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Some models of dynamic cognitive maps with qualitative scales of factors values

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Abstract: Some models of dynamic cognitive maps with linearly ordered qualitative scales of factors values are considered. Notions of vague values and increments in such scales and operations with them are defined. Main effects of behaviour in these models are described. Sources and forms of decrease of data certainty in these models, the means of monitoring of this phenomenon, limits of the modelling process reliability are defined.

1. INTRODUCTION

The cognitive maps are widely used at the analysis of economic, sociological and sociopolitical problems (Axelrod, 1976; Roberts, 1976; Eden, 1988; Kosko, 1992). The *static* cognitive maps are used for determination of power, sign and paths of mutual influence of factors of the considered situation. The *dynamic* cognitive maps are used for forecasting the development of the considered situation in the model time. The models of dynamic cognitive maps of Roberts (1976) and Kosko (1992) are most widely known.

In these models the mutual influence of factors is defined by the transfer coefficient (the weight of influence) with the sign "+" or "-". The weights of influence are expressed by real numbers, which accuracy is not limited. We shall name therefore such models of cognitive maps *exact*. Exact models have the following flaws.

- Exact values of weights could not be received. These values are appointed by an expert, who can specify them only approximately.
- In exact models can occur the complicated effects of behavior which nature can vary at minor alterations of parameters of the model. These effects mismatch the human intuition. Therefore the events in such system are difficult to explain.

In this connection a problem arises, is it correct to use exact cognitive models with inexact data, to what extent we can rely upon the forecast received on the basis of such models. This problem can be resolved by execution in the model the approximate calculations according to accuracy of initial data. However it creates the problems itself. It is known that inaccuracy of numerical data can only increase during the approximate calculations. Loss of accuracy at long calculations can lead to loss of interest to obtained results.

At the same time the problems, to which decision the cognitive maps are applied (research of "weakly structured ", "soft" systems (Checkland and Scholes, 2003)), basically do not require great accuracy. It is enough only to catch a general tendency of development of the considered situation. A natural approach in this case is to use the qualitative models of cognitive maps. All parameters in such models are expressed in finite qualitative scales. Qualitative models have greater stability in comparison with quantitative models. The complicated, difficultly explainable processes are impossible in qualitative models. For the user the qualitative scales form a natural language, in which he defines a model, sets data for it, and receives all explanation in the process of modelling.

2. QUALITATIVE SCALES OF FACTOR VALUES

Various definitions of qualitative scales are possible. The general feature of such scales is the linear order of the values. Non-numerical character of qualitative scales creates difficulty for their use in the computing process together with quantitative numerical data. Therefore sometimes "algebraized" qualitative scales are used. Such scales represent only names of numerical values, with which arithmetic operations are carried out without any restrictions (Averkin, *et al.*, 2006). The qualitative scales often are considered as fuzzy reflection of some numerical domain. Such scales are known as "linguistic" (Zadeh, 1975).

However there is also a wide class of qualitative scales, which have no fuzzy binding to any numerical domain. The scale of school marks, presented in Table 1, may be considered as a typical example of qualitative scale of this kind. The marks are ordered, but it is not defined exactly how far they are located from each other.

Table 1. Qualitative scale of school marks

Failing	Unsatisfactory	Average	Good	Excellent
F	D	C	B	A
1	2	3	4	5

An attempt to reveal main effects of behavior in some models of qualitative cognitive maps (QCM), based on qualitative scales of similar type is presented below. The values of each factor in QCM are defined in its own qualitative scale. Values from different scales are incomparable. Qualitative scale $S = \{s_1, \ldots, s_{n_S}\}$ is a linearly ordered set of symbolical values s_i. Any symbol can be used for designation of value s_i, and index i in particular. A priori scale S has no metrics. However we consider admissible to use the "natural" integer metrics, in which the *distance* between s_i and s_j is defined as $\rho(s_i, s_j) = |j - i|$. The *increment* from s_i to s_j is defined as $\delta(s_i, s_j) = j - i$, where $\delta \in S^\delta = \{-(n_S - 1), \ldots, 0, \ldots, n_S - 1\}$. The increments can be summarized with each other and with values of scale S, regarding the limited ranges of the scales S and S^δ. Summation of values of the scale S is not allowed.

