposed by Spong [1987], given by:

$$\begin{aligned} M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + F_f + K(q_1 - q_2) = 0 \\ J\ddot{q}_2 - K(q_1 - q_2) = u \end{aligned} \quad (1)$$

where q is the n-dimensional vector of generalized coordinates, $M(q)$ is a definite positive $n \times n$ matrix and rep-

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resents the inertia of the n rigid links, $C(q,\dot{q})\dot{q}$ represents Coriolis and centrifugal generalized forces, J is a diagonal matrix of constant inertia of the actuators, K denotes the diagonal matrix of joint stiffness coeffcients, F represents the friction torque and u represents the input generalized force from the actuators. The robot model description and its parameters can be consulted in Ramirez et al. [2002].

3. FRICTION MODELS

In this paper the Generalized Maxwell-Slip model will be used [Lampaert et al., 2003] to describe the friction force F_f. This friction model is chosen because it implements a set of complex friction behaviors, such as stick-slip motion, presliding displacement, stribeck effects, frictional lag, transitions between sliding and presliding regimes, while maintaining a simplicity appropriate to control purposes.

3.1 Maxwell-Slip Generalized Model

The GMS model is a qualitatively new formulation of the rate-state approach of the LuGre [Canudas-De-Wit et al., 1995] and the Leuven models [Swevers et al., 2000]. It is based on three properties of friction [Al-Bender et al., 2005]: i) a Stribeck curve for constant velocities, ii) a hysteresis function with nonlocal memory in the presliding regime, and iii) a frictional lag in the sliding regime.

The GMS consists of a parallel connection of different elementary massless block-spring models, having all the same input, velocity $\dot{q}_1$ (see Fig. 2). The model output is the summation of all individual friction forces F_{b_i} acting on the blocks.

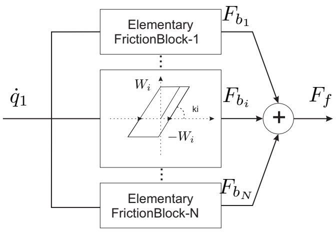

Fig. 2. Parallel connection of N elementary blocks in the GMS model

The adopted rules for friction dynamic behavior are:

- If the elementary block is sticking, the state equation is given by:
$$\frac{dF_{b_i}}{dt} = k_i\dot{q}_1 \tag{2}$$
and the elementary block remains sticking until $|F_{b_i}| > \alpha_i s(\dot{q}_1) = W_i$.
- If the elementary block is slipping, the differential equation is given by:
$$\frac{dF_{b_i}}{dt} = sign(\dot{q}_1)C\left(\alpha_i - \frac{F_{b_i}}{s(\dot{q}_1)}\right) \tag{3}$$
and the elementary block remains slipping until the velocity goes through zero.

In the above rules, k_i is the stiffness of asperities, C is a constant term introduced to directly account for frictional lag dynamics (determines how fast F_{b_i} converges to $\alpha_i s(\dot{q}_1)$) and with the condition of $\sum\alpha_i = 1$ the total friction force for constant velocities will be equals the Stribeck curve, and $s(\dot{q}_1)$ is defined by
$$s(\dot{q}_1) = F_c + (F_s - F_c)e^{-(|\dot{q}_1|/v_s)^2} \tag{4}$$
where F_c, F_s and v_s are, respectively, the Coulomb friction, the static friction and the Stribeck velocity.

The friction force is given as the summation of the outputs of the N elementary state models plus a term that accounts for the viscous friction:
$$F_f = \sum_{i=1}^{N} F_{b_i} + \sigma_2\dot{q}_1(t) \tag{5}$$
where σ_2 is the viscous coefficient.

In the following, we show the approach to obtain the constant dynamic parameters k_i, α_i and C, and the constant static parameters F_s, F_c and v_s included in this model.

4. FRICTION IDENTIFICATION

The identification process consists of three dedicated experiments that emphasize the different behaviors of friction, consequently facilitating the capture of the parameters involved in each behavior. In the first experiment, different signals of constant velocity are imposed on each link. By measuring corresponding friction force, a map friction force versus velocities is constructed. The steady state curve (6), can be identified using LSQCURVEFIT curve fitting technique from MATLAB.
$$F_{f_{ss}} = (F_c + (F_s - F_c)e^{-(|\dot{q}_1|/v_s)^2})sign(\dot{q}_1) + \sigma_2\dot{q}_1 \tag{6}$$
For this experiment it was used a PD controller to guarantee constant velocity at steady-state. The obtained map for link 1 is presented at Fig. 3. This map consists of three tests of 10 different velocities at both directions. Data were also collected in open loop at zero velocity, improving identification of the coefficients F_s and F_c. The result of

