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A Practical Two-Input Single-Output Fuzzy Logic Controller with an Alpha-Beta Prefilter

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Abstract: This paper deals with the design of a practical two-input single-output (TISO) fuzzy logic controller for a nonlinear system. We firstly define two generalized errors for a nonlinear system and develop a stable feedback control scheme. Then a novel fuzzy logic controller is designed and incorporated into this scheme to achieve a better tracking response. The key feature of the proposed control scheme is that a discrete prefilter, called alpha-beta filter is placed in front of the fuzzy logic controller. The significance of using the Kalman-like fixed-gain filter is its capability of suppressing noise and feeding the fuzzy logic controller with smooth signals. From an implementation point of view, the discrete TISO fuzzy logic controller with the alpha-beta prefilter is believed to be quite simple, general and useful for practical applications. The validity of the proposed control scheme is verified through practical testing on the experimental setup called magnetic levitation system. The test results strongly suggest that the newly proposed control scheme is simple yet effective for use in a variety of feedback control systems.

1. INTRODUCTION

Due to its robustness against parameter variations and external disturbance, sliding mode control has been an effective method for controlling nonlinear systems with uncertainties (Utkin, 1977). A key step in the design of the controller is to introduce a transformation of tracking errors to generalized errors so that an n^{th}-order tracking problem can be transformed into an equivalent 1^{st}-order stabilization problem. Since equivalent 1^{st}- order problem is likely to be simpler to handle, a control law may thus be developed to achieve the so-called reaching condition. Unfortunately, an ideal sliding mode controller inevitably has a discontinuous switching function. That will cause a chatting phenomenon and excites un-modeled high frequency dynamics. Such control signal is imperfect and undesirable in practice (Edwards & Spurgeion, 1998)(Slotine & Li, 1991). The widely adopted technique is using a continuous approximation to the discontinuous function to suppress the chattering. Furthermore, there are various compensation strategies have been proposed to reach a better compromise between low chatter and good tracking precision. These include integral sliding control (Baik *et al*., 1996)(Chern & Wu, 1993), sliding control with time-varying boundary layers (Slotine, 1984), and especially fuzzy sliding mode control (Hwang & Li, 1992)(Palm *et al*., 1996). Although being verified to be able to suppress the chattering and result in better tracking performances, they seem not to be feasible for practical implementation.

In this paper, we employed a variant idea from the complementary variable structure control to develop a novel fuzzy sliding mode control scheme (Su & Wang, 2002). By defining two generalized errors as inputs, a two-input single-output (TISO) fuzzy logic controller for n^{th}-order systems will be established. Furthermore, a fixed-gain Kalman-like filter, called alpha-beta filter (Bassem, 2000)(Benedict & Bordner, 1962), is placed in front of the TISO fuzzy controller and used as prefilter to obtain a noise free, smooth signals for the inputs of the fuzzy controller. The proposed control scheme will be shown to result in a stable closed-loop system satisfying a stability condition. To illustrate the effectiveness of the design, an experimental magnetic levitation system will be taken as a test example.

2. A TISO FUZZY CONTROL SCHEME

In this section, we'll present a complementary variable structure control scheme to provide a theoretical basis in which a TSISO fuzzy controller can be based. We consider a nonlinear system and which equations can be written as (Utkin, 1977)

$$\overset{(n)}{y} = f_m\left(y, \dot{y}, \cdots, \overset{(n-1)}{y}\right) + g_m\left(y, \dot{y}, \cdots, \overset{(n-1)}{y}\right)u \qquad (1)$$

where $u, y \in \Re$ are the input and output, respectively; $f_m, g_m : \Re^n \to \Re$, are assumed unknown but bounded by known function F_m and G_m.

