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Saturated LMI Control of Hysteretic Base-Isolated Structures ⋆

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Abstract: The main objective of applying active control forces to base-isolated structures is to protect them in the event of an earthquake. An LMI-based control design is proposed to attenuate seismic disturbances in base-isolated structures under saturation actuators. Using a simplified model of the structure system, a control algorithm design is offered. Performance evaluation of the controller is carried out in a simplified model version of a benchmark building system, which is recognized as a state-of-the-art model for numerical experiments of structures under seismic perturbations. Experimental results show that the proposed algorithm is robust with respect to model and seismic perturbations. Finally, the performance indices show that the proposed controller behaves satisfactorily and with a reasonable control effort. *Copyright© 2008 IFAC*

1. INTRODUCTION

For the purpose of maintaining the seismic response of structures within safety, service and comfort limits, the combination of passive base isolators and feedback controllers (applying forces to the base) has been proposed in recent years. Some groups have proposed active feedback systems, for instance Barbat et al. (2001), Pozo et al. (2006) and Kelly et al. (1987). More recently, semiactive controllers have been proposed in the same setting with the hope of gaining advantage from their easier implementation (see for instance Luo et al. (2001), Ramallo et al. (2002) and Yang and Agrawal (2002)).

The basic concept of base isolation is to make the structure behave like a rigid body through a certain degree of decoupling from the ground motion. In this way it is possible to absorb part of the energy induced by the earthquake, by reducing simultaneously the relative displacements of the structure with respect to the base (damage source) and the absolute accelerations it undergoes (which endanger human comfort and the safety of installations). The idea of adding a feedback control is based on the premise that a control action is to be applied at the base with force magnitudes which are not excessive due to the high flexibility of the isolators. The main benefit of the inclusion of the control is that the assistance of such a force can help prevent large displacements of the base isolator, which could endanger the integrity of the scheme, but it may also introduce an additional effect reducing the interstory drifts, which are already small due to the effect of the isolator. This may be useful, particularly for structures containing sensitive equipment or important resources, such as hospitals, public services, computer facilities, etc.

In this paper, a saturated Linear Matrix Inequality (LMI)-based controller for seismic attenuation is developed and applied to a hysteretic base-isolated eight-storied building, similar to existing building in Los Angeles (California). In recent years, LMI techniques have become quite popular in control design. The main reason for this popularity has been the discovery of interior point methods for convex programming that allow the numerical solution of LMIs in polynomial time. It has been acknowledged that many control problems can be formulated in terms of LMIs (see Boyd et al. (1994), Apkarian et al. (2001), Xie et al. (2000), Oliverira and Peres (2006) and references therein). Moreover, saturation can produce limit cycles even in linear stable systems (Vincent and Grantham, 1997). These limit cycles can induce internal perturbation that can raise instability in structural control systems under seismic perturbation. Because seismic perturbations are unknown, but bounded, there always exists the risk that the actuators reach theirs maximal available force producing saturation. So, the controller has to be well designed to display good performance under seismic perturbation and under saturation effect. To the best knowledge of the authors, almost nothing has been done in this direction applied to structural control systems. However, control of systems with saturation actuators have been extensively studied for several years and it is hard to mention all the work done on it, but we can refer, for instance to Hu and Lin (1983). Furthermore, the maximal available energy of the actuators can be a priori used for control design, which is intuitively correct in civil engineering. The LMI controller design proposed here is based on the results obtained in Nguyen and Jabbari (1999), giving an innovative control algorithm for seismic disturbance attenuation in structures employing saturating actuators. The design is based on a simplified model version of the benchmark building system (Narasimhan et al., 2006), which is recognized as a state-

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of-the-art model for numerical experiments of structures under seismic perturbations. Besides, our controller is robust against model and saturation effect, as can be appreciated in the numerical experiments. Performance of the proposed controller, for seismic attenuation, are evaluated by numerical simulations using seven different earthquakes and eight evaluation criteria, such as the peak base shear, the peak base displacement or the peak absolute floor acceleration.

This paper is structured as follows. The hysteretic base-isolated structure to be controlled is described in Section 2. The saturated LMI-based controller is developed in Section 3. Numerical simulations to analyze the performance of the proposed controller are presented in Section 4. Final comments are given in Section 5.