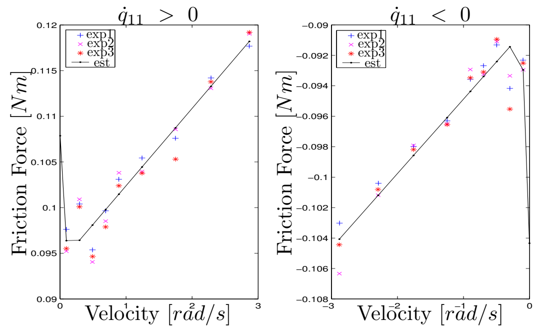

Fig. 3. Stribeck curve - link 1

estimation for both links of static parameters of Stribeck function is presented in table 1.

Table 1. Stribeck curve parameters

	dir.	F_s $[Nm]$	F_c $[Nm]$	v_s $[rad/s]$	σ_2
Link1	$\dot{q}_{11} > 0$	0.10787	0.09393	0.00846	0.06589
	$\dot{q}_{11} < 0$	0.10433	0.08997	0.00489	0.07410
Link2	$\dot{q}_{12} > 0$	0.10697	0.09087	0.07819	0.00374
	$\dot{q}_{12} < 0$	0.09470	0.08363	0.05795	0.00530

The second experiment consist of emphasizing the effects of dynamic parameters k_i and α_i. For this reason, the applied force is slowly ramped up and down with amplitude that keeps the system in presliding regime $F << F_s$. By plotting the friction force as a function of the displacement, the parameters k_i can be estimated using the FMINSEARCH function of MATLAB software, and α_i is chose to be the same for all elementary blocks ($\alpha_i = 1/N$). Fig. 4 illustrates the results obtained for link 1. The number of elementary blocks is defined by a trade-off between processing complexity and accuracy. For this case, it is used $N = 6$ to describe the hysteresis function. The parameters k_i obtained from the estimation are presented in table 2.

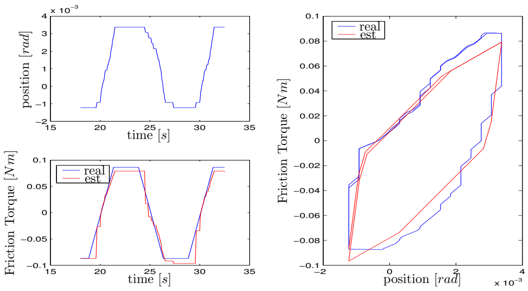

Fig. 4. Identification of parameters k_{1i}

Table 2. Model parameters k_i

	i	1	2	3	4	5	6
Link1	k_{1i}	2.26	5.93	6.95	64.24	11.45	111.41
Link2	k_{2i}	0.12	16.08	19.81	75.86	138.10	150.58

As the third and last experiment the sliding regime is considered. It consist in imposing to the link a non-steady-state without inversions of velocities. In this case, the friction dynamics can be written as in (7).

$$\frac{dF_f}{dt} = sign(\dot{q}_1)C(1 - \frac{F_f}{s(\dot{q}_1)}) \quad (7)$$

By measuring the velocity, the friction force and the estimated steady-state curve $s(\dot{q}_1)$, the attraction parameter C can be estimated using least square techniques. The result of the identification is presented in table 3.

Table 3. Model parameter C

Parameter	**Link1**	**Link2**
C	1.5	1.0

5. CASCADE CONTROL STRATEGY

The Cascade control strategy is an order reduction methodology based on the interpretation of the robotic system as a connection of two subsystems (Fig. 5): Motor Subsystem and Link Subsystem. Thus, based on this interconnection of subsystem, two controllers are explicity derived. These two subsystems are linked by the elastic torque defined by:

$$u_e = K(q_2 - q_1) \quad (8)$$

The objective here is to design an elastic torque such that the link positions q_1 follows q_{1d}. This torque is called desired elastic torque u_{ed}, and allows the definition of rotor desired position q_{2d} using (8) as

$$q_{2d} = K^{-1}u_{ed} + q_1 \quad (9)$$

being the position error of the motors defined as the following expression: $\tilde{q}_2 = q_2 - q_{2d}$.

