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A case study on multiple controller adaptive control

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ABSTRACT
This document reports work on the application of a Multiple Controller Adaptive Control algorithm based on fictitious reference signals. The algorithm has to select a controller in a small set of candidate controllers. The work shows steps towards a good selection of a cost functional and an analysis, based on recent stability results, shows why a cost function that is usual in other control methods is not suitable. With appropriate cost functional and set of controllers, simulations show the method is able to control a time varying process in the presence of measurement noise and loop delay, even outside the design interval for the parameter variation.

KEY WORDS
Multiple controllers, Adaptive, Switching, Unfalsified Control, Stability.

I. Introduction

This work was motivated by the belief that a multiple controller adaptive control (MCAC) of the unfalsified control type could reach results at least as good as those presented by Fekri *et al* in 2006, [1], in a simpler manner. Here, "an algorithm of the unfalsified control type", means an algorithm that uses a fictitious reference signal for the evaluation of each controller in a set of possible controllers and does not make explicit use of a process model.

The task revealed to be harder than initially expected, but the initial hypothesis is not ruled out. The cost functional to be used must be carefully chosen. With an appropriate candidate controllers set it is possible to obtain reasonably good results in a control problem that is not very easy. Stability is guaranteed but some concerns, and the need for further study on performance still remain.

II. The process

The process to be controlled is as shown in figure 1. The

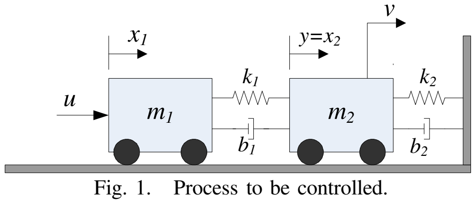

Fig. 1. Process to be controlled.

control input is a force $u(t)$ applied to mass m_1. Initially all parameters will be considered constant in time and their values are in equation (1). Later, in section VIII, k_1 will be considered time varying.

$$m_1 = m_2 = 1, \quad k_2 = 0.15, \quad k_1 = 1, \quad b_1 = b_2 = 0.1 \qquad (1)$$

There is a disturbance force, $v(t)$, acting upon mass m_2. This force is a stationary first-order (colored) stochastic process generated by a low-pass filter, $W_v(s)$, with continuous-time white noise input, $\xi(t)$, with zero mean and a variance equal to 1, that is, $E\{\xi(t)\xi(\tau)\} = \delta(t-\tau)$, as follows:

$$v(s) = W_v(s)\xi(t) = \frac{\alpha}{s+\alpha}\xi(s) \quad , \quad \alpha = 0.1 \qquad (2)$$

The process output, y, is the x_2 cart position. The state and output equations for the process have the standard form with the state space matrices in equations (3) and (4), and the state vector is $x^T(t) = \begin{bmatrix} x_1(t) & x_2(t) & \dot{x}_1(t) & \dot{x}_2(t) \end{bmatrix}$, where x_1 and x_2 are the positions of mass m_1 and m_2, respectively. The first input is the disturbance $v(t)$ and the second input is the control force.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/m_1 & k_1/m_1 & -b_1/m_1 & b_1/m_1 \\ k_1/m_2 & -(k_1+k_2)/m_2 & b_1/m_2 & -(b_1+b_2)/m_2 \end{bmatrix} \qquad (3)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1/m_1 \\ 1/m_2 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \end{bmatrix} \qquad (4)$$

The system is open-loop stable. The open loop response is shown in figure 2.

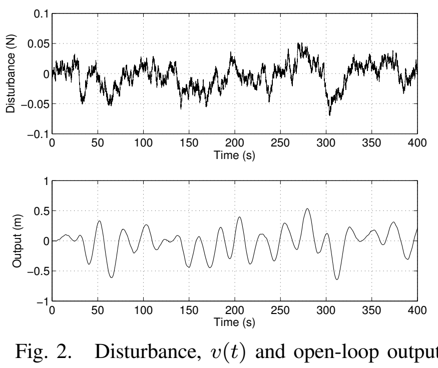

Fig. 2. Disturbance, $v(t)$ and open-loop output.