2. SYSTEM DESCRIPTION

Consider a nonlinear base-isolated building structure as shown in Figure 1. For control design, a dynamic model composed of two coupled subsystems, namely, the main structure or superstructure (S_r) and the base isolation (S_c) is employed:

$$S_r \ : \mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{J}\ddot{x}_g + \bar{\mathbf{C}}\dot{\tilde{\mathbf{r}}} + \bar{\mathbf{K}}\tilde{\mathbf{r}}, \quad (1)$$

$$S_c \ : m_0\ddot{x}_0 + c_0\dot{x}_0 + k_0 x_0 = c_1\dot{r}_1 + k_1 r_1 - \Phi(x_0,t) - m_0\ddot{x}_g + u, \quad (2)$$

where $\ddot{x}_g$ is the absolute ground acceleration, $\mathbf{x} = [x_1, x_2, \ldots, x_8]^{\mathrm{T}} \in \mathbb{R}^8$ represents the horizontal displacements of each floor with respect to the ground. The mass, damping and stiffness of the ith storey is denoted by m_i, c_i and k_i, respectively, $\tilde{\mathbf{r}} = [x_0, \mathbf{r}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^9$ and $\mathbf{r} = [r_1, \ldots, r_8]^{\mathrm{T}} \in \mathbb{R}^8$, represents the horizontal displacements of the i-th floor relative to the $(i-1)$-th storey. The base isolation is described as a single degree of freedom with horizontal displacement x_0. It is assumed to exhibit a linear behavior characterized by mass, damping and stiffness m_0, c_0 and k_0, respectively, plus a nonlinear behavior represented by a hysteretic restoring force $\Phi(x_0,t)$. The matrices $\mathbf{M}, \mathbf{C}, \mathbf{K}, \bar{\mathbf{C}}$ and $\bar{\mathbf{K}}$ of the structure have the following form

$$\mathbf{M} = \mathrm{diag}(m_1, m_2, \ldots, m_8) \in \mathbb{R}^{8\times 8},$$
$$\mathbf{C} = \mathrm{diag}(c_1, c_2, \ldots, c_8) \in \mathbb{R}^{8\times 8},$$
$$\mathbf{K} = \mathrm{diag}(k_1, k_2, \ldots, k_8) \in \mathbb{R}^{8\times 8},$$
$$\mathbf{J} = [1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^8,$$

$$\bar{\mathbf{C}} = (\bar{c}_{ij}) \in \mathbb{R}^{8\times 9}, \ \bar{c}_{ij} = \begin{cases} c_i, & i \le j \\ c_{i+1}, & j-i=2 \\ 0, & \text{otherwise} \end{cases},$$

$$\bar{\mathbf{K}} = (\bar{k}_{ij}) \in \mathbb{R}^{8\times 9}, \ \bar{k}_{ij} = \begin{cases} k_i, & i \le j \\ k_{i+1}, & j-i=2 \\ 0, & \text{otherwise} \end{cases}.$$

The restoring force Φ can be represented by the Bouc-Wen model (Ikhouane et al., 2005; Ikhouane and Rodellar, 2007) in the following form:

$$\Phi(x_0,t) = \alpha K x_0(t) + (1-\alpha) D K z(t) \quad (3)$$

$$\dot{z} = D^{-1}\left(\bar{A}\dot{x}_0 - \beta|\dot{x}_0||\dot{z}|^{n-1}z - \lambda\dot{x}_0|z|^n\right) \quad (4)$$

where $\Phi(x_0,t)$ can be considered as the superposition of an elastic component $\alpha K x_0$ and a hysteretic component $(1-\alpha) DKz(t)$, in which the yield constant displacement is $D > 0$ and $\alpha \in (0,1)$ is the post- to pre-yielding stiffness ratio. $n \ge 1$ is a scalar that governs the smoothness of the transition from elastic to plastic response and $K > 0$.

Finally, u is the control force supplied by an appropriate actuator.

It is well accepted that the movement of the superstructure S_r is very close to the one of a rigid body due to the base isolation (Skinner et al., 1992). Then it is reasonable to assume that the interstory motion of the building will be much smaller than the relative motion of the base (Luo et al., 2001). Consequently, the following simplified equation of motion of the base can be used in the subsequent controller design:

$$\tilde{S}_c \ : m_0\ddot{x}_0 + c_0\dot{x}_0 + k_0 x_0 = -\Phi(x_0,t) - m_0\ddot{x}_g + u. \quad (5)$$

The feasibility of this simplification is justified in a more detailed way in Luo et al. (2001) and Pozo et al. (2007).