The control object is to design a control u such that the output y will approximately track a desired signal, y_d, which is assumed to be n^{th}-order continuously differentiable and all of its derivatives are uniformly bounded. Given the tracking error e as

$$e(t) = y(t) - y_d(t) \tag{2}$$

For any $\lambda > 0$, we can define a generalized error and an additional complementary transformation as follows (Su & Wang, 2002)

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^n \xi \tag{3}$$

$$s_c(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \left(\frac{d}{dt} - \lambda\right) \xi \tag{4}$$

where $\xi(t) = \int^t e(\tau) d\tau$. Eq. (4) likes so-called integral sliding mode control which provides a useful transformation such that the original n^{th}-order tracking problem can be transformed into an equivalent 1^{st}-order stabilization problem. To proceed with the design of the complementary variable structure control which was proposed in Su & Wang, (2002), we can show that the control law

$$u(x) = \frac{G_m}{\rho}(-\hat{w} + v) \tag{5}$$

with

$$G_m = 1\Big/\sqrt{\beta_m^{\min}\beta_m^{\max}} \tag{6}$$

$$v = -\frac{K}{1-k_1} sat\left(\frac{s+s_c}{\Phi}\right) \tag{7}$$

$$k_1 = \sigma/\rho \tag{8}$$

where $sat(\cdot)$ is the saturation function, will result in a closed-loop system satisfying the reaching condition:

$$s\dot{s} + s_c\dot{s}_c \le -(s+s_c)^2 - \eta|s+s_c| \tag{9}$$

provided

$$K \ge \beta_m[\eta + F_m(X)] + |(\beta_m - 1)\hat{w}| + k_1|\hat{w}| \tag{10}$$

where $\hat{w} = \sum_{k=0}^{n-1}\binom{n+1}{k+1}\lambda^{n-k} e^{(k)} + \lambda^{n+1}\xi - y_d^{(n)}$, $\beta_m = \sqrt{\beta_m^{\max}/\beta_m^{\min}}$, $0 < \beta_m^{-1} \le g_m G_m \le \beta_m$, $\Phi > 0$ and $\eta > 0$.

From the definitions (4), (5) of s and s_c, one has

$$s_2 = \frac{s+s_c}{2} = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \tag{11}$$

where s_2 is the conventional sliding function. However, the traditional sliding mode control will cause the chattering phenomenon during the sliding surface, which is disadvantageous to control systems. Therefore, we consider apply the fuzzy control approach to deal with the chattering problem. From the state feedback control law (5), we rewrite (7) to be a fuzzy controller which as

$$v_f = -K_f Fuzzy(s, s_c) \tag{12}$$

with the defined two generalized error as fuzzy inputs.

3. FUZZY CONTROLLER DESIGN WITH AN ALPHA-BETA PREFILTER

With the consideration of realization, we design an alpha-beta filter to produce discrete smoothing input signals to the fuzzy controller, which will helpful in practical implement. A schematic diagram is shown in Fig. 1.

According to the relationship,

$$s_1 = \frac{s-s_c}{2\lambda} = \left(\frac{d}{dt} + \lambda\right)^{n-1} \xi \tag{13}$$

and (11). Clearly,

$$\dot{s}_1 = s_2 \tag{14}$$

$$\dot{s}_2 = v + \Delta(x, v) \tag{15}$$

where Δ is regarded as noise, and the measurement of s_1. Based on the alpha-beta filter algorithm with measurement s_1 is performed, the smoothed estimations at the time instant nT are given by Bassem (2000)

$$\begin{aligned}\begin{bmatrix}\hat{s}_1(n)\\ \hat{s}_2(n)\end{bmatrix} &= \begin{bmatrix}\hat{s}_1(n|n-1)\\ \hat{s}_2(n|n-1)\end{bmatrix} + \hat{K}\left(s_1(n) - \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\hat{s}_1(n|n-1)\\ \hat{s}_2(n|n-1)\end{bmatrix}\right)\\ &= \begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}\begin{bmatrix}\hat{s}_1(n-1)\\ \hat{s}_2(n-1)\end{bmatrix} + \hat{K}\left(s_1(n) - \hat{s}_1(n|n-1)\right)\end{aligned} \tag{16}$$

where T be the sampling interval, $\hat{s}_1(n|n-1)$ and $\hat{s}_2(n|n-1)$ are the prediction signals based on the observations up to the time instant $(n-1)T$, and $(s_1(n)-\hat{s}_1(n|n-1))$ represents the prediction error. The Kalman-like fixed gain, $\hat{K}$, which taken to be

$$\hat{K} = \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix} \tag{17}$$

Fig. 2 depicts the architecture of the alpha-beta filter. The initialization are given as

$$\begin{aligned} &\hat{s}_1(1) = \hat{s}_1(2\,|\,1) = s_1(1) \\ &\hat{s}_2(1) = 0 \\ &\hat{s}_2(2) = [s_1(2) - s_1(1)]/T \end{aligned} \tag{18}$$