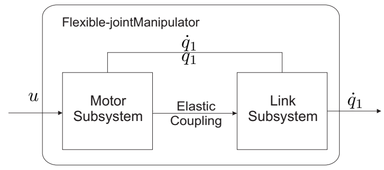

Fig. 5. Interconnection block diagram

This allows us to rewrite expression (1) as

$$\begin{aligned} M(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) + F_f &= u_{ed} + K\tilde{q}_2 \\ J\ddot{q}_2 - K(q_1 - q_2) &= u \end{aligned} \quad (10)$$

By this way, we have now two cascade subsystems which the input of them are u and q_2. The motor dynamics define the unforced subsystem, which drives the link dynamics via q_2. So, the problem is to make q_2 converge to q_{2d} that, applied as input to the link dynamics will drive q_1 towards q_{1d}.

5.1 *Tracking Control of Link Subsystem*

Based on the solution with passivity concepts proposed by Slotine and Li [1991] for rigid robots and including the friction compensation, a control law for link dynamics is defined as

$$u_{ed} = M(q_1)\ddot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + \hat{F}_f - K_{D1}s_1 \quad (11)$$

where $\hat{F}_f$ is the friction force estimation and the position error $\tilde{q}_1$, the "velocity error" s_1, the "reference velocity" $\dot{q}_{1r}$ are respectively defined as

$$\tilde{q}_1 = q_1 - q_{1d} \quad \dot{q}_{1r} = \dot{q}_{1d} - \Lambda_1\tilde{q}_1 \quad s_1 = \dot{q}_1 - \dot{q}_{1r} \quad (12)$$

where K_{D1} and Λ_1 are definite positive diagonal matrix.

Friction Observer: In this paper the GMS model is used for friction compensation. To express the two regimes of the GMS model in a unified framework, we used the form proposed by Nilkhamhang and Sano [2006]. This form is based on the definition of a indicator function $\chi[X]$ of event X as:

$$\chi[X] = \begin{cases} 1 \; if \; X \; is \; true \\ 0 \; if \; X \; is \; false \end{cases} \quad (13)$$

Then the friction force can be written as:

$$F_f = \sum_{i=1}^{N}[\chi_{i,stick}F_{b_i,stick} + \chi_{i,slip}F_{b_i,slip}] + \sigma_2\dot{q}_1(t) \quad (14)$$

where

$\chi_{i,stick} = \chi[F_{b_i}$ is sticking] and $\chi_{i,slip} = \chi[F_{b_i}$ is slipping]

Here $\chi_{i,stick}$ and $\chi_{i,slip}$ are mutually exclusive events. That is, each elementary block must either be sticking or slipping, but cannot be both, at any given time. To construct the observer, (14) is rewritten as a function of estimated parameters $\hat{\chi}_{i,stick}$ and $\hat{\chi}_{i,slip}$.

$$F_f = \sum_{i=1}^{N}[\hat{\chi}_{i,stick}F_{i,stick} + \hat{\chi}_{i,slip}F_{i,slip} + d_i] + \sigma_2\dot{q}_1(t) \quad (15)$$

where

$$d_i = (\chi_{i,stick} - \hat{\chi}_{i,stick})F_{i,stick} + (\chi_{i,slip} - \hat{\chi}_{i,slip})F_{i,slip}$$

In this case d_i is a disturbance term that arises from switching error between the true GMS and the friction observer. The boundedness of d_i can be showed as in Nilkhamhang and Sano [2006] through the lemma 1.

Lemma 1. The unparameterized disturbance d_i is bounded for any $t \geq 0$ if the true and the estimated parameters are bounded.

Proof. There are two main cases to consider:

- If $\chi_{i,stick} = 1$ and $\chi_{i,slip} = 0$, then
$$d_i = \begin{cases} 0 & \text{Se } \hat{\chi}_{i,stick} = 1,\ \hat{\chi}_{i,slip} = 0 \\ d_{i,stick} & \text{Se } \hat{\chi}_{i,stick} = 0,\ \hat{\chi}_{i,slip} = 1 \end{cases} \quad (16)$$
$$d_{i,stick} = F_{i,stick} - F_{i,slip}$$
- If $\chi_{i,stick} = 0$ and $\chi_{i,slip} = 1$, then
$$d_i = \begin{cases} d_{i,slip} & \text{Se } \hat{\chi}_{i,stick} = 1,\ \hat{\chi}_{i,slip} = 0 \\ 0 & \text{Se } \hat{\chi}_{i,stick} = 0,\ \hat{\chi}_{i,slip} = 1 \end{cases} \quad (17)$$
$$d_{i,slip} = -F_{i,stick} + F_{i,slip}$$

So, if the true and estimated parameters of the system are bounded, then the disturbance d_i is also bounded. □