Equation (5) together with (3) and (4) can be expressed in matrix form as:

$$\begin{bmatrix}\dot{x}_0 \\ \ddot{x}_0\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_0+\alpha K}{m_0} & -\dfrac{c_0}{m_0}\end{bmatrix}\begin{bmatrix} x_0 \\ \dot{x}_0\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\dfrac{(1-\alpha)DK}{m_0} & -1\end{bmatrix}\begin{bmatrix} z \\ \ddot{x}_g\end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m_0}\end{bmatrix} u$$
$$= Ax + B_1 w + B_2 u, \quad (6)$$

where $x = [x_0 \ \dot{x}_0]^{\mathrm{T}}$ is the position and velicity ground displacement. The hysteretic component z and the absolute acceleration $\ddot{x}_g$ defines w.

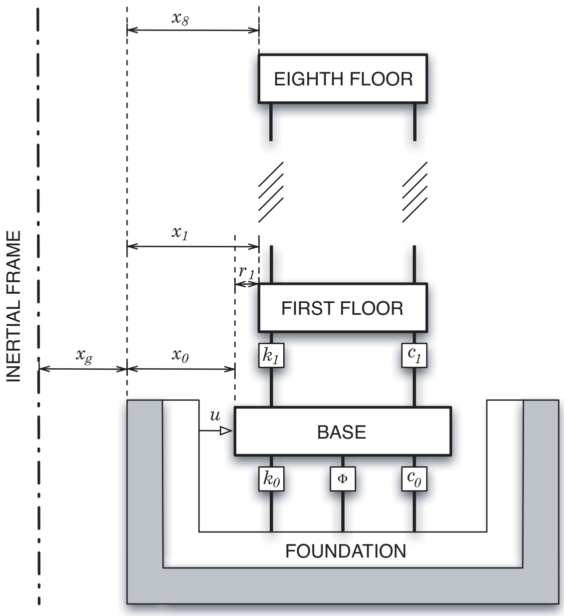

Fig. 1. Base-isolated structure with active control.

3. CONTROL DESIGN

3.1 Controller synthesis

The key of this Section is to obtain a controller through the solution of a Linear Matrix Inequality (LMI) optimization problem.

To design a controller to achieve an optimal performance objective and to guarantee the inputs to remain less than or equal to the saturation limits, we recover the matrix expression of the equation of motion (5) in equation (6) and we define our control objective:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \\ z_\infty &= C_\infty x, \end{aligned} \tag{7}$$

where $x = [x_0 \quad \dot{x}_0]^T \in \mathbb{R}^2$ is the state space vector, composed by the base displacement and velocity; $w \in \mathbb{R}^2$ is the disturbance input; $u \in \mathbb{R}$ is the control input; and $z_\infty \in R$ is the output to be controlled. $A, B_1 \in \mathcal{M}_{2\times 2}(\mathbb{R})$, and $B_2 \in \mathcal{M}_{2\times 1}(\mathbb{R})$, are constant matrices as defined in equation (6). The control objective is to find a state feedback saturated controller that guarantees the $\mathcal{L}_2$ gain of γ_∞ from w to z_∞. Because $w = [z \quad \ddot{x}_g]^T$, the control objective is equivalent to seismic attenuation ($\ddot{x}_g$) and force mitigation of the nonlinear term of the base isolation restoring force (which depends on $z(t)$). In this paper, the H_∞-performance controlled output $z_\infty \in \mathbb{R}$ is defined by the weighted matrix $C_\infty = [0.0001 \quad 1]$. This value of C_∞ is used to give more emphasis to the base velocity rather than to base displacement, because we note that the internal variable $z(t)$, in equation (4), is a function of the velocity.

The control synthesis is based on the result presented in Nguyen and Jabbari (1999), where the level of performance γ_∞ is fixed. In this reference, a high-gain controller design to improve disturbance attenuation for systems with input saturation is presented. It shows that if some set of LMI's are feasible, then there exists a saturate state feedback controller that guarantees the $\mathcal{L}_2$ gain of fixed γ_∞ from w to z_∞. In the present paper, a modification of this result is considered: the level performance γ_∞ is taken as a variable, solving an optimization problem over γ_∞ and obtaining a result less conservative. Consider as a design parameters $\overline{\alpha} > 0$ and $\epsilon > 0$ and define the gain $\gamma^* = \frac{u_{lim}}{(1+\epsilon)w_{\max}} > 0$, where u_{lim} is the maximum value of the saturated control and $w_{\max} = \sqrt{z_{\max} + m\,\ddot{x}_{g-\max}}$ is the maximum of the perturbation.

