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A Distributed Constraint Force Approach for Coordination of Multiple Mobile Robots

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Abstract: A new approach to coordination of multiple mobile robots is presented in this paper. The approach relies on the notion of constraint forces which are used in the development of the dynamics of a system of constrained particles with inertia. A familiar class of dynamic, nonholonomic robots are considered. The goal is to design a distributed coordination control algorithm for each robot in the group to achieve, and maintain, a particular formation while ensuring navigation of the group. The theory of constraint forces is used to generate a stable control algorithm for each mobile robot that will achieve, and maintain, a given formation. The advantage of the proposed method is that the formation keeping forces (constraint forces) cancel only those applied forces which act against the constraints. Another feature of the proposed distributed control algorithm is that it allows to add/remove other mobile robots into/from the formation gracefully with simple modifications of the control input. Further, the algorithm is scalable. To corroborate the theoretical approach, simulation results on a group of six robots are shown and discussed. *Copyright© 2008 IFAC*

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1. INTRODUCTION

Cooperative control of multiple robots has received considerable attention in the last decade due to its wide applications such as moving a large number of objects, environmental monitoring, rescue missions, distributed transportation, and multi-point surveillance. Such tasks generally cannot be accomplished by a single individual robot. The robots are spatially distributed and work together based on either commands given by a supervisor in a centralized control or following some rules and communication strategy designed in advance in a distributed scheme. In many applications, a group of robots is required to follow a predefined trajectory, while maintaining a desired spatial pattern.

The concept of formation control with application to the coordination of multiple mobile robots has been studied extensively in the literature. Some of the existing methods of formation control include potential field methods (Leonard and Fiorello [2001] and Olfati-Saber and Murray [2002]) and optimization-based approaches (Dunbar and Murray [2006]). In the potential function approach, the basic idea is to create an energy like function in terms of the distance constraints between the robots. The negative gradient of the potential function is used as a restoring force on each robot to achieve coordination. In Leonard and Fiorello [2001], an approach for distributed control of multiple agents by using artificial potential functions and virtual leaders was given. The individual agent behaves according to the interaction forces generated by sensing the positions of neighboring agents. In Olfati-Saber and Murray [2002], a specific potential function which is a function of the distance constraints of the desired formation is used. The artificial potential function for obstacle avoidance to multiple vehicles with kinematic models can be found in Dimarogonas et al. [2006]. Instead of relying on repelling potential forces, Chang et al. [2003] present a control law for multiple systems based on gyroscopic forces for collision and obstacle avoidance; the gyroscopic forces were used for obstacle avoidance without affecting the global potential function. Collision avoidance for multi-agent systems using the avoidance control approach was discussed in Stipanovic et al. [2007].

Since the motion of many common mobile robots in practice are subject to nonholonomic constraints, the coordination problem is generally more complicated. Cooperative control of multiple mobile robots with nonholonomic constraints has been addressed in Tanner et al. [2004], Loizou et al. [2004], Liang and Lee [2006], Lawton et al. [2003]. In Tanner et al. [2004], the motion of a group of nonholonomic mobile agents is controlled in a distributed fashion to exhibit flocking behavior. In Loizou et al. [2004], a decentralized navigation function method together with a dipolar potential field is used to stabilize multiple agents with nonholonomic kinematics from an initial configuration to a final configuration. In Liang and Lee [2006], a decentralized formation control scheme for a group of nonholonomic mobile robots based on the potential function method is presented. In Lawton et al. [2003], the mobile robot dynamics are feedback linearized into a double integrator dynamics with output as the robot hand position; a behavior-based approach is proposed to formation maneuvers.

In this work, the notion of constraint forces is used to build a formation from arbitrary initial conditions for multiple mobile robots. The idea of constrained dynamics is that the description of the system includes not only the external forces acting on the particles, but also the constraint forces which limit the motion of the system. An approach to imposing geometric constraints on a system of particles is to add a set of constraint forces to the particles' governing equations which keep the constraints satisfied for all time (see Goldstein [1953] and Udwadia and Kalaba [1996]). The structural distance constraints for a desired formation are converted to constraint forces such that the desired formation can be maintained when the constraint forces are added to the dynamics of the robots. The key idea of the proposed work is to use the notion of constraint forces to determine the total forces required on each robot to achieve, and maintain, the distance constraints of the formation. A centralized control strategy with full information for formation of a group of vehicles using the notion of constraint forces was given in Zou et al. [2007]. In this work we give a distributed control strategy for coordination of multiple mobile robots.