By this way, we propose the observer expressed by
$$\hat{F}_f = \sum_{i=1}^{N} [\hat{\chi}_{i,stick} \hat{F}_{b_i,stick} + \hat{\chi}_{i,slip} \hat{F}_{b_i,slip}] + \sigma_2 \dot{q}_1(t) \quad (18)$$
with the following rules, which are based on previous model rules with addicional terms to guarantee the closed-loop stability:

- If the elementary block is sticking, the state equation is given by:
$$\frac{d\hat{F}_{b_i}}{dt} = k_i \dot{q}_1 \quad (19)$$
and the elementary block remains sticking until $|F_{b_i}| > s(\dot{q}_1)(\alpha_i - (k_e/C)s_1)$.
- If the elementary block is slipping, the differential equation is given by:
$$\frac{d\hat{F}_{b_i}}{dt} = C\left(\alpha_i r(\dot{q}_1) - \frac{F_{b_i}}{s(\dot{q}_1)}\right) - k_e s_1 \quad (20)$$
and the elementary block remains slipping until the velocity goes through zero.

Eq. (20) introduces the function $r(\dot{q}_1)$ to allow the computation of the first and second derivatives of F_f. In this way, the function $sign(\dot{q}_1)$ is smoothed by a function $r(\dot{q}_1)$ (like $r(\dot{q}_1) = tanh(k_v \dot{q}_1)$, where k_v is a positive constant, for example). Introducing the function $r(\dot{q}_1)$, the residual difference $\Delta(\dot{q}_1)$ is defined as
$$\Delta(\dot{q}_1) = sign(\dot{q}_1) - r(\dot{q}_1) \quad (21)$$
With the friction observer defined, the closed-loop dynamics of the link subsystem, first equation of (10), with the control law expressed by (11) result in
$$M(q_1)\dot{s}_1 + C(q_1, \dot{q}_1)s_1 + \tilde{F}_f + K_{D1}s_1 = K\tilde{q}_2 \quad (22)$$
where the friction estimation error $\tilde{F}_f(\dot{q}_1) = F_f(\dot{q}_1) - \hat{F}_f(\dot{q}_1)$ is given by
$$\tilde{F}_f(\dot{q}_1) = \sum_{i=1}^{N} [\hat{\chi}_{i,stick} \tilde{F}_{i,stick} + \hat{\chi}_{i,slip} \tilde{F}_{i,slip} + d_i] \quad (23)$$
Remark 2. Considering the parameters as know the derivative estimation friction in stick is zero $\dot{\tilde{F}}_{i,stick} = 0$. *That can be showed by integrating the state equations* (2) *and* (19) *and evaluating the estimation error as in* (24).
$$F_{b_i,stick} = \int_0^t k_i \dot{q}_1\, dt + F_{b_i,slip} \ ; \ \hat{F}_{b_i,stick} = \int_0^t k_i \dot{q}_1\, dt + \hat{F}_{b_i,slip}$$
$$\tilde{F}_{b_i,stick} = F_{b_i,slip}(t_{ts}) - \hat{F}_{b_i,slip}(t_{ts}) \quad (24)$$
where t_{ts} is the transition between regimes at the instant time. Then, we consider that the estimation error in the stick regime will be equal to estimation error in the slip regime at transition moment. This is treated as another perturbation to the controller.