In general, noise is assumed to be a zero mean random process with variance equal to σ_v^2. Therefore, the variance reduction ration (VRR) ratios are, respectively, given by

$$VRR_{s_1} = \frac{2\alpha^2 - 3\alpha\beta + 2\beta}{\alpha(4-2\alpha-\beta)} \tag{19}$$

$$VRR_{s_2} = \frac{1}{T^2}\frac{2\beta^2}{\alpha(4-2\alpha-\beta)} \tag{20}$$

The VRR ratios are normally used for determining the reduction of measurement noise. However, the maneuverability performance of the filter depends heavily on the choice of the parameters, α and β.

There is a convenience optimum criterion for α and β to be designed to satisfy some performance requirement. The special version was developed by Benedict and Border (1962),

$$\beta = \alpha^2/(2-\alpha) \tag{21}$$

And the VRR rations (19)-(20) can be rewritten as

$$VRR_{s_1} = \frac{\alpha(6-5\alpha)}{\alpha^2 - 8\alpha + 8} \tag{22}$$

$$VRR_{s_2} = \frac{2}{T^2}\frac{\alpha^3/(2-\alpha)}{\alpha^2 - 8\alpha + 8} \tag{23}$$

With the Eq. (21), the appropriate parameters of filter can be optimized by adjusting value of α.

4. EXPERIMENTAL EXAMPLE: A MAGNETIC LEVITATION SYSTEM

4.1 Modelling

The proposed TISO fuzzy control scheme with an alpha-beta prefilter presented in this paper was implemented and tested on an experimental magnetic levitation system. The image and principal scheme of the practical magnetic levitation system are shown in Fig. 3 and Fig. 4, respectively. We can observe that the apparatus is a high nonlinearity cascade-connected system from which mathematical model as descried as follows

$$\dot{x} = v \tag{24a}$$

$$\dot{v} = (-2.3895v - 9.7969) + \left[\frac{0.2581\times10^{-3}}{(0.006-x)^2}\right] i^2 \tag{24b}$$

$$\dot{i} = -\frac{1000}{3} i + 500u \tag{24c}$$

$$y = 200x \tag{24d}$$

where x is the ball position between 0 and 0.005(m), i is the coil current, u is the model input and y is the A/D converter output. The objective of the control is to keep the displacement of the ferric under the influence of magnetic force following a desired trajectory as closed as possible. For a class of cascade-connected system as the magnetic levitation system, we can separately design controller for the driven subsystem (24a)-(24b) and driving subsystem (24c). Once we enable the driven subsystem is asymptotically stable, we just ask the output response of driving subsystem need to converge to stable in a finite time.

4.2 Controller Design

Taking the Laplace transform of (24c), we obtain that the transient response approach to stable in a quite short time, and which state equation can be directly represented as

$$i = 1.5u \tag{25}$$

Therefore, we just need to design an effective controller for the driven subsystem (24a)-(24b). With the control scheme preceding introduced, we can firstly establish that

$$\begin{aligned} &F_m = 2.3895|v| + 9.7969 \\ &\hat{w} = 3\lambda\dot{e} + 3\lambda^2 e + \lambda^3\xi - \ddot{y}_d \\ &G_m = 0.02325 \\ &\beta_m = 6 \end{aligned} \tag{26}$$

and $\phi = i^2$ is taken as control input. According these parameters, we can derive the control law of (24a)-(24b) as

$$\phi(x) = \frac{G_m}{200}\left(-\hat{w} + v_f\right) \tag{27}$$

where v_f was represented in (12). With (25), the full control of magnetic levitation system is

$$u = \sqrt{\phi}/1.5 \tag{28}$$

where ϕ was represented in (27).