In the potential (or penalty) function approach, the square of the constraint function (or some other appropriate positive function of constraints) is treated as potential energy and a formation keeping force that is proportional to the gradient of the potential energy is used. Since these restoring forces, which rely on displacements, are regular forces, they compete with every other applied force. The advantage of the constraint force approach is that the calculated constraint forces cancel only those applied forces that act against the constraints. The main contribution of this paper is in the development of a stable, distributed control algorithm for multiple robots using constraint forces that will simultaneously achieve, and maintain, a given formation together with tracking of a desired group trajectory. Moreover, a safe distance between communicating robots is maintained at all times by using a specific form of the constraint function.

The rest of the paper is organized as follows. Section 2 gives the problem statement. In Section 3, the distributed constraint force approach for the coordination of multiple mobile robots is developed for the hand position dynamics. First, to illustrate the method, a stable control algorithm that will achieve and maintain a desired distance between two robots is designed based on the notion of constraint force between them. Then, a stable, distributed control algorithm is proposed for an arbitrary number of mobile robots with a given information flow pattern between robots. Section 4 gives simulation results on an example of six mobile robots. Conclusions and future work are given in Section 5.

2. PROBLEM STATEMENT

Consider the dynamic model of a mobile robot i (see Figure 1):

$$\begin{aligned} \dot{x}_i &= v_i \cos\theta_i \\ \dot{y}_i &= v_i \sin\theta_i \\ \dot{\theta}_i &= \omega_i \\ \dot{v}_i &= \frac{F_i}{m_i} \\ \dot{\omega}_i &= \frac{\tau_i}{J_i} \end{aligned} \tag{1}$$

where (x_i, y_i) is the inertial position of the i-th robot, θ_i is the orientation, v_i is the translational velocity, ω_i is the angular velocity, F_i is the applied force, τ_i is the applied torque, m_i is the mass, and J_i is the moment of inertia.

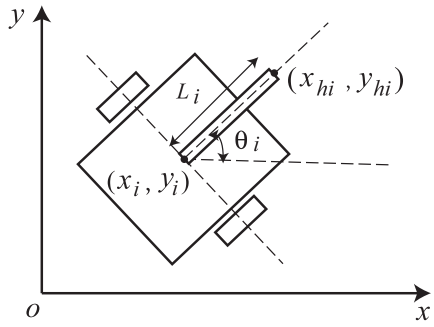

Fig. 1. Mobile Robot

The formation structural topology of the mobile robots can be defined as a formation graph, which will allow us to study the relative position of robots in the group by applying graph theory.

Definition 1. The *formation graph* of n robots is an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is a finite set of vertices (nodes) in correspondence with n robots in the group and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges (i, j) representing inter-robot position specifications.

The neighborhood set of the i-th robot, $\mathcal{N}_i = \{j \mid (i, j) \in \mathcal{E}, j \neq i\}$, includes all the robots which communicate with the i-th robot. The information flow graph and the formation graph are assumed to be identical.

The goal of the paper is to construct a distributed control algorithm for each robot, which depends only on the information available to the robot via the information flow graph, that is capable of driving the group of n robots from any initial configuration to a desired configuration given by the formation graph. Further, we will also require that the group follow a desired navigation trajectory.

3. DISTRIBUTED CONTROL DESIGN

The mobile robot dynamics can be feedback linearized to a two dimensional double integrator, if only an off-wheel axis point of the robot is required to be maintained in formation (see Lawton et al. [2003]). The robot hand position is defined as a point located at a distance $L_i > 0$ from the center of the robot and on the robots' axis of orientation. The coordinates of the hand position (x_{h_i}, y_{h_i}) are

$$x_{h_i} = x_i + L_i \cos\theta_i, \tag{2}$$
$$y_{h_i} = y_i + L_i \sin\theta_i. \tag{3}$$

Differentiating Eqs. (2) and (3) twice and using the mobile robot model (1) gives the relation between the hand position and the applied force/torque on the robot as

$$\begin{bmatrix} \ddot{x}_{h_i} \\ \ddot{y}_{h_i} \end{bmatrix} = G_i + H_i \begin{bmatrix} F_i \\ \tau_i \end{bmatrix} \quad (4)$$

where $G_i = \begin{bmatrix} -v_i\omega_i \sin\theta_i - L_i\omega_i^2 \cos\theta_i \\ v_i\omega_i \cos\theta_i - L_i\omega_i^2 \sin\theta_i \end{bmatrix}$ and $H_i = \begin{bmatrix} \cos\theta_i/m_i & -L_i \sin\theta_i/J_i \\ \sin\theta_i/m_i & L_i \cos\theta_i/J_i \end{bmatrix}$.