To incorporate the friction force of each link in the same equation, the friction variables are written in a matrix equation form. By this way, the following vector are defined based on (18):
$$X_{j,stick} = \begin{bmatrix} \hat{\chi}_{1,stick} \\ \vdots \\ \hat{\chi}_{N_j,stick} \end{bmatrix} \quad X_{j,slip} = \begin{bmatrix} \hat{\chi}_{1,slip} \\ \vdots \\ \hat{\chi}_{N_j,slip} \end{bmatrix}$$
$$F_{j,stick} = \begin{bmatrix} F_{b_1,stick} \\ \vdots \\ F_{b_{N_j},stick} \end{bmatrix} \quad F_{j,slip} = \begin{bmatrix} F_{b_1,slip} \\ \vdots \\ F_{b_{N_j},slip} \end{bmatrix}$$
for $i = 1, ..., N_j$, where M_j is the total elemantary blocks and $j = 1, ..., n$, where n is the joints numbers. Therefore, the estimation friction force error in the stick and slip regimes can be defined as
$$\tilde{\mathbf{F}}_{stick} = \begin{bmatrix} {X_{1,stick}}^T \tilde{F}_{1,stick} \\ \vdots \\ {X_{n,stick}}^T \tilde{F}_{n,stick} \end{bmatrix} \ ; \ \tilde{\mathbf{F}}_{slip} = \begin{bmatrix} {X_{1,slip}}^T \tilde{F}_{1,slip} \\ \vdots \\ {X_{n,slip}}^T \tilde{F}_{n,slip} \end{bmatrix}$$
For stability analysis the following non-negative function is considered
$$V_1 = \frac{1}{2}{s_1}^T M(q_1)s_1 + \frac{1}{2}\tilde{\mathbf{F}}_{slip}^T {k_e}^{-1} \tilde{\mathbf{F}}_{slip} + \frac{1}{2}\tilde{q}_1^T P_1 \tilde{q}_1 \quad (25)$$
Using (22), (23) and the rules (19) and (20), $\dot{V}_1$ is given by
$$\begin{aligned} \dot{V}_1 = &-{s_1}^T K_{D1} s_1 - \tilde{\mathbf{F}}_{slip}^T {k_e}^{-1} \mathbf{C}_s \tilde{\mathbf{F}}_{slip} + \tilde{q}_1^T P_1 \dot{\tilde{q}}_1 \\ &+ {s_1}^T K \tilde{q}_2 - {s_1}^T [\tilde{\mathbf{F}}_{stick} + \mathbf{F}_D] + \tilde{\mathbf{F}}_{slip}^T {k_e}^{-1} \mathbf{C} \mathbf{\Delta}(\dot{q}_1) \end{aligned} \quad (26)$$
where $\mathbf{C}$ and $\mathbf{C}_s$ are, respectivelly, a diagonal matrix of coefficient C_i and $C_i/s_i(\dot{q}_{1i})$ for $i = 1, .., n$, $\mathbf{\Delta}(\dot{q}_1)$ is residual difference vector defined in (21) and $\mathbf{F}_D$ is the vector of disturbance d_i summation given by
$${\mathbf{F}_D}^T = \begin{bmatrix} \sum_{i=1}^{N_1} d_{i1} & \sum_{i=1}^{N_2} d_{i2} & \dots & \sum_{i=1}^{N_n} d_{in} \end{bmatrix}$$

5.2 *Tracking Control of Motor Subsystem*

The problem here is to obtain a control law that makes q_2 to converge to the q_{2d}. To solve this problem, the following control signal is used:
$$u = Ju_0 + K(q_2 - q_1) - K_{D2}s_2 \quad (27)$$
where u_0 is considered as an auxiliary input control, that guarantees the stability of the link's dynamics and contains rigid parameters. K_{D2} is a definite positive diagonal matrix. The variable s_2 has the same meaning of s_1 in (12). So, s_2 is given by
$$s_2 = \dot{\tilde{q}}_2 + \Lambda_2 \tilde{q}_2 \quad (28)$$
where Λ_2 is a definite positive diagonal matrix.

Substituting the input u (27) in (10), a possible choice for u_0 is given by
$$u_0 = \ddot{q}_{2d} - \Lambda_2 \dot{\tilde{q}}_2 \quad (29)$$
thus the closed-loop motor's dynamics can be written as
$$J\dot{s}_2 + K_{D2}s_2 = 0 \quad (30)$$

For stability analysis, the following Lyapunov function is chosen

$$V_2 = (1/2){s_2}^T J s_2 + (1/2)\tilde{q}_2^T P_2 \tilde{q}_2 \qquad (31)$$

where P_2 is a definite positive matrix. The time derivative of (31) along the trajectories of the motor's dynamics is obtained using the closed-loop dynamics (30).

$$\dot{V}_2 = -{s_2}^T K_{D2} s_2 + \tilde{q}_2^T P_2 \dot{\tilde{q}}_2 \qquad (32)$$

5.3 Stability Analysis

Theorem 3. (Stability). When all the system parameters are known, given an initial condition, the controller gains (K_{D1}, Λ_1, K_{D2}, Λ_2, k_e e k_v) can be chosen in order to obtain the convergence of the tracking errors, $\tilde{q}_1$ e $\dot{\tilde{q}}_1$, to a residual set R as $t \to \infty$. The set R depends on the friction characteristics and the controller gains.

