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Path-Tracking Dynamic Model Based Control of an Omnidirectional Mobile Robot ⋆

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Abstract: Considering the dynamic model of an omnidirectional mobile robot (also known of a type (3,0)), in this work is addressed the Path-tracking control problem of this class of systems. The solution of the problem is obtained by considering some suitable modifications of the well known computed-torque control strategy usually used on the field of robot manipulators. The modifications considered in this work are due to the structural differences between a mobile robot and a classical rigid robot manipulator. It is formally proved that the proposed control strategy allows the convergence of the tracking errors and assures the closed loop stability of the system. The tracking strategy is evaluated by simulation, showing an acceptable performance.

1. INTRODUCTION

The classical control problems of regulation and path-tracking have been widely studied in the field of mobile robotics. There has been a deeply analysis for the case of two classes of configuration, the so called differential and omnidirectional mobile robots, also known as been of the type (2,0) and (3,0) respectively (Bétourné and Campion [1996], Kalmár-nagy et al. [2004]).

From a kinematics perspective, the control problem of a mobile robot has been aboard from different perspectives, in this sense, in (Canudas et al. [1996]) and (Campion et al. [1996]) are presented the developments of kinematics models for different types of robots.

A popular class of mobile robot, the (2,0)-type has been subject to several studies considering mainly its kinematics model. For example, in (D'Andrea-Novel et al. [1992]) a dynamic feedback linearization approach is used to solve the path tracking problem, while in (Oriolo et al. [2002]) the same problem is solved and real time implemented. A discrete time approach is considered in (no Suárez et al. [2006]) where a sliding mode control is presented.

For the case of the omnidirectional mobile robot (3,0), the regulation and path-tracking problem has deserved important attention. Considering its kinematics model, in (Watanabe [1998]), several control strategy are proposed. From a different perspective, in (Liu et al. [2003]) it is designed a nonlinear controller based on a Trajectory Linearization strategy and in (Velasco-Villa et al. [2007a]), the remote control of this class of systems is presented by considering a discrete-time strategy assuming a time-lag model of the robot. In (Velasco-Villa et al. [2007b]) the same problem is considered by means of an estimation strategy that predicts the future values of the system based on the exact nonlinear discrete time model of the robot.

A more reduced number of contributions have been focus on the case of an omnidirectional mobile robot based on its dynamic model. For example, in Carter et al. [2001], it is described the mechanical design of a (3,0) robot and based on its dynamic model it is proposed a PID control for each robot wheel independent of the nonlinear model of the vehicle. In (Bétourné and Campion [1996]) the authors consider an Euler-Lagrange formulation and present an output feedback that solves the path-tracking problem. In the same way, in (Williams et al. [2002]) the dynamic model of the robot is considered in order to study the slipping effects between the wheels of the vehicle and the working surface; in (J. H. Chung et al. [2003]) the mobile robot is analyzed in the case of a vehicle supporting castor wheels. In (Kalmár-nagy et al. [2004]) the time-optimization problem of a desired trajectory is considered for a mobile robot subject to structural considerations and admissible inputs in order to obtain feedback laws that are based on the kinematics and dynamic models.

The analysis and control of Euler-Lagrange systems has produced several feedback strategies that has been developed mainly in the area of robot manipulators; for example, passivity based strategies as the PD+ control (Paden and Panja [1998]) or the adaptive controller developed in (Slotine and Weiping [1988]) have been used to solve the path-tracking problem for rigid robots manipulators. The well known computed torque control strategy has been considered in (Wen and Bayard [1988]), (Ortega et al. [1998]) to tackle the regulation and path tracking problems of rigid and flexible manipulators. In (Loria [1996]) it is presented a computed-torque based controller plus a Nonlinear PD to solve the path-tracking problem with output feedback of one degree of freedom.

In this work it is considered the analysis of an omnidirectional mobile robot, that contrary to the differential case it is not affected by non-holonomics restrictions but, instead of this, the slipping effects between the wheels and the working surface are accentuated due to its special wheel configuration. Following the same ideas developed in the field of robot manipulators, it is considered a computer torque control strategy (Loria and Ortega [1995]) to solve the path-tracking control of an omnidirectional mobile robot. As in the case of robot manipulators it will be considered the dynamic model of the vehicle.