Since $\det(H_i) = \frac{L_i}{m_i J_i} \neq 0$, choosing the output feedback linearizing control as

$$\begin{bmatrix} F_i \\ \tau_i \end{bmatrix} = H_i^{-1} \left(\begin{bmatrix} u_{x_i} \\ u_{y_i} \end{bmatrix} - G_i \right) \quad (5)$$

gives the following double integrator dynamics for the hand position:

$$\ddot{r}_i = u_i \quad (6)$$

where $r_i = [x_{h_i}, y_{h_i}]^T \in \mathbb{R}^2$, and $u_i = [u_{x_i}, u_{y_i}]^T \in \mathbb{R}^2$.

We will use the hand position dynamics (6) to describe the distributed control algorithm. First, we will derive the constraint force between a pair of communicating robots; the application of such a force on each robot, in addition to the external forces, will ensure that the constraint is satisfied between the two robots. Then, the distributed control algorithm for the whole group of robots will be developed based on the given information flow graph.

3.1 Constraint Force Between a Pair of Mobile Robots

Consider a pair of robots i and j which share an edge in the information flow graph, i.e., $(i,j) \in \mathcal{E}$. Denote the constraint corresponding to the (i,j) edge in the formation graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ by $\phi_{ij}(r_i, r_j) = 0$ for any $(i,j) \in \mathcal{E}$ and $i \neq j$, with $\phi_{ij}(r_i, r_j)$ in some specific form, for example, the form that corresponds to the Euclidean distance between the two robot hands. Define the composite hand position vector of two communicating robots by $r_{ij} = [r_i^T, r_j^T]^T \in \mathcal{R}^4$. The constraint function is defined as

$$\phi_{ij}(r_{ij}) = \frac{\|r_i - r_j\| - d_{ij}}{\|r_i - r_j\| - r_s}, \ \forall\, (i,j) \in \mathcal{E}, \quad (7)$$

where d_{ij} is the length of the edge (i,j) which is the desired distance between the hand positions of the two robots in the formation and r_s is the safe distance between any two communicating robots. The structural constraint for this pair of robots can be expressed as

$$\phi_{ij}(r_{ij}) = 0. \quad (8)$$

Differentiating Eq. (7) once, we get the constraint velocity as

$$\dot{\phi}_{ij}(r_{ij}, \dot{r}_{ij}) = \frac{\partial \phi_{ij}(r_{ij})}{\partial r_{ij}} \dot{r}_{ij} := A_{ij}(r_{ij}) \dot{r}_{ij} \quad (9)$$

where $A_{ij}(r_{ij}) = \frac{\partial \phi_{ij}(r_{ij})}{\partial r_{ij}}$ is a specially structured 1×4 matrix called the constraint matrix, which is given by

$$A_{ij}(r_{ij}) = [a_{ij}, \ -a_{ij}] \quad (10)$$

with $a_{ij} = \frac{(d_{ij} - r_s)(r_i - r_j)^T}{(\|r_i - r_j\| - r_s)^2 \|r_i - r_j\|}$.

Differentiating $\dot{\phi}_{ij}(r_{ij}, \dot{r}_{ij})$, we get the constraint acceleration as

$$\ddot{\phi}_{ij}(r_{ij}, \dot{r}_{ij}, \ddot{r}_{ij}) = \dot{A}_{ij}(r_{ij}, \dot{r}_{ij}) \dot{r}_{ij} + A_{ij}(r_{ij}) \ddot{r}_{ij} \quad (11)$$

where $\dot{A}_{ij}(r_{ij}, \dot{r}_{ij}) = \frac{\partial \dot{\phi}_{ij}(r_{ij}, \dot{r}_{ij})}{\partial r_{ij}}$.

Assuming that the configuration r_{ij} and the velocity $\dot{r}_{ij}$ both have the desired initial values, i.e., $\phi_{ij}(r_{ij}^0) = \dot{\phi}_{ij}(r_{ij}^0, \dot{r}_{ij}^0) = 0$, the velocity and acceleration, respectively, that are consistent with the constraint are given by

$$\dot{\phi}_{ij}(r_{ij}, \dot{r}_{ij}) = 0, \ \ddot{\phi}_{ij}(r_{ij}, \dot{r}_{ij}, \ddot{r}_{ij}) = 0. \quad (12)$$