4.3 Experimental Results

For experimentation, we define the parameters for controller as $T = 0.002$, $\lambda = 80$, and $\eta = 0.1$. As the procedure illustrated in Fig. 1, we can attain discrete data from $\hat{s}_1$ and $\hat{s}_2$ which were provided by alpha-beta filter. Taking these discrete data as fuzzy inputs, and we define a set of fuzzy rules as listed in Table 1, one of these rules is exampled as

IF s is *PB* and s_c is PB, *THEN* v_f is *PB*

The fuzzy membership functions of input and output are described in Fig. 5. These fuzzy sets are labelled in the linguistic terms of zero (Z), positive small (PS), positive medium (PM), positive big (PB), negative small (NS), negative medium (NM), and negative big (NB). To illustrate the utility of the proposed estimator-based fuzzy control scheme, we make a comparison with complementary variable structure control with saturation function. The comparison results are displayed in Fig. 6 to Fig. 9, where saturation bound $\Phi = 0.1$ and fuzzy scaling gain $K_S = 0.006$, $K_{Sc} = 0.004$. From the experimentations, as indicated in Fig. 6(a)-(b) and Fig. 8(a)-(b), the performance of the proposed TISO fuzzy logic controller is exceptional when compared with another controller. Specially, a better guaranteed steady-state tracking precision as asserted in the text has been justified, as can be observed from these. In addition, the chattering phenomenon of proposed TISO fuzzy logic controller with an alpha-beta prefilter has been eliminated much more than another controller, as shown in Fig. 7 and Fig. 9.

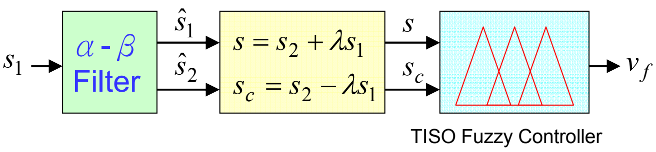

Fig. 1. The structure of a TISO fuzzy controller with alpha-beta filter

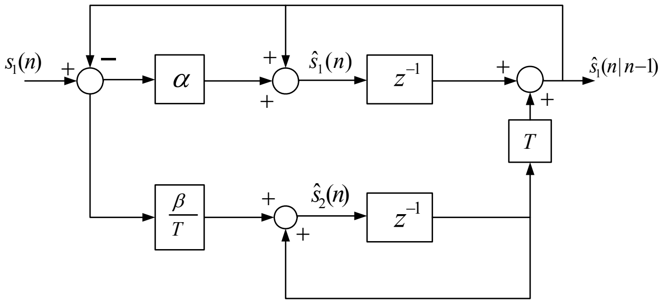

Fig. 2. Architecture of alpha-beta filter

Fig. 3. The image of a practical experimental magnetic levitation system

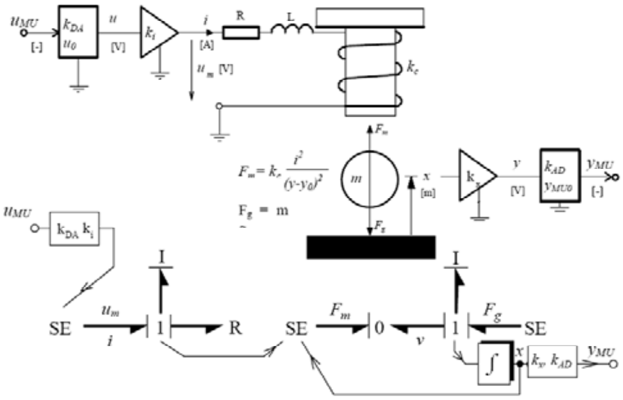

Fig. 4. The principle scheme and bound graph of magnetic levitation system

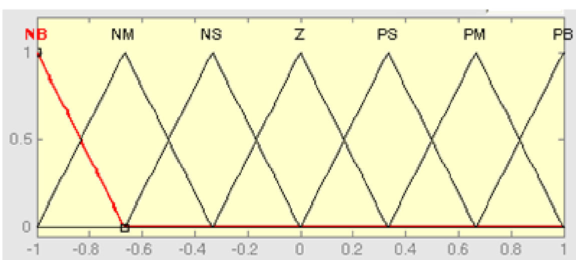

Fig. 5. Fuzzy membership function for input and output

Table 1. The rule table for the fuzzy logic controller

Sc ↓ \ S →	NB	NM	NS	Z	PS	PM	PB
PB	Z	PS	PM	PB	PB	PB	PB
PM	NS	Z	PS	PM	PM	PB	PB
PS	NM	NS	Z	PS	PS	PM	PB
Z	NB	NM	NS	Z	PS	PM	PB
NS	NB	NM	NS	NS	Z	PS	PM
NM	NB	NB	NM	NM	NS	Z	PS
NB	NB	NB	NB	NB	NM	NS	Z