At practical use of qualitative scales it is difficult to do without *intermediate* values. In the scale of school marks such values can be expressed by modifiers "+" or "-". However, because of absence of metrics, it is impossible to define the exact position of intermediate values in scale S. We define the *intermediate* value **v**, located between two next values s_i and s_{i+1}, as fuzzy sum $\mu_\mathbf{v}(i)/i + \mu_\mathbf{v}(i+1)/(i+1)$, where $\mu_\mathbf{v}(i)$ and $\mu_\mathbf{v}(i+1)$ reflect a measure of similarity of value **v** to the values s_i and s_{i+1} respectively.

2. VAGUE QUALITATUVE VALUES AND INCREMENTS

As a result of decrease of data accuracy in QCM, the *vague* qualitative values can appear. The reason of it is inexactness of definition of QCM or conflicts of influence on a factor from another factors arising in the modelling time. A vague value is characterized not by one value $s_i \in S$, but by some set of such values, each with own degree of certainty.

A *vague value* **v** in scale S is defined as fuzzy sum

$$\mathbf{v} = \sum_{i=1}^{n_S} \mu_\mathbf{v}(i)/i\,,$$

where $\mu_\mathbf{v}(i) \in [0, 1]$ is understood as a degree of certainty, that s_i belongs **v**. The nonzero members of value **v** we name its *components*. The value $\mu_\mathbf{v}(i)$ we name *weight* of the component s_i. Unlike the vague values, we name the values $s_i \in S$ *basic* or *exact*. The value $\mathbf{v}_{ex} = (1, 0, 0.3, 0, 0, 0, 0.5)$ may be considered as an example of a vague value for $n_S = 7$.

A *vague increment* in scale S is defined as fuzzy sum

$$\delta = \sum_{i=-(n_S-1)}^{n_S-1} \mu_\delta(i)/i\,.$$

Although summation of basic values is illegal, the summation of vague values is allowed and is defined by the scheme of fuzzy OR.

$$\mathbf{u} + \mathbf{v} = \sum_{i=1}^{n_S} \mu_{\mathbf{u}+\mathbf{v}}(i)/i\,, \qquad (1)$$

where $\mu_{\mathbf{u}+\mathbf{v}}(i) = \max(\mu_\mathbf{u}(i), \mu_\mathbf{v}(i))$. Note that operation (1) can give the vague result even at exact arguments.

The arithmetic operations with vague values and increments are defined by analogy to operations with fuzzy numbers (Dubous, *et al.*, 1978), taking into account the limits of scale range:

$$\mathbf{v} + \delta = \sum_{i=1}^{n_S} \mu_{\mathbf{v}+\delta}(i)/i\,, \qquad (2)$$

where $\mu_{\mathbf{v}+\delta}(i) = \max_{k,l} \min(\mu_\mathbf{v}(k), \mu_\delta(l))$, provided that $i = \max(1, \min(n_S, k + l))$.

$$\delta_1 + \delta_2 = \sum_{i=-(n_S-1)}^{n_S-1} \mu_{\delta_1+\delta_2}(i)/i\,, \qquad (3)$$

where $\mu_{\delta_1+\delta_2}(i) = \max_{k,l} \min(\mu_{\delta_1}(k), \mu_{\delta_2}(l))$, provided that $i = \max(-(n_S-1), \min((n_S-1), k + l))$.

The definitions (1)-(3) show that the scale borders have *absorbing effect*: all values beyond the scale range are mapped into its borders.

The efficient increment δ', which value **v** receives as a result of operation (2), can differ from the increment δ due to the absorbing effect of borders. The true value δ' can be obtained from next formula

$$\delta' = \sum_{i=-(n_S-1)}^{n_S-1} \mu_{\delta'}(i)/i\,, \qquad (4)$$

where $\mu_{\delta'}(i) = \max_{k,l} \min(\mu_\mathbf{v}(k), \mu_\delta(l))$, provided that $i = \max(1, \min(n_S, k + l)) - k$. At such definition $\delta' = 0$, if $\delta = 0$.

Intermediate value of scale S is a special case of the vague qualitative value. Other special cases are a basic value s_i and the *fuzzy singleton* μ_i/i, $\mu_i \in [0, 1]$.

Vague qualitative value contains two types of uncertainty: on distribution of components in a scale and on their weights. At the operations with vague values, just as with inexact numbers, there is a tendency of lessening of the data accuracy. This tendency is expressed in the increase of number of components and in the reduction of their weights. However, because of the absorbing effect of the scale borders, the result of individual operation can, both to increase, and to decrease vagueness of data.

3. DEFAZZIFICATION OF VAGUE VALUES

A vague qualitative value **v** finally requires defazzification - choice of a single value $d(\mathbf{v}) \in S$ (defazzifier), which presents value **v** as a whole. Basically, for this purpose can be used any concept of *center* of fuzzy set (centroid, median, center of maxima, etc.) rounded to nearest basic value.