That is, if the two robots, i and j, begin their motion at the position and velocity that is initially consistent with the constraint, subsequent motion of the robots satisfies the position and velocity constraints if we ensure that the acceleration constraint equation in (12) is satisfied. Now, the pertinent question is, how do we ensure that the acceleration constraint equation is met for all time? The answer lies in finding the constraint forces to make this possible. The constraint forces will limit the motion of the system such that the constraints are satisfied. In addition to depending on the state of the system, the constraint forces also depend on the other applied forces. The dynamics of the constrained system can be written as

$$\ddot{r}_{ij} = F_{n_{ij}} + F_{c_{ij}} \quad (13)$$

where $F_{c_{ij}} = [F_{c_i}^T, F_{c_j}^T]^T$ is the constraint force that keeps the accelerations consistent with the acceleration constraint equation and $F_{n_{ij}} = [F_{n_i}^T, F_{n_j}^T]^T$ is a composite vector of applied forces on the two robots. The constraint force satisfies the following equation:

$$A_{ij}(r_{ij}) F_{c_{ij}} = -\dot{A}_{ij}(r_{ij}, \dot{r}_{ij}) \dot{r}_{ij} - A_{ij}(r_{ij}) F_{n_{ij}}. \quad (14)$$

Equation (14) alone does not uniquely determine the constraint force, since we have only one equation and four unknowns (the 4 components of $F_{c_{ij}}$). The widely used procedure in dynamics is to use the principle of virtual work (Goldstein [1953]) to obtain the constraint forces, which states that the constraint forces do not add or remove energy. Therefore, to ensure that the constraint force does no work, we require that $F_{c_{ij}}^T \dot{r}_{ij}$ be zero for every $\dot{r}_{ij}$ satisfying $\dot{\phi}_{ij}(r_{ij}, \dot{r}_{ij}) = 0$, that is,

$$F_{c_{ij}}^T \dot{r}_{ij} = 0, \ \forall\, \dot{r}_{ij} \in \{\dot{r}_{ij} \mid A_{ij}(r_{ij}) \dot{r}_{ij} = 0\}. \quad (15)$$

From Eq. (15), it is clear that $F_{c_{ij}}$ must be orthogonal to the velocity vector $\dot{r}_{ij}$. Since $\dot{r}_{ij}$ must lie in the null space of $A_{ij}(r_{ij})$, the constraint force $F_{c_{ij}}$ must lie in the null space complement of $A_{ij}(r_{ij})$. Thus, the vector $F_{c_{ij}}$ satisfying Eq. (15) can be expressed in the form

$$F_{c_{ij}} = A_{ij}^T(r_{ij}) \lambda_{ij} \quad (16)$$

where λ_{ij} is the Lagrange multiplier, which is obtained by substituting (16) into (14),

$$A_{ij}(r_{ij}) A_{ij}^T(r_{ij}) \lambda_{ij} = -\dot{A}_{ij}(r_{ij}, \dot{r}_{ij}) \dot{r}_{ij} - A_{ij}(r_{ij}) F_{n_{ij}}. \quad (17)$$

The constraint force for robots i and j are then given by $F_{c_i} = [I_2, \ \emptyset_2] F_{c_{ij}}$ and $F_{c_j} = [\emptyset_2, \ I_2] F_{c_{ij}}$ where I_2 and $\emptyset_2$ are the identity and zero matrices, respectively, of dimension two.

The discussion on the development of constraint forces so far was based on the assumption that at the start of the motion of the robots, the constraint equations are satisfied. To consider arbitrary initial conditions for the robots, which do not satisfy the constraint equations, we will use the notion of feedback in the constraint acceleration equation; this will account for the mismatch in the initial condition and appropriately compute the constraint force. This idea was used to prevent numerical drift in the simulation of dynamic equations with constraints in Witkin et al. [1990]. In the application of cooperative control of a group of robots, we generally require desired formation of the group as well as navigation. Instead of solving for $\ddot{\phi}_{ij} = 0$ to determine the constraint force, as it was done earlier, the following equation will be used:

$$\ddot{\phi}_{ij} = -k_d\dot{\phi}_{ij} + g_{ij} \tag{18}$$

where k_d is a positive constant and the function g_{ij} is included to couple the constraint force and constraint acceleration; note that g_{ij} acts like a force on a robot with position given by ϕ_{ij}. Therefore, the constraint force vector for the two robots is calculated based on the following equations:

$$F_{c_{ij}} = A_{ij}^T\lambda_{ij}, \tag{19}$$

$$A_{ij}A_{ij}^T\lambda_{ij} = -\dot{A}_{ij}\dot{r}_{ij} - A_{ij}F_{n_{ij}} - k_d\dot{\phi}_{ij} + g_{ij}. \tag{20}$$

Next, we will derive the function g_{ij} based on the two robots achieving, and maintaining, a formation with a desired navigation trajectory. The navigational feedback control F_{n_i} acting on the i-th robot is chosen as

$$F_{n_i} = \ddot{r}_{d_i} - c_1e_i - c_2\dot{e}_i \tag{21}$$

where c_1 and c_2 are positive constants, $e_i = r_i - r_{d_i}$ and $\dot{e}_i = \dot{r}_i - \dot{r}_{d_i}$ are navigational tracking errors, and $r_{d_i}, \dot{r}_{d_i}$ and $\ddot{r}_{d_i}$ are the desired position, velocity, and acceleration, respectively. The navigational force and tracking errors for the j-th robot are given by replacing the index i with j.