This work is organized as follows: Section 2 presents the kinematics and dynamic model of the omnidirectional mobile robot where an Euler-Lagrange formulation is used

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to derive the dynamic model and pointing out some of the structural properties of the model. Immediately, In Section 3 the path-tracking control problem is stated and the proposed solution is derived showing the closed loop stability of the scheme and the appropriate convergence of the tracking errors. In Section 4, the performance of the proposed strategy is evaluated by means of numerical simulation showing its adequate performance and finally, in Section 5, some conclusions are presented.

2. OMNIDIRECTIONAL MOBILE ROBOT

Considering wheeled mobile robots, a widely used classification of this type of vehicles is based on the degrees of mobility δ_m and steerability δ_s (Canudas et al. [1996]). The mobile robot considered in this work is of the type $(\delta_m, \delta_s) = (3, 0)$. It means that, it has three degrees of mobility and zero degrees of steerability that allows the displacements of the vehicle in all directions instantaneously. This fact represents the main characteristic of this type of vehicles producing a significant advantage over the rest of the wheeled mobile robots.

A top view of the configuration of the robot considered in this work is depicted in Figure 1 where it is possible to see the moving reference frame $X_m - Y_m$ located at the center of the vehicle with the X_m axis aligned with respect to the wheel 3. Wheels 1 and 2 are placed symmetrically with an angle $\delta = 30°$ with respect to the Y_m axis. In the same way, in Figure 1, it is shown the fixed reference frame $X - Y$ which provides the absolute localization of the vehicle on the workspace.

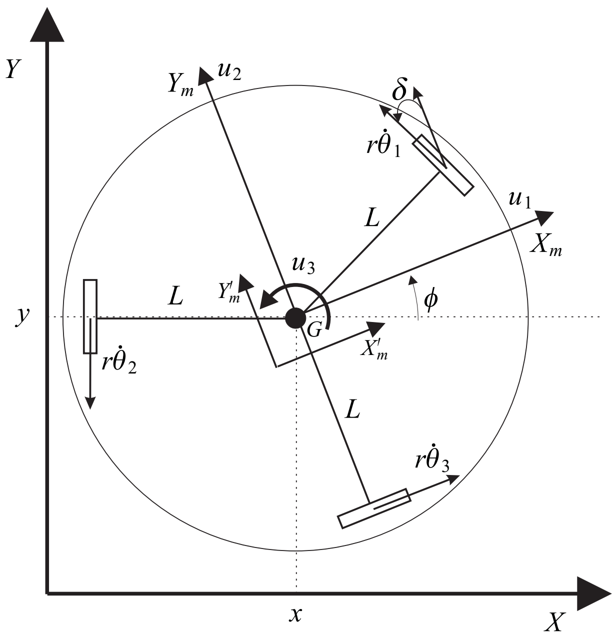

Fig. 1. Robot móvil omnidireccional

The kinematics model of an omnidirectional mobile robot can be easily obtained by considering the geometric relations given in figure 1. The velocity components with respect to the axis $X - Y$ are obtained (Campion et al. [1996]) as,

$$\begin{aligned} \dot{x} &= u_1 \cos\phi - u_2 \sin\phi \\ \dot{y} &= u_1 \sin\phi + u_2 \cos\phi \\ \dot{\phi} &= u_3. \end{aligned} \tag{1}$$

where point (x, y) is the position of the center of the robot on the plane $X - Y$ and ϕ in the angular position with respect to the X-axis. The input signals are given by u_1, u_2 and u_3, with u_1, u_2 two orthogonal vectors and where u_1 is aligned with respect to the reference axis of the robot and u_3 is the rotational velocity of the robot. Considering also the velocity relations depicted on Figure 1 it is possible to obtain the inverse kinematics relations of the omnidirectional robot with respect to the wheels angular velocity as,

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -\sin(\delta+\phi) & \cos(\delta+\phi) & L \\ -\sin(\delta-\phi) & -\cos(\delta-\phi) & L \\ \cos(\phi) & \sin(\phi) & L \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} \tag{2}$$

where θ_1, θ_2, θ_3 represent the angular displacements of wheels one, two and three, respectively; δ is the orientation of the i-wheel with respect to its longitudinal axis; L is the distance between the center of each wheel and the center of the vehicle and r is the radius of each wheel. The kinematics model is obtained as the inverse map of (2).

2.1 Dynamic model

The dynamics of an omnidirectional mobile robot has been analyzed in (Balakrishna and Ghosal [1995]) where the model is derived by the use of the Euler Lagrange formalism and in (Watanabe [1998]) where the dynamic model is obtained by considering a Newton-Euler strategy.