However, presentation of a multi-component vector **v** by a one-component object $d(\mathbf{v})$ entails loss of information, hence such representation can be only approximate. To not "forget" the vagueness of value **v** at defazzification it is desirable not only to assign $d(\mathbf{v})$, but also to estimate a measure of its conformity to value **v** as a whole. With this aim we consider the result of defazzification of value **v** in the form of fuzzy singleton **d**(**v**) = μ(**v**)/d(**v**), where μ(**v**) is understood as the integrated estimation of certainty of vague value **v**. The estimation μ(**v**) should take into consideration the set of all components of value **v**, their weights and allocation. We suppose that μ(**v**) meets the next requirements:

- the more closely the components of value **v** are located on the scale, the more μ(**v**) is,
- μ(**v**) weakly depends from small components of **v**,
- μ (**v**) $\le$ *max*(**v**), where *max*(**v**) = $\max_i \mu_v(i)$, and in particular:

  μ (**v**) = μ_i, if **v** is a singleton μ_i/i,

  μ (**v**) < *max*(**v**), if **v** is not a singleton.

Taking these requirements into consideration, we define function μ (**v**) in the next form:

$$\mu(\mathbf{v}) = max(\mathbf{v})\cdot(1 - \frac{w(\mathbf{v})-1}{n_S}),$$

where $w(\mathbf{v}) \in [1, n_S]$ – *width* of value **v**, which estimates the distribution of its components on scale S. The definition of w(**v**) below reflects a measure of consolidation of components around some center $c \in S$. At first we define the *radius* r_c of value **v** regarding c as

$$r_c = \frac{\sum_{i=1}^{n_S} |i-c| \cdot \mu_{\mathbf{v}}(i)}{\sum_{i=1}^{n_S} \mu_{\mathbf{v}}(i)}.$$

Radius r_c reflects the average nearness of the components of value **v** to the center c. Therefore the width w_c(**v**) regarding center c can be defined as

$$w_c(\mathbf{v}) = 2 \cdot r_c + 1$$

The definition of w_c(**v**) is suitable for any type of center c. However it allows to define one more concept of center of the value **v** as a point $cm(\mathbf{v}) \in S$, which radius $r_{cm(\mathbf{v})}$ is minimal. The center *cm* may not coincide with *centroid* or *median*. For instance, for the value $\mathbf{v}_{ex}$ *centroid* =3, *median* =2, and *cm* =1.

Further we consider the defazzificator **d**(**v**) in the form of **d**(**v**) = μ(**v**)/*cm*(**v**), where

$$\mu(\mathbf{v}) = max(\mathbf{v})(1 - \frac{2r_{cm(\mathbf{v})}}{n_S}). \quad (5)$$

For example,

$$\mathbf{d}(\mathbf{v}_{ex}) = \mu(\mathbf{v}_{ex})/d(\mathbf{v}_{ex}) = 0{,}47/1. \quad (6)$$

The more μ(**v**), the more certain is vague value **v**. In particular, if μ(**v**) = 1, then **v** is exact value of the scale S. This property allows us to use μ(**v**) as a *criterion of certainty* of vague data during all process of calculations in QCM.

4. QUALITATIVE FUNCTIONS OF INFLUENCE

The numerical coefficients used in exact cognitive maps for expression of influence power, is not applicable to qualitative values. Therefore in QCM the functional representation is used for this purpose. Functional representation is the universal form for expression of influence, which, basically, allows to express influence of both qualitative, and numerical factors. A typical example of QCM is presented on Fig.1.

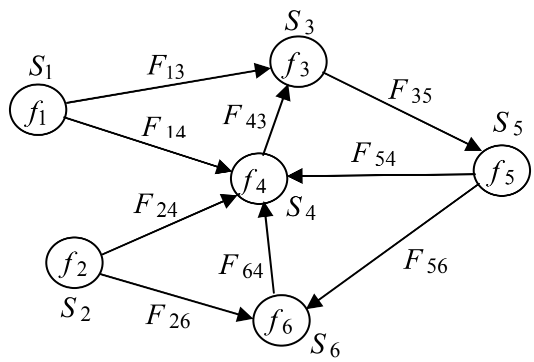

Fig.1 Qualitative cognitive map.