The constraint forces F_{c_i} and F_{c_j} are calculated using (19) and (20). The overall control input u_i for the i-th robot in Eq. (6) is given by

$$u_i = F_{n_i} + F_{c_i}. \tag{22}$$

We select the function g_{ij} in (18) based on whether this control input tracks the desired navigation trajectories and achieves, and maintains, the desired distance between the two robots. We consider the following Lyapunov function candidate

$$E_{ij} = \frac{1}{2}k_1e_{ij}^Te_{ij} + \frac{1}{2}k_2\dot{e}_{ij}^T\dot{e}_{ij} + \frac{1}{2}\dot{\phi}_{ij}^2 \tag{23}$$

where k_1 and k_2 are positive constants and $e_{ij} = [e_i^T, e_j^T]^T$. The derivative of E_{ij} with respect to time is given by

$$\begin{aligned}\dot{E}_{ij} &= k_1\dot{e}_{ij}^Te_{ij} + k_2\dot{e}_{ij}^T\ddot{e}_{ij} + \dot{\phi}_{ij}\ddot{\phi}_{ij}\\ &= k_1\dot{e}_{ij}^Te_{ij} + k_2\dot{e}_{ij}^T(F_{n_{ij}} + F_{c_{ij}} - \ddot{r}_{d_{ij}}) + \dot{\phi}_{ij}(-k_d\dot{\phi}_{ij} + g_{ij})\\ &= -k_2c_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 - (-k_1 + c_1k_2)\dot{e}_{ij}^Te_{ij}\\ &\quad + k_2\dot{r}_{ij}^TF_{c_{ij}} - k_2\dot{r}_{d_{ij}}^TF_{c_{ij}} + \dot{\phi}_{ij}g_{ij}.\end{aligned}$$

Choosing $k_1 = k_2c_1$ we obtain

$$\begin{aligned}\dot{E}_{ij} &= -k_2c_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 - k_2\dot{r}_{d_{ij}}^TA_{ij}^T\lambda_{ij}\\ &\quad + k_2\dot{r}_{ij}^TA_{ij}^T\lambda_{ij} + \dot{\phi}_{ij}g_{ij}.\end{aligned}$$

Since $\dot{r}_{d_{ij}} = [\dot{r}_{d_i}^T,\ \dot{r}_{d_j}^T]^T$ is the desired navigation velocity, it must satisfy $\dot{r}_{d_{ij}}^TA_{ij}^T = 0$. Further, choosing

$$g_{ij} = -k_2\lambda_{ij} \tag{24}$$

we get

$$\dot{E}_{ij} = -k_2c_2\dot{e}_{ij}^T\dot{e}_{ij} - k_d\dot{\phi}_{ij}^2 \le 0. \tag{25}$$

Therefore, E_{ij} is a Lyapunov function. As a result, e_{ij}, $\dot{e}_{ij}$, and $\dot{\phi}_{ij}$ are bounded. From (23) and (25), we can conclude that $\dot{e}_{ij}$ and $\dot{\phi}_{ij}$ are square integrable signals. Further, from the dynamics of the constraint and the tracking error,

$$\ddot{\phi}_{ij} = -k_d\dot{\phi}_{ij} - k_2\lambda_{ij} \tag{26}$$

$$\ddot{e}_{ij} = -c_1e_{ij} - c_2\dot{e}_{ij} + A_{ij}^T\lambda_{ij}, \tag{27}$$

we can conclude that both $\ddot{\phi}_{ij}$ and $\ddot{e}_{ij}$ are bounded. Therefore, $\dot{e}_{ij}$ and $\dot{\phi}_{ij}$ converge to zero asymptotically by invoking Barbalat's lemma. Further, we can show via direct calculation that $\dddot{\phi}_{ij}$ and $\dddot{e}_{ij}$ are bounded by differentiating Eqs. (26) and (27). Therefore, the signals $\ddot{\phi}_{ij}$ and $\ddot{e}_{ij}$ asymptotically converge to zero. From Eqs. (26) and (27), we can conclude that λ_{ij} and e_{ij} converge to zero asymptotically. Moreover, from the definition of the constraint vector $\phi_{ij}(r_{ij}) = \frac{\|r_i - r_j\| - d_{ij}}{\|r_i - r_j\| - r_s} = \frac{\|(e_i - e_j) + (r_{d_i} - r_{d_j})\| - d_{ij}}{\|r_i - r_j\| - r_s}$, asymptotic convergence of e_{ij} to zero implies asymptotic convergence of $\phi_{ij}(r_{ij})$ to $\frac{\|(r_{d_i} - r_{d_j})\| - d_{ij}}{\|r_i - r_j\| - r_s}$, which is zero. Furthermore, since $\dot{\phi}_{ij}$ is bounded and