Control goals are attenuation of the effect of the disturbance on the position of mass m_2 and good reference following. The frequency range of interest will be the low frequencies, until around $0.1 rad \cdot s^{-1}$, where the disturbance

$v(t)$ has most of its power. The following is a study on the ability of a MCAC algorithm to achieve these objectives with a small set of candidate controllers.

III. Three controllers and a cost functional

An important characteristic that a controller must possess to be used in an unfalsified control setting is that a unique fictitious reference signal can be determined, given the controller and past measurement data, [2]. In that case the controller is said to be *causally left invertible*.

Consider controllers with the structure described in equation (5). The symbol y represents the measured output and r is the reference signal. It is the same PID structure used in [3] and it is causally left invertible. The proportional gain is k_P, k_I is the integral gain, k_D is the derivative gain and ε is a small number (our choice was $\epsilon = 1/20$).

$$u = \left(k_P + \frac{k_I}{s}\right)(r - y) - \frac{sk_D}{\varepsilon s + 1} y \qquad (5)$$

Three controller parameter sets were selected by an ad-hoc method and are shown in table I. Examples of closed loop

TABLE I

	K_P	K_I	K_D
controller 1:	1.0	0.01	0.05
controller 2:	0.2	0.01	0.05
controller 3:	0.01	0.01	0.05

output signals, obtained with each controller in the loop, for a particular instance of the disturbance input, are shown in figure 3. All simulation conditions were equal except for the

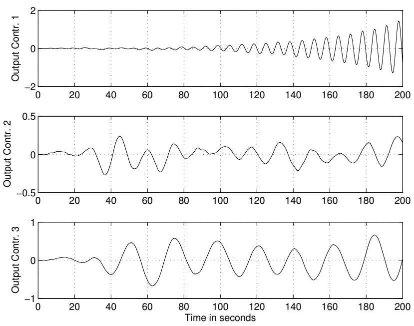

Fig. 3. Output signal with each PID controller.

controller. Observing the output plots in figure 3 it may be apparent that the best controller for disturbance attenuation is controller 2. A possible measure of the quality of each of these controllers is the 2-norm of each output signal in figure 3. That norm confirms controller 2 is better then the others after some simulation time. The first controller results in an unstable closed loop.

Consider that the performance of each controller is evaluated by the cost functional in equation (6). It is selected because it weights control error and control effort and is quite common in optimal control. For the present case it helps showing how some cost functionals can lead to bad behavior in this kind of algorithms, based on fictitious references, enforcing the need for careful cost functional selection.

$$V_1 = \int_0^t [(y(\tau) - r(\tau))^2 + u(\tau)^2] d\tau \qquad (6)$$

The cost for each controller, obtained with that controller in the loop during all the simulation time, with a null reference and the disturbance described in section II,is plotted in figure 4. It can be seen from the plots that, although the controller with lower cost changes over time in the first part of the simulation, after around 120 seconds the best controller becomes controller 2 and that remains for the rest of the time. The cost functional in equation (6) could be thought to be a reasonable cost since it weighs output error and control effort.

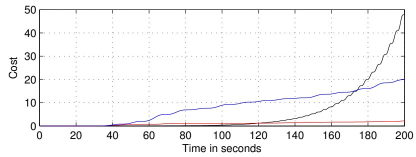

Fig. 4. Cost values for controllers 1 (black), 2 (red) and 3 (blue).

IV. Algorithm for controller selection

The performance of each controller in a candidate controllers set is evaluated using a cost functional. If the controller being evaluated is in the loop one can compute the cost value using input output data and the real reference signal for the system. The controllers that are not in the loop can be evaluated using a fictitious reference signal. That is the reference signal that would have produced the same measured data, had that controller been in the loop.