Following (Balakrishna and Ghosal [1995]), the kinetic energy of the robot is given by the wheel rotational energy and the translational and rotational energy of the robot. Therefore, the Lagrangian of the system is obtained as,

$$\mathcal{L} = \frac{1}{2}[M_p(V_{Gx}^2 + V_{Gy}^2) + I_p\dot{\phi}^2] + \frac{1}{2}\sum_{i=1}^{3} I_{ri}\dot{\theta}_i^{\,2} \tag{3}$$

where V_{Gx}, V_{Gy} are the velocity along the axis X_m, Y_m respectively. M_p is the mass and I_p the moment of inertia about the Z axis of the vehicle, I_{r_i} the moment of inertia of each wheel about its axis.

Considering that the kinematics restrictions (2) are satisfied for all t, it is possible to neglect the friction and slip effects between the wheels and the working surface. Assuming also that the inertia at each wheel I_{r_i} are equal, the dynamic model of the system can be obtained as,

$$D\ddot{q}_m + C(\dot{q}_m)\dot{q}_m = B\tau \tag{4}$$

where,

$$q = \begin{bmatrix} x_m \\ y_m \\ \phi \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix},$$

$$D = \begin{bmatrix} \frac{3I_r}{2r^2} + M_p & 0 & 0 \\ 0 & \frac{3I_r}{2r^2} + M_p & 0 \\ 0 & 0 & I_p + \frac{3L^2 Ir}{r^2} \end{bmatrix},$$

$$C(\dot{q}_m) = \begin{bmatrix} 0 & -M_p\dot{\phi} & 0 \\ M_p\dot{\phi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \frac{1}{r}\begin{bmatrix} -\sin(\delta) & -\sin(\delta) & 1 \\ \cos(\delta) & -\cos(\delta) & 0 \\ L & L & L \end{bmatrix}.$$

Dynamic model (4) provides the representation of the omnidirectional mobile robot on the moving axes $X_m - Y_m$. To obtain the absolute localization of the mobile robot on the fixed reference frame, consider now the non singular transformation $\dot{q}_m = T(\phi)\dot{q}$,

$$T = T(\phi) = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (5)$$

that allows rewriting model (4) in the form,

$$DT\ddot{q} + [D\dot{T} + C(\dot{q})T]\dot{q} = B\tau. \quad (6)$$

where

$$q = [x, y, \phi]^T.$$

It easy to verify that the inertia matrix DT of the system it is not symmetric not positive definite,

$$DT = \frac{1}{2rb_1}\begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & \dfrac{2Lb_1}{b_2} \end{bmatrix},$$

$$b_1 = \frac{r}{3I_r + 2r^2M_p}, \quad b_2 = \frac{Lr}{3L^2I_r + r^2I_p}.$$

In order to obtain the solution of the path tracking problem by means of a computed-torque strategy consider the new representation of system (6) in the form,

$$\mathcal{D}\ddot{q} + \mathcal{C}(\dot{q})\dot{q} = \mathcal{B}\tau \quad (7)$$

where,

$$\mathcal{D} = T^{-1}DT = \begin{bmatrix} M_p + \dfrac{3I_r}{2r^2} & 0 & 0 \\ 0 & M_p + \dfrac{3I_r}{2r^2} & 0 \\ 0 & 0 & I_p + \dfrac{3I_rL^2}{r^2} \end{bmatrix}$$

$$\mathcal{C}(\dot{q}) = T^{-1}D\dot{T} + T^{-1}C(\dot{q})T = \frac{3I_r}{2r^2}\begin{bmatrix} 0 & -\dot{\phi} & 0 \\ \dot{\phi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B} = T^{-1}B = \frac{1}{r}\begin{bmatrix} -\sin(\delta+\phi) & -\sin(\delta-\phi) & \cos\phi \\ \cos(\delta+\phi) & -\cos(\delta-\phi) & \sin\phi \\ L & L & L \end{bmatrix}.$$

2.2 Structural properties

Before presenting the control strategy, some remarks are pointed out and some structural properties of the dynamic model of the omnidirectional mobile robot (7) are stated. These properties will be necessary to get a control strategy based on a computed-torque methodology equivalent to the one use in the case of robot manipulators.

Remark 1. Since matrix $\mathcal{D}$ in the model (7) is diagonal with positive elements, then it is symmetric positive definite.

Remark 2. Since matrix $\mathcal{D}$ is constant, the classic skew symmetric property on robot manipulators $N(q,\dot{q}) = \dot{\mathcal{D}} - 2\mathcal{C}(\dot{q})$ it is trivially satisfied.