Let $f_1, \ldots, f_N$ be the factors of a QCM model M and $S_1, \ldots, S_N$ the scales of their values. The influence of factor f_i on factor f_j is expressed by function F_{ij}, associated with the edge directed from input node f_i to output node f_j. The function F_{ij} maps a value of scale S_i into a value of scale S_j. In the simplest case function F_{ij} maps a basic value of scale S_i to a basic value of scale S_j. If it takes place we name scales S_i and S_j *consistent*. However such consistency of the scales is not always possible. So in general we suppose that function F_{ij} maps a basic value of scale S_i into a basic or an *intermediate* value of scale S_j.

The power of influence corresponds to the "speed" of change of the function F_{ij} depending on change of argument. The sign of influence corresponds to the type of change: the increase or the decrease.

Table 2. Qualitative function of influence

	1	2	3	4	5	6	7
1					0,35	0,65	
2					0,90	0,35	
3				0,45	0,55		
4				1,00			
5			0,55	0,45			
6		0,35	0,90				
7		0,65	0,35				

Formally, the function F_{ij} is defined by the matrix $\mathbf{F}_{ij}$ = $\|f_{ij}(k,l)\|$, where $k = \overline{1, n_{S_i}}$, $l = \overline{1, n_{S_j}}$, $f_{ij}(k,l) \in [0,1]$. The value $\mathbf{v}_j$ of factor f_j, depending on the value $\mathbf{v}_i$ of factor f_i, is calculated as matrix product

$$\mathbf{v}_j = \mathbf{v}_i \cdot \mathbf{F}_{ij},$$

or, componentwise, in the form of

$$\mu_{v_j}(l) = \sum_{k=1}^{n_{S_i}} \mu_{v_i}(k) \cdot f_{ij}(k,l),$$

where multiplication is the fuzzy AND (min), and sum is the fuzzy OR (max).

An example of "negative" influence function F for $n_S = 7$ is presented in Table 2. Zero entries are omitted. The function F, for example, maps the value $\mathbf{v}_{ex}$ into the value $\mathbf{v}_{ex}' = F(\mathbf{v}_{ex})$:

$$\mathbf{v}_{ex}' = (1, 0, 0.3, 0, 0, 0, 0.5) \cdot \mathbf{F} = (0, 0.5, 0.35, 0.3, 0.35, 0.65, 0). \quad (7)$$

This example shows, that operation $\mathbf{v} \cdot \mathbf{F}$ can decrease $max(\mathbf{v})$.

Inconsistency of scales is one of the reasons of appearing the vague values in QCM. Other reason - the disagreement of mutual influences of factors upon each other.

5. GENERAL ASPECTS OF BEHAVIOUR IN QCM

The values $\mathbf{v}_i$ of factors f_i, $i = \overline{1, N}$, form the *state* $\mathbf{s} = \{\mathbf{v}_i\}$ of QCM model M. Initial state of model M represents the initial situation. We suppose that initial values of factors are the basic or the intermediate values of the corresponding scales. In each step of the model time the state s is recalculated according to mutual influence of the factors. The transient process begins with initial state and terminates in case of stabilization of the state, cycling, or loss of accuracy, when the certainty of data falls below an admissible limit. As a *criterion of certainty* of state $\mathbf{s}$ as a whole we use the value $\mu(\mathbf{s}) = \max_i(\mathbf{v}_i)$, $i = \overline{1, N}$.

We consider the disagreement of influences on a factor from another factors, exerted in the same step of the modelling time, as a *conflict* in the model M. At occurrence of the conflict the following variants of behaviour are possible:

1. To cease further modelling, considering such situation as erroneous.
2. To integrate the disagreed influences, considering this disagreement simply as a new addition to uncertainty of data, which was in the model due to inexact definition of the model, vagueness of initial data and integration of previous conflicts.

Second approach is considered further. The defazzification of integrated vague values can be executed both in each step of modelling time and on the termination of calculations. It is necessary to note, however, that the defazzification, executed in a cycle of recalculation of the model state, can change the course of calculations. Let's consider for example the functional operation $\mathbf{v}_{ex}' = \mathbf{v}_{ex} \cdot \mathbf{F}$. We have $\mathbf{d}(\mathbf{v}_{ex}') = \mu(\mathbf{v}_{ex}')/d(\mathbf{v}_{ex}') = 0{,}39/4$ by (5, 7). On the other hand, $\mathbf{d}(\mathbf{v}_{ex}) = 0{,}47/1$ by (6). Applying function $\mathbf{F}$ to $\mathbf{d}(\mathbf{v}_{ex})$ we have:

$\mathbf{v}_{ex}'' = \mathbf{d}(\mathbf{v}_{ex}) \cdot \mathbf{F} = 0.35/5 + 0.47/6$ and

$\mathbf{d}(\mathbf{v}_{ex}'') = \mu(\mathbf{v}_{ex}'')/d(\mathbf{v}_{ex}'') = 0.42/6$.