Proposition 1. Consider system (7). Given $\gamma_* = \frac{u_{\lim}}{w_{\max}} > 0$, assume there exists a positive definite constant matrix $Q > 0$, the scalars $\gamma_\infty > 0$ and $\rho > 0$, for some $\bar{\alpha} > 0$ such that the following LMI's are feasible:

$$\begin{bmatrix} AQ + QA^T - \rho B_2 B_2^T & B_1 & QC_\infty^{\mathrm{T}} \\ B_1^T & -\gamma_\infty & 0 \\ C_\infty Q & 0 & -I \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} AQ + QA^T + \overline{\alpha} Q - \rho B_2 B_2^T & B_1 \\ B_1^T & -\overline{\alpha} \end{bmatrix} \leq 0 \tag{9}$$

Table 1. Parameters and control law

$u_{\lim} = 10m_0$	$\delta = 5 \cdot 10^6$
$u = -\mathrm{sat}(0.05980x_0 + 0.14892\dot{x}_0)$	
$w_{\max} = \sqrt{(8.58m)^2 + 2^2}$	
$\gamma_\infty = 15.8676$	

$$\begin{bmatrix} 4Q & \rho B_2 \\ \rho B_2^T & \gamma_*^2 \end{bmatrix} > 0 \tag{10}$$

Then, the high-gain nonlinear state feedback controller

$$u = -\mathrm{sat}\left(\frac{\delta\,\rho}{2} B_2^T Q^{-1} x\right), \tag{11}$$

where

$$\mathrm{sat}(x) = \begin{cases} u_{\lim}, & x > u_{\lim} \\ x, & |x| \leq u_{\lim} \\ -u_{\lim}, & x < -u_{\lim} \end{cases},$$

guarantees quadratic stability with L_2-gain level of $\sqrt{\gamma_\infty}$. The term δ in (11) is any constant larger than one and $u_{\lim}$ is the desired saturation limit.

Proof. The proof is based on results over LMI theory presented in Boyd et al. (1994), where it is shown that (8)-(9) is a necessary and sufficient condition for the existence of sub-optimal H_∞ state feedback controller, defined here by the specific structure $u = -\frac{\rho}{2} B_2^{\mathrm{T}} Q^{-1}$ ensuring internal stability. Inequality (10) is a necessary and sufficient condition for $\|u\|_\infty \leq u_{lim}$. Then, by Nguyen and Jabbari (1999), equation (11) defines a high-gain nonlinear state feedback controller, and the proposition is proved.

Proposition 1 shows that if there exists a constant matrix $Q > 0$ and a scalar $\rho > 0$, it is possible to optimize the $\mathcal{L}_2$-gain γ_∞ solving a set of LMI's. This is the main difference with Nguyen and Jabbari (1999), where the gain level γ_∞ is given. As in Nguyen and Jabbari (1999), within $x : x^T Q_{-1} x \leq w_{\max}^2$, the control input is guaranteed to be less or equal to the saturation level u_{lim}.

3.2 LMI-based control algorithm

In this section, the steps to solve the disturbance attenuation control problem are discussed. The final goal is to find a feasible solution for the gain matrices and to get a bound for the performance criterion.

`Step 0`: Verify that (A, B_2) is controllable.

`Step 1`: Define the saturation limit $u_{\lim}$ and the largest disturbance amplitude $w_{\max} = \sqrt{z_{\max} + m\ddot{x}_{g-\max}}$. Define matrix C_∞, that is, which state variable has to be controlled.

`Step 2`: Fix $\bar{\alpha} > 0$. The choose of this value depends on each problem. We have fixed $\bar{\alpha} = 0.01$. Then, for this value:

`Step 3`: Solve the LMI system (8)-(10) via LMI optimization[1] on γ_∞, with LMI variables $Q > 0$, $\rho > 0$ and $\gamma_\infty > 0$.

`Step 4`: Is this LMI feasible?