Property 3. Notice that $\mathcal{C}(\dot{q})\dot{q}$ as is usual, does not possess a unique representation, in particular,

$$\begin{aligned} \mathcal{C}(\dot{q})\dot{q} &= \frac{3I_r}{2r^2}\begin{bmatrix} 0 & -\dot{\phi} & 0 \\ \dot{\phi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\dot{q} \\ &= \frac{3I_r}{4r^2}\begin{bmatrix} 0 & \dot{\phi} & \dot{y} \\ -\dot{\phi} & 0 & -\dot{x} \\ -\dot{y} & \dot{x} & 0 \end{bmatrix}\dot{q} \\ &= \mathcal{C}_a(\dot{q})\dot{q} \end{aligned}$$

that allows to get a new representation that satisfies the skew symmetric property.

Property 4. Considering matrix $\mathcal{C}_a(\dot{q})$ it is possible to establish that for any vector $z \in R^n$,

$$\begin{aligned} \mathcal{C}_a(\dot{q})z &= \frac{3I_r}{4r^2}\begin{bmatrix} 0 & \dot{\phi} & \dot{y} \\ -\dot{\phi} & 0 & -\dot{x} \\ -\dot{y} & \dot{x} & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \\ &= \frac{3I_r}{4r^2}\begin{bmatrix} \dot{\phi}z_2 + z_3\dot{y} \\ -\dot{\phi}z_1 - z_3\dot{x} \\ z_2\dot{x} - z_1\dot{y} \end{bmatrix} \\ &= \frac{3I_r}{4r^2}\begin{bmatrix} 0 & z_3 & z_2 \\ -z_3 & 0 & -z_1 \\ -z_2 & z_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} \\ &\quad + \frac{3I_r}{4r^2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2z_2 & -2z_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} \end{aligned} \quad (8)$$

this is,

$$\mathcal{C}_a(\dot{q})z = \mathcal{C}_a(z)\dot{q} + \mathcal{C}_r(z)\dot{q}. \quad (9)$$

Property 5. From the new structure of matrix $\mathcal{C}_a(\dot{q})$ and the definition of $\mathcal{C}_r(\dot{q})$ it is possible to show that they are bounded in the form,

$$||C_a(\dot{q})|| \leq k_c||\dot{q}||, \;\; ||C_r(\dot{q})|| \leq k_r||\dot{q}||, \quad (10)$$

where it is easy to show that k_c and k_r can be chosen as,

$$k_c = \frac{3I_r}{4r^2} \text{ and } k_r = \frac{3I_r}{2r^2}.$$

3. PATH-TRACKING PROBLEM

In this work it is considered the classical path-tracking control problem for omnidirectional mobile robot. This is, it is required that the output of system (7) follows a desired trajectory $q_d(t)$ by means of a feedback law of the form,

$$\tau(t) = \alpha(q^{(i)}(t), q_d^{(i)}(t)),$$

such that, in closed loop with the mobile robot (7) the tracking error $\tilde{q} = q - q_d$ converges to zero,

$$\lim_{t\to\infty}[q(t) - q_d(t)] = 0.$$

To tackle the tracking problem described above, consider the feedback,

$$\mathcal{B}\tau = \mathcal{D}\ddot{q}_d + \mathcal{C}(\dot{q}_d)\dot{q}_d - K_p\tilde{q} - K_d\dot{\tilde{q}} + \mathcal{C}_r(\dot{q}_d)\dot{q}, \quad (11)$$

or equivalently,

$$\tau = \mathcal{B}^{-1}\left[\mathcal{D}\ddot{q}_d + \mathcal{C}(\dot{q}_d)\dot{q}_d - K_p\tilde{q} - K_d\dot{\tilde{q}} + \mathcal{C}_r(\dot{q}_d)\dot{q}\right].$$

As mentioned earlier, q_d correspond to the desired trajectory and the tracking error is given by $\tilde{q} = q - q_d$. K_p and K_d are matrices of proportional and derivative gains that additionally are diagonal and positive definite; the matrix $\mathcal{C}_r(\dot{q}_d)$ is defined in (8).