In a pure unfalsified control algorithm, [2], [3], the evaluation of controllers in a set is used to exclude the ones that have been proven not to be able to meet the performance specifications. Those controllers are said to be falsified because their ability to meet specification has been proven false. In that case, when the controller in the loop is falsified, one of the unfalsified controllers in the set is put in the closed loop.

In another family of different, but closely related algorithms, [4], [5], [6], the evaluation of controllers is used to select, from all controllers in the candidate controllers set, which at each moment, should be put in the loop, because it has a better evaluation. No controller is falsified. The controller switching can be considered periodically, at any moment, or based on the cost values of the controllers.

This last approach is tested here for the control of the process in figure 1 considering as candidate controllers the three controllers presented in section III. The algorithm selects, at each sampling instant, the controller to be put in the loop using the cost functional shown in equation (6).

The fictitious reference signal for each of our three candidate controllers can be computed using equation (7). That is, given the actually measured output, $y(t)$, and control signal, $u(t)$, the only reference signal that could be compatible to those signals, if controller i was in the loop, is $\tilde{r}_i$, as computed by equation (7). Equation (7) can be deduced from equation (5).

$$\tilde{r}_i = y + \frac{s}{K_{P_i} s + K_{I_i}} \left(u + \frac{sK_{D_i}}{\varepsilon s + 1} y\right) \qquad (7)$$

In the present case, using the cost functional in equation (6) the algorithm selects controller 1 as the best controller to put in the loop. It does so very shortly after the simulation is started and independently of which of the three controllers is active in control. This is an unexpected result since, from the plots in the last section, controller 2 is understood as a better controller and controller 1 is in fact a destabilizing controller.

Figure 5 shows the cost of each controller using data collected when controller 1 is in the loop. Using the input and output data from the process with controller 1 in the loop, a fictitious reference signal is computed for each controller and that reference signal and the input and output data from the process are used in the computation of the cost associated with the controller. The cost of controller 3 is always much bigger then the costs of controllers 1 and 2 so only this last two are shown in the plot in figure 5. Controller one has the lower cost and that is why it is selected.

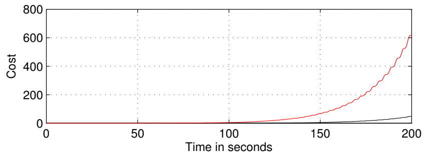

Fig. 5. Cost values for controllers 1 (black) and 2 (red) with data obtained with controller 1 in the loop.

The fact that this controller selection algorithm does not recognize the best controller is a matter of some concern and deserves attention. Although nothing in the theory of unfalsified control guarantees the best controller, in this sense, is always chosen, it was expected, based on previously seen cases, that the algorithm could perform well. The non adequacy of the cost functional just used is the cause for the algorithm failure and is analyzed in the next section.

V. Particular mechanism causing the algorithm failure with the first cost functional

Consider the first cost functional, defined in equation (6). It has two terms inside the square brackets. The second of this terms is in fact equal for all controllers because $u(t)$ is the control signal effectively applied to the process. This second term does not change the value of a controller cost relative to the others.

The first term is the integral of $(y(\tau) - \tilde{r}_i(\tau))^2$, and it uses the same measured output and the particular fictitious reference signal for the controller being evaluated. It may be relevant to look at equation (7) and observe the particular transfer function from y to $\tilde{r}$ for each controller. A Bode plot of each of these transfer functions show, see figure 6, that their gains are close to one for the low frequencies and the phase shift is also small for the low frequencies. It can be observed that the gain and phase are smaller for controller 1, then for controller 2 and then for controller 3. This means the absolute value of $(y - \tilde{r})$ is smaller for controller 1 then for controllers 2 or 3. That is the reason why controller 1 has a lower cost and is selected as the better controller when the cost functional in equation 6 is used.