Different result of the same operation is obtained. It is the consequence of the fact that defazzification entails loss of information, as it was noted in section 3. To prevent this effect it is preferable to keep the present vague values intact as long as possible, and use the defazzification only for supervision over the course of calculations and giving out the final values of factors.

Below we consider two models of QCM, which differ in type of the influence functions and a way of integration their values. We consider the tendencies and the forms of decrease of data certainty in these models.

6. MODEL *V-V* (value- value)

In the *V-V* model M the influence function F_{ij} maps a value of input factor f_i into a value of output factor f_j. On each entering edge of a factor f of the model M the instruction comes to replace old state of this factor on a new one. If these instructions differ, conflict occurs. This conflict is integrated. This means, that new value $\mathbf{v}$ of factor f is calculated as sum (1) of all input influences. As was noted above this operation can give a vague result even for exact arguments. It means that the vague values can appear in the *V-V* model even if the scales of all factors are consistent and initial values of all factors are the exact values of the corresponding scales.

The estimate $\mu(\mathbf{v})$ is used as a measure of certainty of value $\mathbf{v}$. As old state of factor f is replaced, the estimate $\mu(\mathbf{v})$ for this factor can both to increase, and to decrease. However, as it follows from (1), if the estimate $\mu(\mathbf{s})$ of certainty of entire state of the model M has gone down to some level, then it cannot rise above it any more. Therefore the lessening of data certainty in the *V-V* model occurs for the model as a whole. This lessening can stabilize on some nonzero level, or fall up to zero level.

7. MODEL *Δ-Δ* (increment- increment)

In the *Δ-Δ* model M the influence function F_{ij} maps an increment of the input factor f_i to an increment of the output factor f_j. The increment may be positive or negative, exact or vague. The distinction of increments, coming on various inputs edges of factor f_j also can be considered as a conflict. In the *Δ-Δ* model this conflict is solved automatically due to summation (2, 3) of the increments with each other and with the old value of f_j. The efficient increment of factor f_j is calculated by (4). For the model to be steady at zero increments, the influence functions should preserve zero values of the corresponding increment scales.

In the *Δ-Δ* model the influence function F_{ij} does not transfer explicit influence of a state on a state. However, the state of a factor f_i has effect on the state of factor f_i due to nonlinear character of the summation operation (2). Since in the *Δ-Δ* model the disagreement of influences is solved automatically, the only source of vagueness in the model is vagueness of initial data and inconsistency of the scales. If such vagueness is absent, it does not appear during calculation. However as a whole, the tendencies of lessening of data accuracy in the *Δ-Δ* model are more various, than in the *V-V* model. Besides the general decrease of certainty, expressed by the estimate $\mu(\mathbf{s})$, in this model is also possible steady decrease of certainty of separate factors. As it follows from (2 - 4) if $max(\mathbf{v}_i)$, for some factor f_i has decreased to some level, then it cannot increase above this level any more. At significant decrease of $max(\mathbf{v}_i)$ the factor f_i actually loses the information on its state and ceases to participate in transfer of influences. Such "fading away" of factors is a specific type of stabilization of transient process in a Δ-Δ model.

CONCLUSION

The qualitative approach has appeared as an attempt of decision of the problems inherent in exact numerical models of dynamic cognitive maps. These problems are connected with inability of the expert to exactly indicate parameters of the model. The requirement of excessive accuracy confuses the expert, mentally depresses him, and forces him to be mistaken. In addition, complex effects of behavior (instability, cycles, chaotic processes, etc.), which are possible in exact dynamic models, create difficulties in understanding and explaining the processes happening in such models.

In this connection the question arises, insofar it is possible to trust the conclusions based on exact model if its parameters are chosen arbitrarily in significant measure. In QCM we avoid the arbitrariness in the model parameters, however meet with a new problem - the data vagueness arising in the process of calculations, which can make impossible the forecasting the remote consequences of initial influence.

The QCM is a rough enough, but logically consecutive instrument of the study. Though it cannot provide determination of the long-term forecast, they are quite suitable for determination of short-term trends of the situation development. The level of trust to the conclusions, based on QCM, is determined by the proposed estimate of certainty of the vague qualitative data. Even at presence of vagueness the correct analysis is possible before the data certainty is not lowered below admissible limit. The computer tests on various examples of QCM show that there are at least tens of steps before this level is achieved.

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