Consider now the model of the mobile robot (7) in closed loop the feedback (11), under these conditions, it is obtained the system,

$$\mathcal{D}\ddot{\tilde{q}} + \mathcal{C}(\dot{q})\dot{q} - \mathcal{C}(\dot{q}_d)\dot{q}_d + K_p\tilde{q} + K_d\dot{\tilde{q}} - \mathcal{C}_r(\dot{q}_d)\dot{q} = 0. \quad (12)$$

With the aim to obtain an expression in terms of the tracking error, notice that taking into account the property given in (9) it is possible to show that,

$$\begin{aligned}\mathcal{C}(\dot{q})\dot{q} - \mathcal{C}(\dot{q}_d)\dot{q}_d &= \mathcal{C}(\dot{q})\dot{q} - \mathcal{C}(\dot{q}_d)\dot{q}_d \pm \mathcal{C}(\dot{q})\dot{q}_d \\ &= \mathcal{C}(\dot{q})\dot{\tilde{q}} - \mathcal{C}(\dot{q}_d)\dot{q}_d + \mathcal{C}(\dot{q}_d)\dot{q} + \mathcal{C}_r(\dot{q}_d)\dot{q} \\ &= [\mathcal{C}(\dot{q}) + \mathcal{C}(\dot{q}_d)]\dot{\tilde{q}} + \mathcal{C}_r(\dot{q}_d)\dot{q}\end{aligned}$$

From the above developments, equation (12) can now be rewritten as,

$$\mathcal{D}\ddot{\tilde{q}} + [\mathcal{C}(\dot{q}) + \mathcal{C}(\dot{q}_d)]\,\dot{\tilde{q}} + K_d\dot{\tilde{q}} + K_p\tilde{q} = 0. \quad (13)$$

that describes the dynamics of the tracking error.

With the intention to simplify the later developments, and where no confusion arise, in the rest of the paper it will be considered the notation $\mathcal{C} = \mathcal{C}(\dot{q})$, $\mathcal{C}_d = \mathcal{C}(\dot{q}_d)$ and $\tilde{\mathcal{C}} = \mathcal{C}(\dot{\tilde{q}})$.

3.1 Closed loop stability

The dynamics of the omnidirectional mobile robot (7) in closed loop with the feedback (11) can be analyzed by mean of the tracking error equation (13). First of all, notice that $(\tilde{q}, \dot{\tilde{q}}) = (0, 0)$ is an equilibrium point of the error system (13), for this reason, the stability of the error system can be analyzed by considering the candidate Lyapunov function,

$$V(\tilde{q}, \dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T\mathcal{D}\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_p\tilde{q} + \epsilon\tilde{q}^T\mathcal{D}\dot{\tilde{q}} + \frac{1}{2}\epsilon\tilde{q}^T K_d\tilde{q} \quad (14)$$

where $\mathcal{D}$ is defined in (7) and matrices K_p, K_d are given in (11). It is an easy task to verify that function $V(\tilde{q}, \dot{\tilde{q}})$ defined in (14) is positive definite for all $\epsilon > 0$ sufficiently small.

Considering now the time derivative of V, it is obtained,

$$\dot{V} = \dot{\tilde{q}}^T\mathcal{D}\ddot{\tilde{q}} + \tilde{q}^T K_p\dot{\tilde{q}} + \epsilon\tilde{q}^T\mathcal{D}\ddot{\tilde{q}} + \epsilon\dot{\tilde{q}}^T\mathcal{D}\dot{\tilde{q}} + \epsilon\tilde{q}^T K_d\dot{\tilde{q}},$$

this is,

$$\dot{V} = -\dot{\tilde{q}}^T[\mathcal{C}+\mathcal{C}_d+K_d-\epsilon\mathcal{D}]\dot{\tilde{q}} - \epsilon\tilde{q}^T K_p\tilde{q} - \epsilon\tilde{q}^T(\mathcal{C}+\mathcal{C}_d)\dot{\tilde{q}}. \quad (15)$$

From the fact, that matrices $\mathcal{C}$, $\mathcal{C}_d$ are skew symmetric, equation (15) can be rewritten as,

$$\dot{V} = -\dot{\tilde{q}}^T[K_d - \epsilon\mathcal{D}]\dot{\tilde{q}} - \epsilon\tilde{q}^T K_p\tilde{q} - \epsilon\tilde{q}^T(\mathcal{C} + \mathcal{C}_d)\dot{\tilde{q}}. \quad (16)$$