VI. Analysis

The analysis of the problem above is presented in this section in light of the stability results by Wang *et al* (2005),

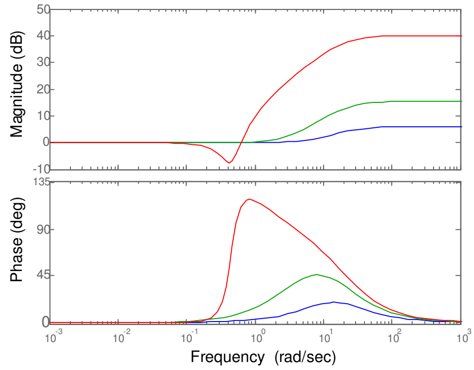

Fig. 6. Bode diagrams for the transfer functions from y to $\tilde{r}$ for each controller, $G_{y,\tilde{r}_1}$ (blue), $G_{y,\tilde{r}_2}$ (green) and $G_{y,\tilde{r}_3}$ (red).

in [7]. In that paper stability of a multiple controller adaptive control (MCAC) of the kind used here is ensured by the following Theorem from [7], where d stands for data, K for controller, and τ for the present instant of time.

Theorem: ([7]) If the following Assumptions 1, 2 and 3 hold, the unfalsified MCAC system is stable.

Assumption 1: The cost function $V(d, K, \tau)$ is $\mathcal{L}_{2e}$-gain-related.

Assumption 2: Each candidate controller $K \in \boldsymbol{K}$ is SCLI.

Assumption 3: The safe adaptive control problem is feasible.

The analysis will proceed with the verification of each of these Assumptions.

Assumption 3 means that there is at least one stabilizing controller in the candidate controllers set that can control the process with the desired performance. In our case study, no specific performance requirement is stated, stability is enough, for now.

In the present problem Assumption 3 can be proven, or refuted, since the process and the controllers are known. (5) and the process transfer function $W_p(s)$. Computing the closed loop poles for each of the three candidate controllers in table I and the process in equation (4) shows only controller 1 leads to unstable poles. One can conclude that Assumption 3 is satisfied, the adaptive control problem is feasible.

Assumption 2 states that all the candidate controllers should be Stable Causally Left Invertible (SCLI). This means that the fictitious reference generator associated with each candidate controller must exist, must be causal and must be incrementally stable. In the present case, the fictitious reference generators are given by equation (7) that can be written as

$$\tilde{r} = \frac{s}{K_P s + K_I} u + \frac{(K_D + \epsilon K_P)s^2 + (K_P + \epsilon K_I)s + K_I}{\epsilon K_P s^2 + (K_P + \epsilon K_I)s + K_I} y \tag{8}$$

Computing the transfer functions in (8) for each controller in table I readily shows that the fictitious reference generators exist and are causal and stable for all the three controllers.

It can be easily shown that if a linear system is stable then it is also incrementally stable. Assumption 2 is true for the present case.

A. Verification of Assumption 1

Assumption 1 is that the cost function $V(d, K, \tau)$ is $\mathcal{L}_{2e}$-gain-related. Recall the following definition of $\mathcal{L}_{2e}$-gain-related from [7].

Definition: Given a cost/candidate controller-set pair $(V, \boldsymbol{K})$, we say that the cost V is $\mathcal{L}_{2e}$-gain-related if for each $d \in \mathcal{L}_{2e}$ and $K \in \boldsymbol{K}$,
1) $V(K, d, \tau)$ is monotone in τ,
2) the fictitious reference signal $\tilde{r}_\tau(K, d) \in \mathcal{L}_{2e}$ exists and,
3) for every $K \in \boldsymbol{K}$ and $d \in \mathcal{L}_{2e}$, $V(K, d, \tau)$ is bounded as τ increases to infinity if, and only if, stability is unfalsified by the input-output pair $(\tilde{r}_\tau(K, d), d)$.

The first condition in the above definition of $\mathcal{L}_{2e}$-gain-related is readily verified since the cost function in equation (6) is the integral of the sum of two squared functions. The second condition was already verified since, by equation (8), the signal $\tilde{r}$ always exist for the selected set of controllers. The verification or invalidation of the third condition will now be studied.