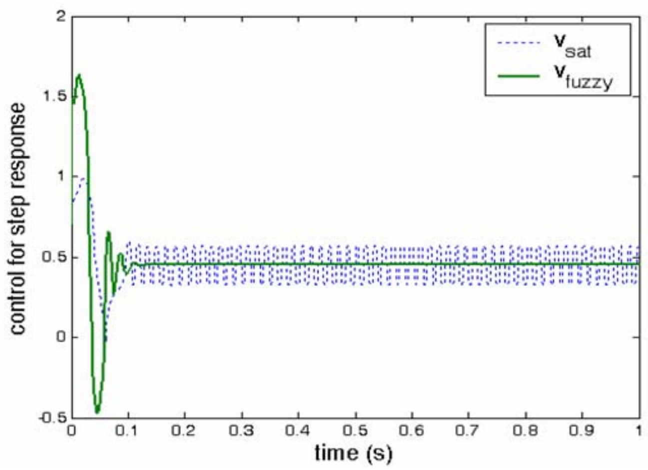

Fig. 7. Controller outputs comparison (step input)

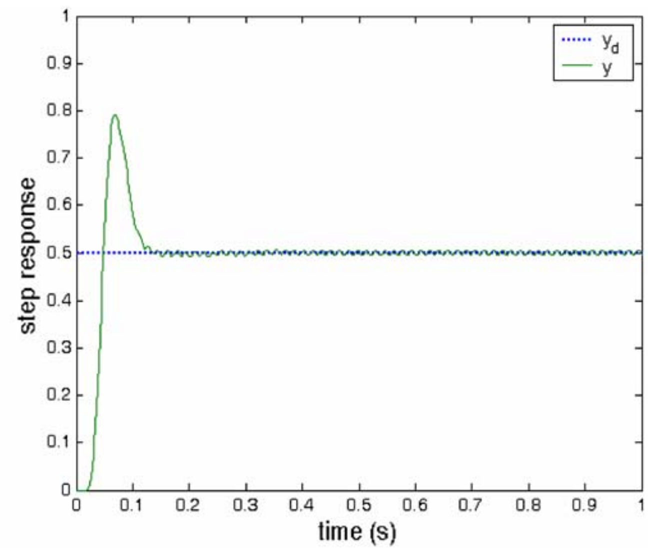

Fig. 6(a). Step response of complementary variable structure control with saturation ($\Phi = 0.1$)

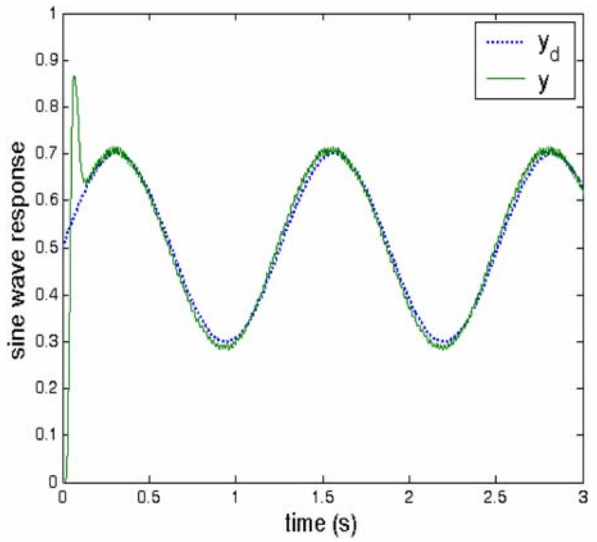

Fig. 8(a). Sine wave response of complementary variable structure control with saturation ($\Phi = 0.1$)