Now from matrix $\mathcal{C}$ it is possible to see that,

$$\begin{aligned}\mathcal{C}(\dot{q}) + \mathcal{C}(\dot{q}_d) &= \mathcal{C}(\dot{\tilde{q}} + \dot{q}_d + \dot{q}_d) \\ &= \mathcal{C}(\dot{\tilde{q}}) + \mathcal{C}(2\dot{q}_d) \\ &= \tilde{\mathcal{C}} + 2\mathcal{C}_d,\end{aligned} \quad (17)$$

that allow to rewrite equation (16) as,

$$\dot{V} = -\dot{\tilde{q}}^T K_d\dot{\tilde{q}} +^T \epsilon\dot{\tilde{q}}\mathcal{D}\dot{\tilde{q}} - \epsilon\tilde{q}^T K_p\tilde{q} - \epsilon\tilde{q}^T\mathcal{C}(\dot{\tilde{q}})\dot{\tilde{q}} - 2\epsilon\tilde{q}^T\mathcal{C}(\dot{q}_d)\dot{\tilde{q}}. \quad (18)$$

Noting that the terms in the preceding equation can be bounded as,

$$\begin{aligned}-\dot{\tilde{q}}^T K_d\dot{\tilde{q}} &\le -\underline{\lambda}(K_d)||\dot{\tilde{q}}||^2 \\ \epsilon\dot{\tilde{q}}\mathcal{D}\dot{\tilde{q}} &\le \epsilon\bar{\lambda}(\mathcal{D})||\dot{\tilde{q}}||^2 \\ -\epsilon\tilde{q}^T K_p\tilde{q} &\le -\epsilon\underline{\lambda}(K_p)||\tilde{q}||^2 \\ -\epsilon\tilde{q}^T\mathcal{C}(\dot{\tilde{q}})\dot{\tilde{q}} &\le \epsilon k_c||\tilde{q}||||\dot{\tilde{q}}||^2 \\ -2\epsilon\tilde{q}^T\mathcal{C}(\dot{q}_d)\dot{\tilde{q}} &\le 2\epsilon k_c||\dot{q}_d||||\tilde{q}||||\dot{\tilde{q}}||\end{aligned}$$

where $\underline{\lambda}(A)$ and $\bar{\lambda}(A)$ represent the smallest and largest eigenvalue of a given matrix A respectively and k_c is given in (10). The above properties, allows to rewrite (18) in the form,

$$\begin{aligned}\dot{V} \le\ & -\underline{\lambda}(K_d)||\dot{\tilde{q}}||^2 + \epsilon\bar{\lambda}(\mathcal{D})||\dot{\tilde{q}}||^2 - \epsilon\underline{\lambda}(K_p)||\tilde{q}||^2 \\ & + \epsilon k_c||\tilde{q}||||\dot{\tilde{q}}||^2 + 2\epsilon k_c||\dot{q}_d||||\tilde{q}||||\dot{\tilde{q}}||.\end{aligned} \quad (19)$$

In order to find the stability conditions of the closed loop systems, notice that equation (19) is equivalent to,

$$\dot{V} \le -\left[\, ||\tilde{q}||\ ||\dot{\tilde{q}}||\, \right] P \begin{bmatrix} ||\tilde{q}|| \\ ||\dot{\tilde{q}}|| \end{bmatrix} \quad (20)$$

where,

$$P = \begin{bmatrix} \epsilon\underline{\lambda}(K_p) & -\epsilon k_c||\dot{q}_d|| \\ -\epsilon k_c||\dot{q}_d|| & \underline{\lambda}(K_d) - \epsilon\bar{\lambda}(\mathcal{D}) - \epsilon k_c||\tilde{q}|| \end{bmatrix}.$$

It is clear, from the preceding developments, that the error system (13) will be stable if the matrix P is positive definite. This latter condition is satisfied by means of,

$$\begin{aligned} i)&\ \ \epsilon\underline{\lambda}(K_p) > 0 \\ ii)&\ \ \det\{P\} > 0.\end{aligned}$$

Condition $i)$ is trivially satisfied since matrix K_p is symmetric positive definite. Condition $ii)$ can also be written as,

$$\epsilon\underline{\lambda}(K_p)\left[\underline{\lambda}(K_d) - \epsilon\bar{\lambda}(\mathcal{D}) - \epsilon k_c||\tilde{q}||\right] - \epsilon^2 k_c^2||\dot{q}_d||^2 > 0,$$

that is equivalent to:

$$\frac{\underline{\lambda}(K_p)\underline{\lambda}(K_d)}{k_c^2||\dot{q}||^2 + \underline{\lambda}(K_p)[\bar{\lambda}(\mathcal{D}) + k_c||\tilde{q}||]} > \epsilon,$$

that can be always satisfied for a sufficiently small ϵ. Therefore, the asymptotic stability of the system is established.