`Step 4.1`: If no, return to step *2* and change the value of α. By step 0, the problem is feasible, but the choice of α is crucial.

[1] The MATLAB LMI toolbox can be used for this purpose.

Table 2. Model coefficients of the base-isolated structure

	mass (kg)	**stiffness** (N/m)	**damping** (Ns/m)
base	3565.7	919422	101439
1st floor	2580	12913000	11363
2nd floor	2247	10431000	10213
3rd floor	2057	7928600	8904
4th floor	2051	5743900	7578
5th floor	2051	3292800	5738
6th floor	2051	1674400	4092
7th floor	2051	496420	2228
8th floor	2051	49620	704

Table 3. Parameters of the hysteresis model in equations (3)-(4)

$\alpha = 0.5$	$\bar{A} = 1$
$K = 61224.49$ N/m	$\beta = 0.5$
$D = 0.0245$ m	$\lambda = 0.5$

`Step 4.2`: If yes, check if the performance level γ_o can be improved: return to step *2* and increase α. If the new γ_n is worst, keep $\gamma_\infty = \gamma_0$. If not, $\gamma_\infty = \gamma_n$.

`Step 5`: With γ_∞ fixed, using Proposition 1, the high-gain nonlinear state feedback controller is defined in (11).

`Step 6`: A simulation is made sweeping through $\delta > 1$.

In Step 4.2, γ_0 represents the L_2 level used to observe the disturbance attenuation performance in the interactive algorithm process. Because in Proposition 1, δ can be any constant greater than one, Step 6 is incorporated to look for, by numerical simulations, a sub-optimal value of δ.

4. NUMERICAL RESULTS

The results of the saturated LMI controller in Table 1 – based on equation (11)– are summarized in Table 4, for the fault normal (FN) component and the fault parallel (FP) components. The evaluation is reported in terms of the performance indices described in the Appendix. The controlled structure –whose parameters are described in Tables 2-3– is simulated for seven earthquake ground accelerations as defined in Narasimhan et al. (2006) (Newhall, Sylmar, El Centro, Rinaldi, Kobe, Ji-Ji and Erzinkan). All the excitations are used at the full intensity for the evaluation of the performance indices. The performance indices larger than 1 indicate that the response of the controlled structure is bigger than that of the uncontrolled structure. These quantities are highlighted in bold.

In this paper, the controllers are assumed to be fully active. These actuators are used to apply the active control forces to the base of the structure. In this control strategy most of the response quantities are reduced substantially from the uncontrolled cases.

The base and structural shears are reduced between 8 and 55% in a majority of earthquakes (except El Centro and Sylmar). The reduction in base displacement is between 17 and 69% in all cases. Reductions in the inter-storey drifts between 4 and 40% are achieved when compared to the uncontrolled case. The floor accelerations are also reduced by 4-44% in a majority of earthquakes (except Newhall (FP-y), Sylmar (FP-y), El Centro and Ji-ji (FP-x)).

The benefit of the active control strategy is the reduction of base displacements (J_3) and shears (J_1, J_2) of up to 69% without increase in drift (J_4) or accelerations (J_5). The reduction of the peak base displacement J_3 of the base-isolated building is one of the most important criteria during strong earthquakes. Moreover, the index J_6 in the proposed scheme reach to small values, which means that the force generated by all control devices with respect to the base shear of the structure is acceptable.

For the base-isolated buildings, superstructure drifts are reduced significantly compared to the corresponding fixed-buildings because of the isolation from the ground motion. Hence, a controller that reduces or does not increase the peak superstructure drift (J_4), while reducing the base displacement significantly (J_3), is desirable for practical applications (Xu et al., 2006). In this respect, the proposed active controller performs well.

4.1 Time-history plots and discussion of the results

Figures 2-3 show the time-history plots of various response quantities for the uncontrolled building, and the building with active controllers using some of the seven earthquakes. More precisely, Figure 2 presents the plots for the base displacement under Erzinkan (FP-y, FN-x), for both the uncontrolled and the controlled situations. The plotted quantities in Figure 3 is the absolute acceleration of the base, also for both the uncontrolled and the controlled situations. The resulting saturated control force under Sylmar is depicted in Figure 4.

Looking at Figure 2, it can be seen that the controlled relative displacement of the base are significantly reduced compared to the uncontrolled case. Figure 3 shows that the reduction in the absolute base acceleration is not as drastic, but it is still significant.