$$\lim_{\|r_i - r_j\| \to r_s+} \dot{\phi}_{ij}(r_{ij}, \dot{r}_{ij}) = \infty,\ \forall\ (i,j) \in \mathcal{E}, \tag{28}$$

any pair of communicating robots will never enter the unsafe region given by $\Omega = \{r_{ij} : \|r_i - r_j\| \le r_s\}$.

Note that the choice of $g_{ij} = -k_2\lambda_{ij}$ will result in the constrained force to be given by

$$F_{c_{ij}} = A_{ij}^T\lambda_{ij}, \tag{29}$$

$$\lambda_{ij} = \frac{1}{k_2 + A_{ij}A_{ij}^T}\left(-\dot{A}_{ij}\dot{r}_{ij} - A_{ij}F_{n_{ij}} - k_d\dot{\phi}_{ij}\right). \tag{30}$$

The constraint force for robots i and j are then given by $F_{c_i} = [I_2,\ \emptyset_2]F_{c_{ij}}$ and $F_{c_j} = [\emptyset_2,\ I_2]F_{c_{ij}}$. Note that the constraint forces on the two robots satisfy $F_{c_i} = -F_{c_j}$, that is,

$$[I_2,\ \emptyset_2]A_{ij}^T\lambda_{ij} = -[\emptyset_2,\ I_2]A_{ij}^T\lambda_{ij}. \tag{31}$$

Since they are internal forces, addition of the i-th and j-th dynamics will result in the cancelation of these forces for the two robots case.

3.2 Distributed Algorithm for Multiple Mobile Robots

As in the previous section, we consider the navigational feedback control F_{n_i} acting on the i-th robot to be given by (21). The constraint force acting on the i-th robot is chosen as the total of the constraint force contribution from all the robots which directly communicate with it, i.e., F_{c_i} is given by

$$F_{c_i} = \sum_{(i,j)\in\mathcal{E}} [I_2,\ \emptyset_2]\, A_{ij}^T\lambda_{ij}, \tag{32}$$

$$\lambda_{ij} = \frac{1}{k_2 + A_{ij}A_{ij}^T}\left(-\dot{A}_{ij}\dot{r}_{ij} - A_{ij}F_{n_{ij}} - k_d\dot{\phi}_{ij}\right). \tag{33}$$

For example, consider the formation shown in Fig. 2. The constraint force applied on robot 2 is the summation of the constraint forces contributed from robots 1, 3, 4, and 5, that is,

$$F_{c_2} = [I_2,\ \emptyset_2]\,(F_{c_{21}} + F_{c_{23}} + F_{c_{24}} + F_{c_{25}}).$$

The overall control input u_i for the i-th robot in Eq. (6) is given by

$$u_i = F_{n_i} + F_{c_i}. \tag{34}$$

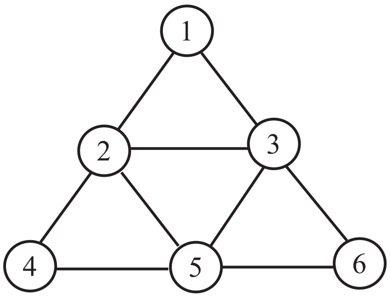

Fig. 2. Delta formation of a group of six robots

To show that this control input tracks the desired navigation trajectories and achieves, and maintains, the given formation, we consider the following composite Lyapunov function candidate:

$$E = \frac{1}{2}\sum_{i=1}^{n} k_2 c_1 e_i^T e_i + \frac{1}{2}\sum_{i=1}^{n} k_2 \dot{e}_i^T \dot{e}_i + \frac{1}{2}\sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}^2. \tag{35}$$

The derivative of E with respect to time is given by

$$\dot{E} = k_2 c_1 \sum_{i=1}^{n} \dot{e}_i^T e_i + k_2 \sum_{i=1}^{n} \dot{e}_i^T \ddot{e}_i + \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\ddot{\phi}_{ij}$$