To prove that the third condition holds one has to prove that the following two statements are equivalent:
a) $V(K, d, \tau)$ is limited as τ increases to infinity;
b) stability is unfalsified by the input-output pair $(\tilde{r}_\tau(K, d), d)$.

The definiton of unfalsified stability is as follows, [7]: Given an input-output pair (r, d) of a system, we say that stability of the system is unfalsified by (r, d) if there exist $\beta, \alpha \geq 0$ such that

$$\|d_\tau\| < \beta\|r_\tau\| + \alpha, \forall t \geq 0 \tag{9}$$

holds; otherwise, we say stability of the system is falsified by (r, d).

The norm of a scalar signal, $\|r_\tau\|$, used here is defined as $\|r_\tau\| = \left(\int_0^\tau |r(t)|^2 dt\right)^{\frac{1}{2}}$. Since $d_\tau = \left[\begin{array}{cc} u_\tau & y_\tau \end{array}\right]^T$ is a vector signal with two components one can use the norm definition used by Wang *et al* in [8], which states that, for a vector of functions $x(t) = \left[\begin{array}{cccc} x_1(t) & x_2(t) & \ldots & x_n(t) \end{array}\right]$ the $\mathcal{L}_{2e}$ norm is defined as $\|x\| = \max_{i=1,2,\ldots,n} \|x_i\|$. In the present case, expression (9) can then be written as $\max(\|u_\tau\|, \|y_\tau\|) < \beta\|\tilde{r}_\tau\| + \alpha, \quad \forall\tau \geq 0$.

It is apparent that our first cost functional, in equation (6), is not $\mathcal{L}_{2e}$-gain-related because even when the closed loop system is stable the cost can grow to infinity when τ grows. To see that, consider a periodic input, not identically zero, for which the control signal is also periodic. In that case, even if reference following is perfect, the value of $V_1(K, d, \tau)$ tends to infinity as time grows without, however, stability being falsified by $\tilde{r}((K, d_\tau), d_\tau)$. This proves the selected cost is a candidate for causing the failure of the control algorithm.

VII. Selection of a cost functional

To have stability guaranties we look for a cost detectable cost functional. It seems natural, in view of equation (9), to select such a cost to be of the form $V = \frac{\|d\|}{\|r\|}$, since stability means there are constants $\beta, \alpha \geq 0$ such that

$$\|d\| < \beta\|r\| + \alpha \quad \Rightarrow \quad \frac{\|d\|}{\|r\|} < \beta + \frac{\alpha}{\|r\|} \tag{10}$$

As argued above, $\|d_\tau\| = max(\|y_\tau\|, \|u_\tau\|)$ so the following cost could be suggested

$$V_2(K, d, \tau) = \frac{max(\|y_\tau\|, \|u_\tau\|)}{\|\tilde{r}_\tau\|} \tag{11}$$

This is, by design, a cost function that has the property of cost detectability. However it is not monotone which is one of the conditions for guaranteed stability, present in Assumption 1 above. Cost functional $V_3(K, d, \tau) = max_{[0,\tau]} \frac{max(\|y_\tau\|^2, \|u_\tau\|^2)}{\|\tilde{r}_\tau\|^2}$ guarantees the $\mathcal{L}_{2e}$-gain-related property. With V_3, the algorithm selects a stabilizing controller but performance is not good. Cost functional V_3 could perhaps be expected to generate poor behavior because its numerator is equal for all controllers in the set, it depends only on the data. It is only the denominator, the norm of the fictitious reference, that "decides about" the relative cost of the controllers.

Aiming at a better control behaviour another cost functional, $V_4(K, d, \tau) = max_{[0,\tau]} \frac{\|y(t) - \tilde{r}(t)\|^2}{\|\tilde{r}\|^2}$, was tested. Cost V_4 is an attempt to make the cost explicitly dependent on the reference following qualities of the closed loop. With cost functional V_4 and this set of controllers the algorithm can not be used for disturbance rejection in this system with a null reference.