Remark 1. In the results presented in this Section, we have consider *the same* expression for the control law, no matter what earthquake we are considering for simulation. This results can be improved if the parameters $u_{\lim}$ and δ in Table 1 are chosen for each earthquake in an independent way, according to the characteristics of the different seismic zones.

5. CONCLUDING REMARKS

From a structural point of view, the objective of an active control component, as part of a hybrid seismic control system for buildings (and other structures), is to keep the base displacement relative to the ground, the interstory drift and the absolute base acceleration within a reasonable range (which can be affected by the design of the base isolator). In this work, we have proposed and applied a saturated LMI-based controller for seismic attenuation of base-isolated structures. The simulation results illustrate that the base and structural shears, the base displacement, the interstory displacements and the floor accelerations have been significantly reduced by using the proposed saturated controllers as compared with the purely passive isolation scheme. One of the key points of the proposed control scheme is the simplicity of the control law.

Table 4. Numerical results for the proposed saturated LMI controler

Earthquake	Case	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
Newhall	FP-x, FN-y	0.700	0.668	0.697	0.791	0.751	0.174	0.533	0.708
	FP-y, FN-x	0.689	0.717	0.401	0.658	**1.155**	0.499	0.282	0.814
Sylmar	FP-x, FN-y	0.765	0.751	0.637	0.801	0.797	0.288	0.598	0.816
	FP-y, FN-x	0.956	**1.013**	0.702	0.892	**1.067**	0.358	0.497	0.906
El Centro	FP-x, FN-y	0.881	**1.032**	0.307	0.601	**1.362**	0.606	0.230	**1.219**
	FP-y, FN-x	0.878	0.933	0.349	0.818	**1.279**	0.859	0.274	**1.052**
Rinaldi	FP-x, FN-y	0.913	0.917	0.833	0.891	0.852	0.157	0.599	0.827
	FP-y, FN-x	0.714	0.656	0.570	0.744	0.828	0.354	0.413	0.763
Kobe	FP-x, FN-y	0.493	0.454	0.493	0.718	0.559	0.206	0.458	0.506
	FP-y, FN-x	0.705	0.664	0.664	0.716	0.726	0.195	0.407	0.529
Ji-Ji	FP-x, FN-y	**1.003**	0.972	0.776	0.815	**1.231**	0.284	0.534	0.742
	FP-y, FN-x	0.678	0.667	0.564	0.961	0.958	0.467	0.535	0.815
Erzinkan	FP-x, FN-y	0.827	0.806	0.636	0.883	0.824	0.373	0.559	0.877
	FP-y, FN-x	0.520	0.485	0.408	0.767	0.749	0.428	0.311	0.493

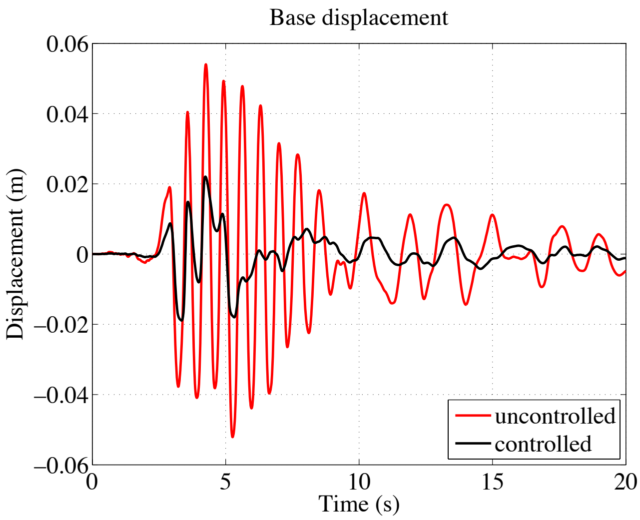

Fig. 2. Time-history response of the isolated building under Erzinkan excitation (FP-y, FN-x). Displacement of the base, for both the uncontrolled (black) and the controlled (red) situations.