Proof. Consider a Lyapunov function $V = V_1 + V_2$, composed by (25) e (31).

$$\begin{array}{c} V = (1/2){s_1}^T M(q_1) s_1 + (1/2)\tilde{\mathbf{F}}_{slip}^T {k_e}^{-1} \tilde{\mathbf{F}}_{slip} \\ +(1/2)\tilde{q}_1^T P_1 \tilde{q}_1 + (1/2){s_2}^T J s_2 + (1/2)\tilde{q}_2^T P_2 \tilde{q}_2 \end{array} \qquad (33)$$

This expression can be written in the following matrix equation form

$$V = (1/2)\rho_1^T N_1 \rho_1 \qquad (34)$$

where the state error vector is defined as

$$\rho_1^T = \left[\, \tilde{q}_1^T \;\; \dot{\tilde{q}}_1{}^T \;\; \tilde{q}_2^T \;\; \dot{\tilde{q}}_2{}^T \;\; \tilde{\mathbf{F}}_{slip}^T \,\right]$$

and the definite positive matrix N_1 is defined as

$$N_1 = \begin{bmatrix} \Lambda_1 M \Lambda_1 + P_1 & 2\Lambda_1 M & 0 & 0 & 0 \\ 2\Lambda_1 M & M & 0 & 0 & 0 \\ 0 & 0 & \Lambda_2 J \Lambda_2 + P_2 & 2\Lambda_2 J & 0 \\ 0 & 0 & 2\Lambda_2 J & J & 0 \\ 0 & 0 & 0 & 0 & k_e^{-1} \end{bmatrix}$$

taking the derivative of V along the trajectories of (22) and (30), we get

$$\begin{array}{c} \dot{V} = -{s_1}^T K_{D1} s_1 - \tilde{\mathbf{F}}_{slip}^T {k_e}^{-1} [\mathbf{C}_s \tilde{\mathbf{F}}_{slip} - \mathbf{C}\boldsymbol{\Delta}(\dot{q}_1)] \\ -{s_2}^T K_{D2} s_2 + \tilde{q}_2^T P_2 \dot{\tilde{q}}_2 - {s_1}^T [\tilde{\mathbf{F}}_{stick} + \mathbf{F}_D - K\tilde{q}_2] \end{array} \qquad (35)$$

with $P_1 = 2\Lambda_1 K_{D1}$ and $P_2 = 2\Lambda_2 K_{D2}$. Using (12) and (28) we obtain the matricial form given by

$$\dot{V} = -\rho_1^T N_2 \rho_1 + \rho_1 D(\rho_1) \qquad (36)$$

where N_2 and $D(\rho_1)$ are defined as:

$$N_2 = \begin{bmatrix} \Lambda_1 K_{D1} \Lambda_1 & 0 & -\Lambda_1 K/2 & 0 & 0 \\ 0 & K_{D1} & -K/2 & 0 & 0 \\ -\Lambda_1 K/2 & -K/2 & \Lambda_2 K_{D2} \Lambda_2 & 0 & 0 \\ 0 & 0 & 0 & K_{D2} & 0 \\ 0 & 0 & 0 & 0 & k_e^{-1}\mathbf{C}_s \end{bmatrix} \qquad (37)$$

$$D(\rho_1) = \left[\, \Lambda_1(\tilde{\mathbf{F}}_{stick} + \mathbf{F}_D) \;\; \tilde{\mathbf{F}}_{stick} + \mathbf{F}_D \;\; 0 \;\; 0 \;\; {k_e}^{-1}\mathbf{C}\boldsymbol{\Delta}(\dot{q}_1) \,\right]^T \qquad (38)$$

From (38), $D(\rho_1)$ is defined as a perturbation due to the switching error $\mathbf{F}_D$, stick regime error $\tilde{\mathbf{F}}_{stick}$ and smoothness error of signal function $\Delta(\dot{q}_1)$. The last term will be zero when the velocity $\dot{q}_1 = 0$ and $|\Delta(\dot{q}_1)| < 1$ when $\dot{q}_1 \neq 0$.

From Gershgorin's theorem [Lewis et al., 1993] some condition must be satisfied to guarantee N_2 as a definite positive matrix. These conditions given some restriction to the adjust of controller gains and are defined as:

$$\Lambda_2 K_{D2} \Lambda_2 > (1/2)K(\Lambda_1 + I_n) \quad K_{D1} > (1/2)K \qquad (39)$$

With the satisfied conditions N_2 results uniformly definite positive, i.e.:

$$N_2 \geq \alpha I_n \qquad (40)$$

where α is a positive constant given by:

$$\alpha = \inf_{t \in [0,T]} \lambda_{min}(N_2) \;\; \forall T \geq 0 \qquad (41)$$

With the assumption (40) and the Rayleigh-Ritz theorem [Lewis et al., 1993], (36) can be written as:

$$\dot{V} \leq -\alpha \|\rho_1\|^2 + \|\rho_1\| \, \|D(\rho_1)\| \qquad (42)$$

From the definition of $D(\rho_1)$ in (38), can be establish a superior limit for $\|D(\rho_1)\| \leq \bar{D}$. Based on (42) the condition to $\dot{V}$ negative is given by:

$$\|\rho_1\| > \bar{D}(\rho_1)/\alpha \qquad (43)$$

With the objective to verify that there is a region where $\dot{V}$ is negative and limited by a constant, we consider the condition (43) and (34). Through (34) and (42) and Rayleigh-Ritz theorem is possible to show that the errors norm $\|\rho_1\|$ tends to a residual set as $t \to \infty$. Therefore, the tracking errors $\tilde{q}_1$ and $\dot{\tilde{q}}_1$ tend to a residual set as $t \to \infty$. According to (43), the residual set will depend on $\bar{D}(\rho_1)$ and α defined in (41).