4. SIMULATION RESULTS

The control strategy proposed in this work is evaluated in this section by means of simulation experiments. It is considered a circular path as a desired trajectory. The experiments are carried out by the use of a MATLAB-Simulink platform.

The desired trajectory is obtained by means of the equations,

$$\begin{aligned} x_d(t) &= r_w \sin(t_p(t)) \\ y_d(t) &= -r_w \cos(t_p(t)) \end{aligned} \qquad (21)$$

where r_w is the radius of the generated circle and t_p describes the angle of the point (x_d, y_d) on the plane $X-Y$ with respect to the X axis. The dependence over the time variable is obtained by considering the parameterization of $t_p(t)$ by the time polynomial,

$$t_p(t) = 20d_m(\frac{t}{t_f})^3 - 30d_m(\frac{t}{t_f})^4 + 12d_m(\frac{t}{t_f})^5 \qquad (22)$$

where t_f is the final execution time and $2d_m$ it is a constant value providing its final magnitude. The polynomial (21) satisfies the initial conditions $t_p(0) = 0$, $\frac{dt_p}{dt}(0) = 0$, $t_p(t_f) = 2d_m$, $\frac{dt_p}{dt}(t_f) = 0$.

Finally, the desired orientation angle ϕ_d of the mobile robot is obtained independently of the trajectory described in the plane (21), by means of a sinusoidal signal of the form

$$\phi_d(t) = \phi_{\max} \sin(ft) \qquad (23)$$

where $\phi_{\max}$ is the magnitude of the desired orientation angle and f is a desired orientation frequency.

The control scheme considered in this work is showed in the Figure 2 where the main elements of the proposed structure are described.

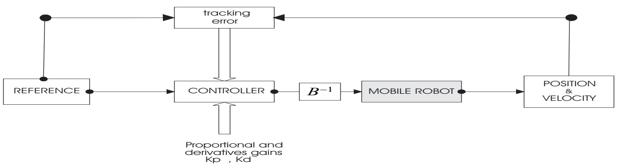

Fig. 2. General control scheme

All simulations experiments were carried out by considering a set of physical parameters for the dynamic model (7) given by,

$$\begin{gathered} Mp = 9.58 \text{ Kg}, \;\; Ir = 0.52 \text{ Kgm}^2, Ip = 0.17 \text{ Kgm}^2 \\ L = 0.205 \text{ m}, \;\; r = 0.03965 \text{ m}. \end{gathered}$$

The design parameters involved in the feedback control law (11), were considered as,

$$\begin{gathered} K_{p1} = 600, K_{p2} = 450, K_{p3} = 100 \\ K_{d1} = 700, K_{d2} = 650, K_{d3} = 200 \end{gathered}$$

It is considered for the desired trajectory a radius $r = 0.5$ m. with initial conditions for the mobile robot given as $(x, y, \phi, \dot{x}, \dot{y}, \dot{\phi}) = (0.1, -0.4, 0, 0, 0, 0)$ and a total execution time of $t_f = 40$ sec. In Figure 3 is shown the input torque applied to each wheel of the omnidirectional robot, while in Figure 4, the tracking errors $e_x = x - x_d$, $e_y = y - y_d$ and $e_\phi = \phi - \phi_d$ are depicted. The convergence of the velocity errors are presented in Figure 5 and finally, in Figure 6 it is shown the evolution of the robot over the $X-Y$ plane.

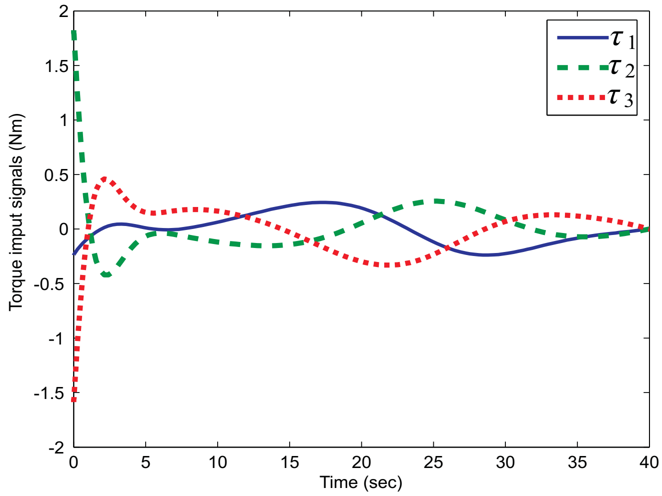

Fig. 3. Imput torque applyied to each wheel.