Other cost functionals, similar to V_4, where tried, without success. There is a need for frequency weighting to get better performance in the frequencies of interest. This can be accomplished with cost functional V_5 in equation (12).

$$V_5(K, d, \tau) = max_{[0,\tau]} \frac{\|y(t) - w_m * \tilde{r}(t)\|^2}{\|\tilde{r}\|^2} \tag{12}$$

In equation (12), w_m is the inverse Laplace transform of a transfer function specifying a desired frequency behavior for the closed loop. This allows for the specification of different performance requirements in different frequencies. In the rest of this document w_m is selected as $w_m(t) = \mathcal{L}^{-1}\{\frac{1}{s+1}\}$.

Cost functional V_5 results in a more robust control scheme then cost functional V_4. It can also be said that with cost functional V_5 the dependence of the cost on the initial instants of simulation becomes less severe. With this cost functional the algorithm can be used for disturbance rejection.

VIII. A set of five controllers

The three controllers set used above is a poor set and the performance of its best controller is not very good as can be concluded with a few simulation runs. This can be confirmed comparing the simulation results shown in figure 3 with results reported in Fekri *et al* (2006), [1]. The weakness of the controllers set just considered and the intent for study of the performance capabilities of the algorithm motivates the use of another set of candidate controllers.

The adaptive control algorithm's performance and robustness should be tested in the presence of measurement noise, parameter variations and delay in the feedback loop. A small set of candidate controllers will be considered. The testing conditions are similar to those described in [1] by Fekri *et al* (2006), as follows. Parameter k_1 changes in the interval $[0.25, 1.75]$. A delay $\tau \leq 0.05s$ affects the loop. An additive measurement noise, $\theta(t)$, affecting the measurement $y = x_2(t)$, independent of $\xi(t)$, is present in the tests. The measurement noise is defined by

$$E\{\theta(t)\} = 0, \quad E\{\theta(t)\theta(\tau)\} = 10^{-6}\delta(t-\tau) \tag{13}$$

Parameter k_1 can change in the interval $[0.25, 1.75]$ giving origin to a continuous set of different models for the process. Particular points in the range of k_1 are selected for the design of controllers, say five points. The interval $[0.25, 1.75]$ is divided in five smaller intervals and the middle point of each of this intervals is selected for the design of a controller. We hope that the algorithm can select, at each moment, an adequate controller, as the model changes in time.

Each interval has a wideness of 0.3 and the middle points of the intervals are $p_1 = 0.40$, $p_2 = 0.7$, $p_3 = 1.0$, $p_4 = 1.3$, $p_5 = 1.6$. Corresponding to each of this points controllers C_{p1} to C_{p5} are designed. The MCAC algorithm with this five controllers will be tested.

A. Controllers design

The requirements on each controller are good disturbance rejection and zero steady state error. The process to be controlled is completely controllable and observable. The design of each controller is done via pole placement. The control structure is as shown in figure 7. In the diagram, K_{pp}

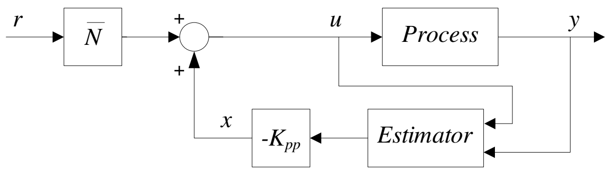

Fig. 7. Control structure.

is a vector of gains for state feedback and $\overline{N}$ is a constant that is computed for zero steady state error using equation (15) after solving equation (14), [9]. In equation (15) the matrices A, b, C and D are the process state space matrices from the control input to the measured output. The control signal is computed as in the second equation in (15).

$$\begin{bmatrix} A & b \\ C & D \end{bmatrix} \times \begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{14}$$

$$\overline{N} = N_u + K_{pp}N_x \quad , \quad u(t) = \overline{N}r - K_{pp}x \tag{15}$$

The state estimator poles and the closed loop poles are selected in the same places for all controllers. The state estimator poles are at -10, -15, -20 and -25 and the closed loop poles are placed at -0.5, $-2 \pm i$, and -5. Considering only the controlled input, $u(t)$, the state estimator is described by equation (16) where b is the second column of matrix B in equation (4).