6. EXPERIMENTAL RESULTS

In order to validate the proposed control approach, experiments were performed with the robot presented in Section 2. The experimental tests are formulated using a sinusoidal and a polynomial desired trajectory. The objective is to verify the robot performance to velocity inversions and steady-state regime. For both cases the control gains are adjusted as follows:

$$\begin{array}{cc} K_{D1} = diag\,\{0.8, 0.8\} & K_{D2} = diag\,\{0.2, 0.22\} \\ \Lambda_1 = diag\,\{3.0, 3.0\} & \Lambda_2 = diag\,\{30, 40\} \end{array}$$

For the experiments (Fig. 6 and 8) the Cascade controller is designed without friction compensation, to show the friction influence on the robot dynamics and the tracking error that appears with the reference trajectories.

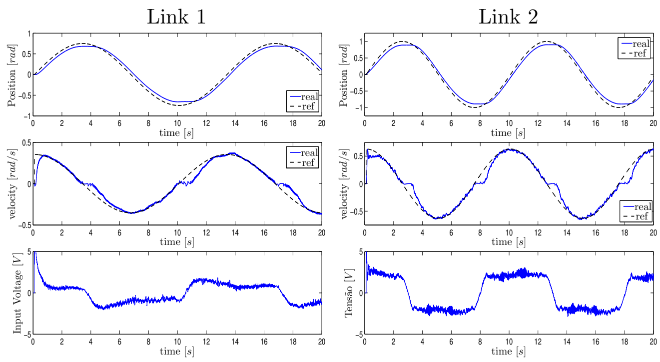

Fig. 6. Sinusoidal trajectory - without compensation

To verify the convergence of the tracking errors and the performance improvement in the Fig. 7 and 9 are presented the trajectories with Cascade control considering the GMS friction observer. In the case of friction compensation the parameter k_e and k_v are respectively 0.01 and 50 for both links.

The tracking errors for both trajectories are presented in Fig. 10. In this results, it is confirmed the importance of the friction compensation for the reduction of the tracking errors.