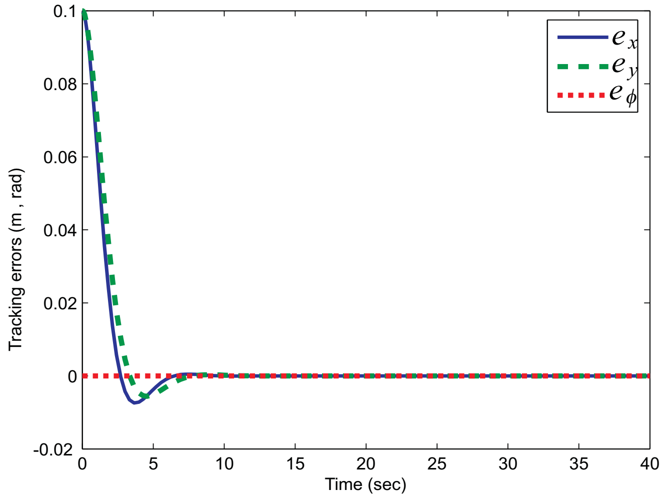

Fig. 4. Tracking errors $e_x = x - x_d$, $e_y = y - y_d$ and $e_\phi = \phi - \phi_d$.

5. CONCLUSIONS

In this work it is presented an extension of the well known computed torque control commonly used in the field of robot manipulators to the case of an omnidirectional mobile robot. With this purpose, the dynamic model of the omnidirectional mobile robot it is rewritten in order to obtain a constant positive definite inertia matrix. This fact allows a clear and simple extension of the computed torque control strategy to the case of a (3,0) robot. The use of the dynamic model allows the consideration of several physics effects that are not contemplated by a kinematics based control perspective. Although in this work, the friction and slipping effects between the wheels and the working surface are neglected, the obtained results show an acceptable performance. The closed loop stability of the system is formally analyzed stating the appropriate error convergence.

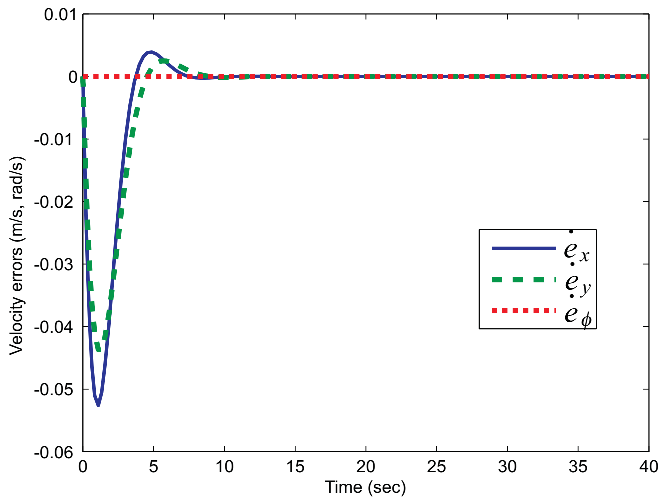

Fig. 5. Velocity errors $\dot{e}_x = \dot{x} - \dot{x}_d$, $\dot{e}_y = \dot{y} - \dot{y}_d$ and $\dot{e}_\phi = \dot{\phi} - \dot{\phi}_d$.

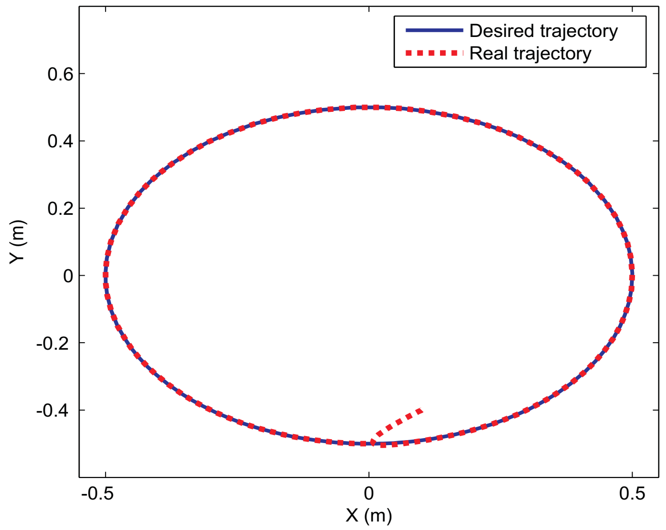

Fig. 6. Evolution on the $X - Y$ plane.

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