$$\dot{\hat{x}} = (A - LC)\hat{x}(t) + \begin{bmatrix} b & L \end{bmatrix} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \tag{16}$$

The gain, L, of the estimator is selected using the *place* function of *Matlab* so that the eigenvalues of $(A - LC)$, the estimator poles, are at the chosen locations. According to this design the values of L, K and $\overline{N}$ computed for each controller are shown in tables II and III. Each controller is able to control the process in its respective design interval

TABLE II

CONTROLLER GAINS (K) AND ESTIMATOR GAINS (L)

Controller	Gain vector
K_1 :	$\begin{bmatrix} 19.8616 & 6.2903 & 9.2400 & 71.9041 \end{bmatrix}$
K_2 :	$\begin{bmatrix} 22.3763 & -9.4640 & 9.2000 & 37.2374 \end{bmatrix}$
K_3 :	$\begin{bmatrix} 23.4086 & -13.5699 & 9.2400 & 24.4343 \end{bmatrix}$
K_4 :	$\begin{bmatrix} 23.2839 & -16.5052 & 9.2000 & 16.1606 \end{bmatrix}$
K_5 :	$\begin{bmatrix} 23.1525 & -17.6605 & 9.2000 & 11.4753 \end{bmatrix}$
L_1 :	$\begin{bmatrix} 1180.20 & 69.70 & 185739.75 & 1753.13 \end{bmatrix}^T$
L_2 :	$\begin{bmatrix} 12124.08 & 69.70 & 105383.21 & 1.752.53 \end{bmatrix}^T$
L_3 :	$10^4 \times \begin{bmatrix} 1.1680 & 0.0070 & 7.3241 & 0.1752 \end{bmatrix}^T$
L_4 :	$\begin{bmatrix} 10300.02 & 69.70 & 55933.86 & 1751.33 \end{bmatrix}^T$
L_5 :	$\begin{bmatrix} 9031.76 & 69.70 & 45117.15 & 1750.73 \end{bmatrix}^T$

with good performance and stability. Note however that the controllers designed for lower values of k_1 lead the closed loop to be unstable if k_1 surpasses certain values inside its interval of variation.

TABLE III

REFERENCE GAIN

$\overline{N}_1$	$\overline{N}_2$	$\overline{N}_3$	$\overline{N}_4$	$\overline{N}_5$
31.2500	17.8571	12.5000	9.6154	7.8125

IX. SIMULATION RESULTS

The MCAC algorithm was implemented with the above five candidate controllers for control of the process in figure 1 with the parameter values in equations (1) except for k_1 that was made time-variant. Several simulation experiments where done and the results of two of them are presented next. In both an actuation delay of $0.05s$ and measurement noise, as described in the beginning of section VIII, are present. The sampling time for the measurements is 0.01s and the minimum time between controller switches is 0.25s.

a) Experiment 1: Parameter k_1 changes according to $k_1(t) = 1 - 0.75 \times cos(0.01t)$. The algorithm was able to control the process in a stable way. Disturbance rejection results for a null reference and the disturbance already described in section II can be seen in figure 8. The output peak value a few seconds before $t = 200s$ is -0.4.

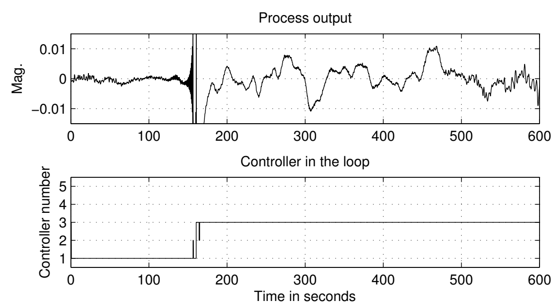

Fig. 8. Output magnitude and controller in the loop (experiment 1).