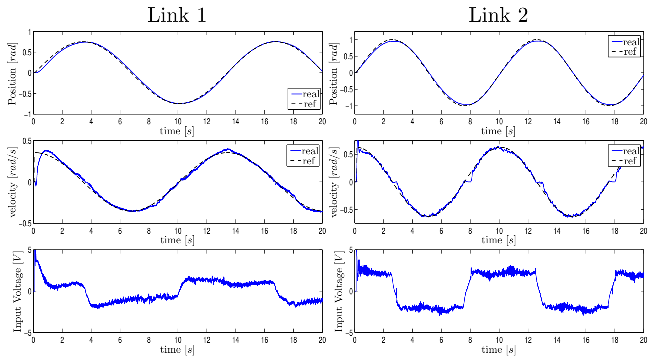

Fig. 7. Sinusoidal trajectory - with compensation

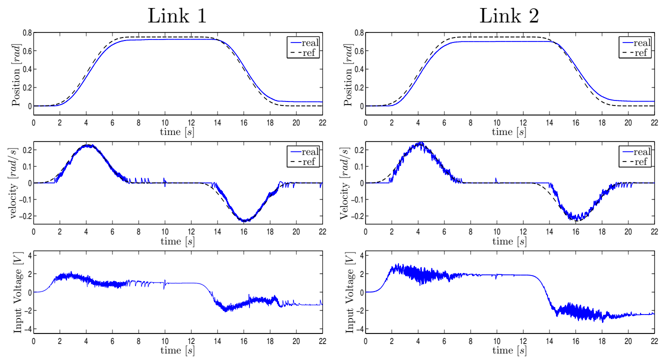

Fig. 8. Polynomial trajectory - without compensation

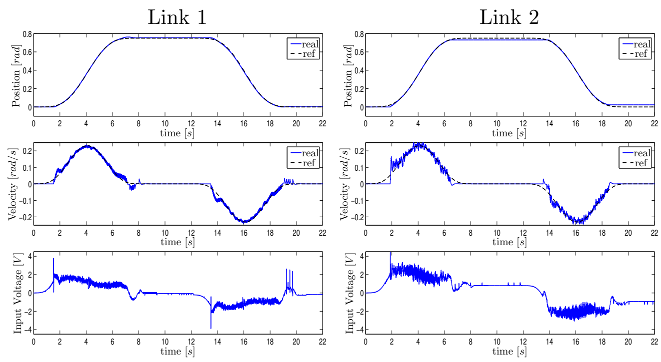

Fig. 9. Polynomial trajectory - with compensation

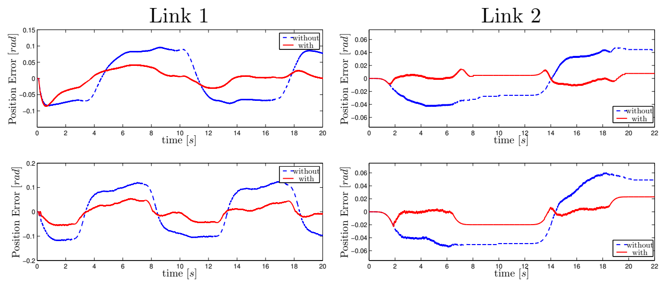

Fig. 10. Position errors

With the values obtained in table 4, it is possible to verify the system with Cascade control considering the GMS friction observer presents better performance than a system without friction consideration.

7. CONCLUSIONS

A Cascade control strategy using the GMS friction model and flexibility considerations was proposed in this work.

Table 4. Mean Square Quadratic Error - MSE

MSE	Sinusoidal Trajectory		Polynomial Trajectory	
	with	without	with	without
Link1	0.0007866	0.0048029	0.0000352	0.0010912
Link2	0.0009879	0.0083391	0.0002063	0.0019605

Through three kinds of experiments the off-line estimation was carried out for each independent joint. The convergence of the tracking errors was theoretically and experimentally demonstrated through two different types of trajectories. In the Cascade control project the system parameters are considered as known. Future research will take account with the comparision to others friction models and the consideration of parametric uncertainties.

REFERENCES

F. Al-Bender, V. Lampaert, and J. Swevers. The generalized Maxwell-slip model: a novel model for friction Simulation and compensation. *IEEE Transactions on Automatic Control*, 50(11):1883–1887, 2005.

A. Albu-Schäffer and G. Hirzinger. State feedback controller for flexible joint robots: A globally stable approach implemented on dlr´s light-weight robots. *In Proceedings IROS Japan,*, CD-ROM, 2000.

C. Canudas-De-Wit, K. J. Astrom, and P. Lischinsky. A new model for control of systems with friction. *IEEE Transactions on Automatic Control*, 40(3):419–425, 1995.

P. S. Gandhi, F. H. Ghorbel, and J. Dabney. Modeling, identification, and compensation of friction in harmonic drives. *Proc. of the 41st IEEE Conf. on Decision and Control*, 2002.

S. Jeon and M. Tomizuka. Limit cycles due to friction forces in flexible joint mechanisms. *Advanced Intelligent Mechatronics. Proceedings, 2005 IEEE/ASME International Conference on*, pages 723–728, 2005.

V. Lampaert, F. Al-Bender, and J. Swevers. A generalized Maxwell-slip friction model appropriate for control purposes. *Proc. of the 2003 Int. Conf. on Physics and Control,Saint-Petersburg,Russia*, pages 1170–1178, 2003.

F. L. Lewis, C. T. Abdallah, and D. M. Dawson. *Control of Robot Manipulators*. McMillan Publishing Company, New York, 1993.

I. Nilkhamhang and A. Sano. Adaptive compensation of a linearly-parameterized gms friction model with parameter projection. *45th IEEE Conference on Decision and Control*, pages 6271–6276, 2006.

A. R. G. Ramirez, E. R. De Pieri, and R. Kinceler. Protótipo de robô planar para o estudo e controle da flexibilidade nas transmissões. *Congresso Brasileiro de Automatica*, Setembro:2138–2143, 2002.

J. J. Slotine and W. Li. *Applied Nonlinear Control*. Prentice Hall International, 1991.

M. Spong. Modelling and control of elastic joint robots. *ASME, Journal of Dyn. Syst. Meas. and Control*, 109: 310–319, 1987.

J. Swevers, F. Al-Bender, C. Ganseman, and T. Projogo. An integrated friction model struture with improved presliding behavior for accurate friction compensation. *IEEE Transactions on Automatic Control*, 45(4):675–686, 2000.