Substituting the control law (34) and the dynamics of the robots (6), with F_{c_i} given by (32) and F_{n_i} given by (21), into $\dot{E}$, and simplifying, we get

$$\dot{E} = k_2 c_1 \sum_{i=1}^{n} \dot{e}_i^T e_i + k_2 \sum_{i=1}^{n} \dot{e}_i^T \left(F_{n_i} + F_{c_i} - \ddot{r}_{d_i}\right) + \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}(-k_d\dot{\phi}_{ij} - k_2\lambda_{ij}).$$

Upon simplification, we can write $\dot{E}$ as

$$\dot{E} = -k_2 c_2 \sum_{i=1}^{n} \dot{e}_i^T \dot{e}_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}^2 - k_2 \sum_{i=1}^{n} \dot{r}_{d_i}^T F_{c_i} + k_2 \left(\sum_{i=1}^{n} \dot{r}_i^T F_{c_i} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\lambda_{ij} \right). \tag{36}$$

In Eq. (36) , we can show that the third and fourth terms are identically equal to zero. Note that

$$\sum_{i=1}^{n} \dot{r}_i^T F_{c_i} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\lambda_{ij} = \sum_{i=1}^{n} \dot{r}_i^T \sum_{(i,j)\in\mathcal{E}} [I_2,\ \emptyset_2] A_{ij}^T \lambda_{ij} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} A_{ij}\dot{r}_{ij}\lambda_{ij}.$$

Since the constraint force components between any pair of robots satisfy (31), and since $A_{ij} = -A_{ji}$ and $\lambda_{ij} = \lambda_{ji}$, we have

$$[\dot{r}_j^T,\ \emptyset_2] A_{ji}^T \lambda_{ji} = -[\emptyset_2,\ \dot{r}_j^T] A_{ji}^T \lambda_{ji} = [\emptyset_2,\ \dot{r}_j^T] A_{ij}^T \lambda_{ij}.$$

Then,

$$\sum_{i=1}^{n} \dot{r}_i^T \sum_{(i,j)\in\mathcal{E}} [I_2,\ \emptyset_2] A_{ij}^T \lambda_{ij} = \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{r}_{ij}^T A_{ij}^T \lambda_{ij}.$$

Thus, we have

$$\sum_{i=1}^{n} \dot{r}_i^T F_{c_i} - \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}\lambda_{ij} = 0. \tag{37}$$

Further, since $\dot{r}_{d_i}$ is the desired velocity, it must satisfy $\dot{r}_{d_{ij}}^T A_{ij}^T = 0$ for any $i \neq j$. Hence, $\sum_{i=1}^{n} \dot{r}_{d_i}^T F_{c_i} = \sum_{i=1}^{n} \sum_{\substack{(i,j)\in\mathcal{E} \\ j>i}} \dot{r}_{d_{ij}}^T A_{ij}^T \lambda_{ij} = 0.$

Therefore,

$$\dot{E} = -k_2 c_2 \sum_{i=1}^{n} \dot{e}_i^T \dot{e}_i - k_d \sum_{i=1}^{n} \sum_{(i,j)\in\mathcal{E},j>i} \dot{\phi}_{ij}^2 \leq 0. \tag{38}$$

Using the same arguments as in the previous section, we can conclude asymptotic convergence of e_i, $\dot{e}_i$, ϕ_{ij} and $\dot{\phi}_{ij}$ to zero. Further, any pair of communicating robots will never enter the unsafe region, under the assumption that the initial distance between any pair of communicating robots is larger than the safe distance. The results of this section are summarized in the following theorem.

Theorem 1. For a group of mobile robots given by the dynamics (1), the choice of the following control algorithm:

$$\begin{bmatrix} F_i \\ \tau_i \end{bmatrix} = H_i^{-1}\left(u_i - G_i\right) \tag{39}$$

$$u_i = F_{n_i} + F_{c_i} \tag{40}$$

$$F_{n_i} = \ddot{r}_{d_i} - c_1 e_i - c_2 \dot{e}_i \tag{41}$$

$$F_{c_i} = \sum_{(i,j)\in\mathcal{E}} \frac{[I_2,\ \emptyset_2]\, A_{ij}^T}{k_2 + A_{ij}A_{ij}^T} \left(-\dot{A}_{ij}\dot{r}_{ij} - A_{ij}F_{n_{ij}} - k_d\dot{\phi}_{ij} \right) \tag{42}$$

will ensure that all signals are bounded and the signals e_i, $\dot{e}_i$, ϕ_{ij}, and $\dot{\phi}_{ij}$ asymptotically converge to zero.

Remark 1. Note that if new robots are added to the formation with new edges in the formation graph, the constrained force F_{c_i} given by (42) will include additional terms according to the new edge set.