b) Experiment 2: In this experiment k_1 takes values inside and outside the range of design. The results indicate that reasonably good behavior can be expected if one of the controllers can stabilize and control the process for such values of k_1. Suppose that k_1 changes according to

$k_1(t) = 1.5 - 0.75 \times cos(0.02t)$, that is k_1 changes in the interval $[0.75, 2.25]$ with a frequency of twice that considered before. If k_1 changes this way the algorithm behaves as shown in the plots in figure 9. It can be seen that the MCAC scheme can find a reasonable controller for the process, even outside the initial range of variation of k_1. The lower

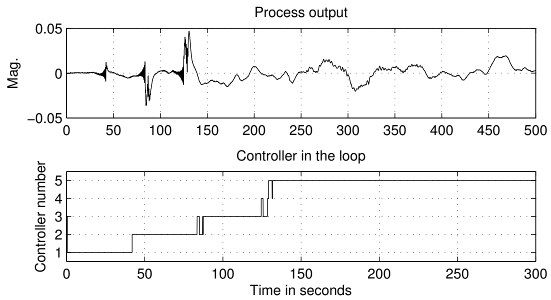

Fig. 9. Output magnitude and controller in the loop (experiment 2).

numbered controllers become unstabilizing as the value of k_1 grows. It can be seen that the algorithm changes from the initial controller 3 to controller 1 because controller 1 has a better performance. After some time, as k_1 grows, that controller becomes a destabilizing one and controllers 2, 3, 4 and 5 are put in the loop. Note however that controller 5 remains in the loop, even in times when other controllers would result in better performance.

X. Discussion

This work enforces the idea that the selection of a cost functional for an MCAC algorithm of the unfalsified control type described must be done with care. It also launches some doubts on the goodness of the basic ideas behind the algorithm for controller selection in this kind of MCAC control schemes.

Although there is a solid logical reason for "pure" unfalsified control to work, [2], [3], proving that some controllers are not able to satisfy some performance criteria and keeping the others, we have not found a good reason for a MCAC of the present type, the one exposed in [7], to be able to guarantee performance, even if good performance controllers are included in the candidate controllers set.

The MCAC switching algorithm, selects the best controller to be put in the loop, according to a cost functional that uses fictitious reference signals. This is similar to the optimal unfalsified control algorithm or best-fit controller in [4], [5], [6]. It seems possible, however, that there in no good reason to believe that a cost computed using a fictitious reference signal is a good indicator of controller performance, although this fictitious signal must be compatible with the data collected in the functioning system.

The best reason for such a belief we have found are the results described in some papers by Safonov and his coworkers showing, by means of some simple examples, that this idea can work, namely in [4] and [5]. In this references however the candidate controller sets were of the model reference adaptive control (MRAC) type and the controller sets have infinite cardinality. We have also obtained good results, in other works, using the same algorithm and the same kind of candidate controller sets to other processes.

If, before selecting the optimal controller, an unfalsification step is done, guaranteeing that the remaining set only contains controllers at a certain unfalsified cost level, then, performance should be guaranteed. This however is not always easy to apply because it may be hard to select an appropriate cost level. The adequate cost functional may also be difficult to select. On the other hand if the system is time varying it may be inappropriate to exclude controllers that latter may be needed.

XI. Conclusions

This multiple controller adaptive algorithm is able to stabilize a time varying process, both in cases of slow parameter variations and in cases when there are jumps in the parameter values, even in the presence of measurement noise and delay in the feedback loop.

Care must be taken in the selection of the cost functional since for stability guarantees it must be *cost detectable* and monotone, as shown by Wang *et al*, [7].

Performance can be compared to that of more complicated algorithms, namely that of the robust multiple model adaptive control (RMMAC) architecture presented in [1]. It should also be emphasized that the present algorithm has the advantage of requiring less assumptions on the process and the signals that may affect it. We are close to confirm the initial hypothesis that this MCAC algorithm can be used to attain results comparable to those of the RMMAC, at least for this process.

Further study is needed on the performance of this type of best-fit control algorithm with finite cardinality controller sets and on ways to improve it.

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