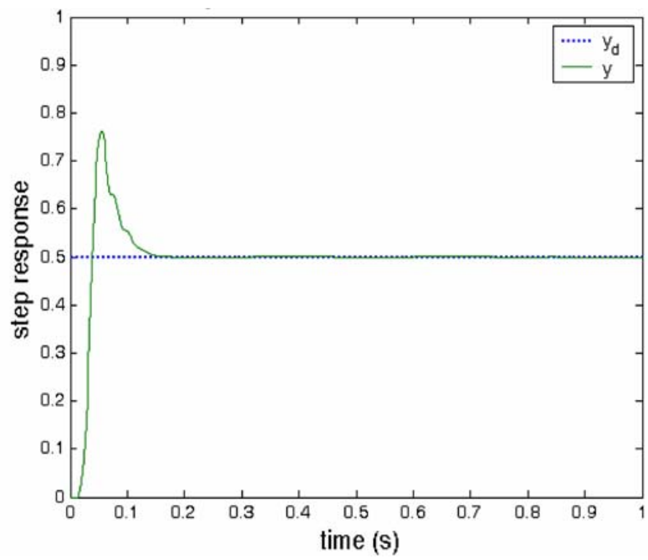

Fig. 6(b). Step response of TISO fuzzy logic control with an alpha-beta prefilter

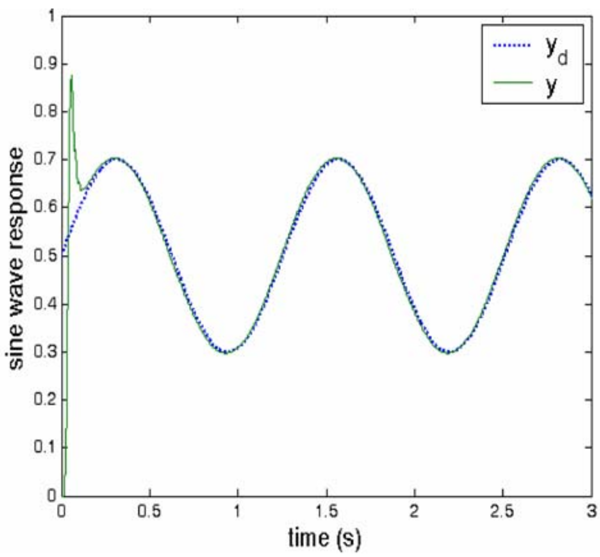

Fig. 8(b). Sine wave response of TISO fuzzy logic control with an alpha-beta prefilter

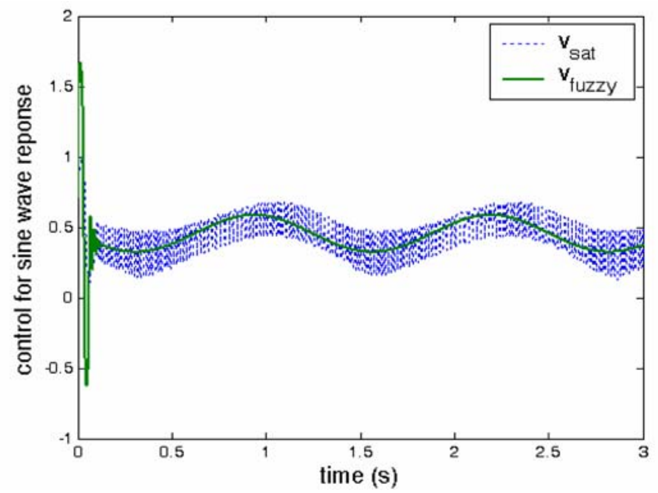

Fig. 9. Controller outputs comparison (sine input)

5. CONCLUSIONS

Under the consideration of practical implementation, we have successfully developed a TISO fuzzy logic controller with an alpha-beta prefilter. The main purpose of using the Kalman-like fixed-gain filter, alpha-beta filter, is not only utilized to suppress the noise, but also provide discrete smooth input signals for the TISO fuzzy controller. The most important features of the newly proposed TISO fuzzy logic control scheme are its simplicity and generic in structure for practical applications. The effectiveness of the designed has been demonstrated through the test on the PC-based magnetic levitation system. Experimental results indicate the discrete TISO fuzzy logic controller with an alpha-beta prefilter is practically implementable and seems to be outstanding in tracking smooth reference signals.

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