4. SIMULATIONS

This section presents simulation results for a group of six robots to achieve and maintain a delta formation as shown in Fig. 2. The distributed control law given in Theorem 1 with constraint functions of the form (7) is applied with a safe distance of $r_s = 0.22$ m. Each robot in the group starts from an arbitrary location which does not satisfy the constraint equations. The desired distance of each edge in the delta formation is chosen as 1 m. The inertial parameters are selected as $m_i = 10$ kg, $J_i = 0.13$ kg-m^2 and $L_i = 0.1$ m. The initial positions of the six robots are given by $[x_1(0), y_1(0)] = [-1, 1.4]$, $[x_2(0), y_2(0)] = [-0.7, -0.9]$, $[x_3(0), y_3(0)] = [-0.2, 0.3]$, $[x_4(0), y_4(0)] = [1, 2]$, $[x_5(0), y_5(0)] = [0.1, 1]$, $[x_6(0), y_6(0)] = [-0.18, 0]$, where the distance units are all in meters; the starting orientation θ_i, linear speed v_i and angular speed ω_i of the six robots is chosen to be zero. The desired navigation trajectory for each robot within the delta formation is taken as a straight line with constant velocity. The control gain parameters for each robot in the distributed control algorithm are selected as $c_1 = 0.4$, $c_2 = 0.6$, $k_d = 1.8$, and $k_2 = 5$. The result is shown in Fig. 3; each robot in the group starts at the initial position denoted by $\circ$ and the robots reach a desired delta formation while approaching the desired navigation trajectory (dotted line). The corresponding inter-robot distance is shown in Fig. 4, which indicates that the safe distance between two communicating robots is always maintained.

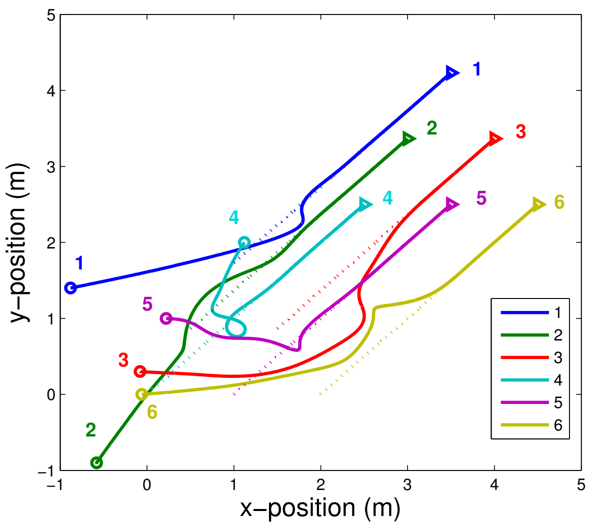

Fig. 3. Delta formation of six robots

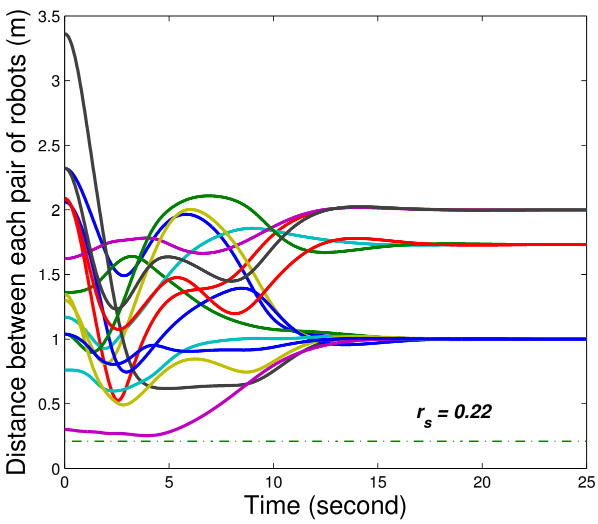

Fig. 4. Distance between pairs of robots

5. CONCLUSIONS

Based on the notion of constraint forces, we have developed a stable, distributed control algorithm for coordination of a group of mobile robots. The dynamics of each mobile robot with nonholonomic constraints is feedback linearized with the output as the off-wheel axis position (hand position) along the longitudinal orientation of the robot. Given a formation, an information flow pattern, and a desired trajectory, the distributed control algorithm developed for each robot in the group is capable of achieving and maintaining the formation along the desired trajectory while maintaining a safe distance between communicating robots. The algorithm is modular and scalable. Simulation results on an example formation of a group of six mobile robots were shown to corroborate the proposed algorithm.

Potential future work includes the use of a sensing region around each mobile robot in which the robot is capable of sensing the presence of other robots. This will facilitate introducing the safe distance notion to all the robots which come within the sensing region.

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