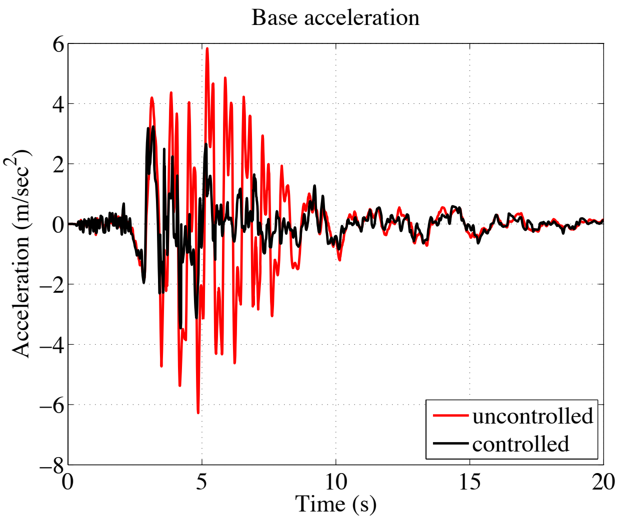

Fig. 3. Time-history response of the isolated building under Erzinkan excitation (FP-y, FN-x). Absolute acceleration of the base, for both the uncontrolled (black) and the controlled (red) situations.

Appendix A. EVALUATION CRITERIA

The following eight evaluation criteria are used for performance evaluation and are based on both maximum and RMS responses of the building. In the following discussion, the term *uncontrolled* refers to the isolation system with no supplemental control devices. These evaluation criteria are reproduced here to assist the reader in comprehending this paper.

(1) Peak base shear (isolation level) in the controlled structure normalized by the corresponding shear in the uncontrolled structure

$$J_1(q) = \frac{\max_t \|V_0(t,q)\|}{\max_t \|\hat{V}_0(t,q)\|}$$

(2) Peak structure shear (at first storey level) in the controlled structure normalized by the corresponding shear in the uncontrolled structure

$$J_2(q) = \frac{\max_t \|V_1(t,q)\|}{\max_t \|\hat{V}_1(t,q)\|}$$

(3) Peak base displacement or isolator deformation in the controlled structure normalized by the corresponding displacement in the uncontrolled structure

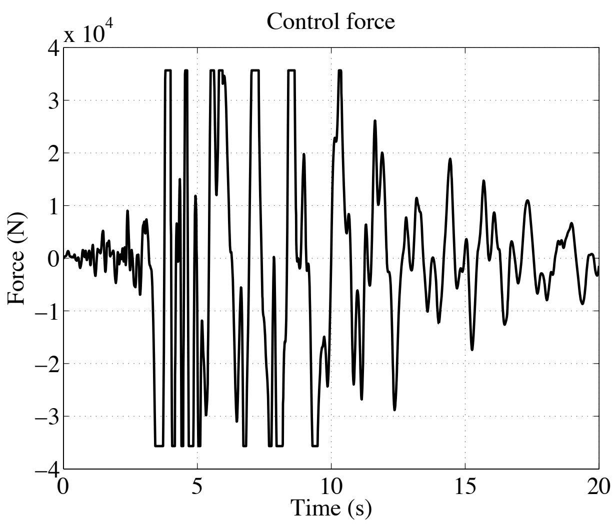

Fig. 4. Saturated LMI-based control force under Sylmar excitation (FP-x, FN-y).

$$J_3(q) = \frac{\max_t \|x_0(t,q)\|}{\max_t \|\hat{x}_0(t,q)\|}$$

(4) Peak inter-storey drift in the controlled structure normalized by the corresponding inter-storey in the uncontrolled structure

$$J_4(q) = \frac{\max_{t,f} \|d_f(t,q)\|}{\max_{t,f} \|\hat{d}_f(t,q)\|}$$

(5) Peak absolute floor acceleration in the controlled structure normalized by the corresponding acceleration in the uncontrolled structure

$$J_5(q) = \frac{\max_{t,f} \|a_f(t,q)\|}{\max_{t,f} \|\hat{a}_f(t,q)\|}$$

(6) Peak force generated by all control devices normalized by the peak base shear in the controlled structure

$$J_6(q) = \frac{\max_t \| \sum_k F_k(t,q)\|}{\max_t \|V_0(t,q)\|}$$

(7) RMS base displacement in the controlled structure normalized by the corresponding RMS base displacement in the uncontrolled structure

$$J_7(q) = \frac{\sqrt{\frac{1}{T_f}\int_0^{T_f} x_0^2 dt}}{\sqrt{\frac{1}{T_f}\int_0^{T_f} \hat{x}_0^2 dt}}$$

(8) RMS absolute floor acceleration in the controlled structure normalized by the corresponding RMS acceleration in the uncontrolled structure

$$J_8(q) = \frac{\sqrt{\frac{1}{T_f}\int_0^{T_f} a^2 dt}}{\sqrt{\frac{1}{T_f}\int_0^{T_f} \hat{a}^2 dt}}$$

where, f = floor number, $1,\dots,8$; q = earthquake number, $1,\dots,7$; t = time, $0 \leq t \leq T_q$.

These eight criteria are calculated to evaluate the controller performance according with international performance criteria in Civil Engineering (Narasimhan et al., 2006).

REFERENCES

P. Apkarian, H.D. Tuan, and J. Bernussou. Continous-time analysis, eigenstructure assignment and H_2 synthesis with enhanced LMI characterizations. *IEEE Transactions on Automatic Control*, 46(12), 1941–1946, 2001.

A. Barbat, J. Rodellar, E. Ryan, and N. Molinares. Active control of nonlinear base-isolated buildings. *ASCE Journal of Engineering Mechanics*, 121(6), 676–684, 1995.

S. Boyd, L.E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM Studies in Applied Mathematics,1994.

T. Hu, and Z. Lin. *Control Systems with Actuator Saturation: Analysis and Design.* Birkhauser, Boston, 2001.

F. Ikhouane, V. Mañosa, and J. Rodellar. Adaptive control of a hysteretic structural system. *Automatica*, 41(2):225–231, 2005.

F. Ikhouane, and J. Rodellar. *Systems with Hysteresis: Analysis, Identification and Control Using the Bouc-Wen Model.* John Wiley & Sons, Inc., 2007.

L. Jansen, and S. Dyke. Semiactive control strategies for MR dampers: comparative study. *ASCE Journal of Engineering Mechanics*, 126(8), 795–803, 2000.

J. Kelly, G. Leitmann, and A. Soldatos. Robust control of base-isolated structures under earthquake excitation. *Journal of Optimization Theory and Applications*, 53(2), 159–180, 1987.

N. Luo, J. Rodellar, J. Vehí, and M. De la Sen. Composite semiactive control of a class of seismically excited structures. *Journal of The Franklin Institute*, 338:225–240, 2001.

S. Narasimhan, S. Nagarajaiah, E.A. Johnson, and H.P. Gavin. Smart base-isolated benchmark building. Part I: problem definition. *Structural Control and Health Monitoring*, 13(2-3):573–588, 2006.

T. Nguyen, and F. Jabbari, Disturbance Attenuation for Systems with Input Saturation: An LMI Approach. *IEEE Transactions on Automatic Control*,44(4), 852–857, 1999.

R.C.L.F. Oliveira, and P.L.D. Peres. LMI conditions for robust stability analysis based on polynomially parameter-dependent Lyapunov functions, *Systems and Control Letters*, 55(1):52–61, 2006.

F. Pozo, F. Ikhouane, G. Pujol, and J. Rodellar. Adaptive backstepping control of hysteretic base-isolated structures. *Journal of Vibration and Control*, 12(4):373–394, 2006.

F. Pozo, P.M. Montserrat, J. Rodellar, and L. Acho. Robust active control of hysteretic base-isolated structures: Application to the benchmark smart base-isolated structures. *Structural Control and Health Monitoring*, under review.

J.C. Ramallo, E.A. Johnson, and B.F. Spencer. Smart base isolation systems. *Journal of Engineering Mechanics - Proceedings of the ASCE*, 128(10):1088–1099, 2002.

R.I. Skinner, G.H. Robinson, and G.H. McVerry. *An Introduction to Seismic Isolation.* John Wiley & Sons, Inc., Chichester, UK, 1992.

T.L. Vincent, and W.J. Grantham. *Nonlinear and Optimal Control Systems.* Wiley, New York, 1997.

S. Xie, L. Xie, Y. Wang and G. Guo, Decentralized control of multimachine power systems with guaranteed performance, *IEE Proceedings on Control Theory and Applications*, 147(3), 355–365, 2000.

Z. Xu, A.K. Agrawal, and J.N. Yang. Semi-active and passive control of the phase I linear base-isolated benchmark building model. *Structural Control and Health Monitoring*, 13(2-3):626–648, 2006.

J.N. Yang, and A.K. Agrawal. Semi-active hybrid control systems for nonlinear buildings against near-field earthquakes. *Engineering Structures*, 24(3):271–280, 2002.