Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Fault diagnosis for switching system using Observer Kalman filter IDentification

A. Akhenak * **L. Bako** *,** **E. Duviella** * **K. M. Pekpe** ** **S. Lecoeuche** *,**

* *Ecole des Mines de Douai, Département Informatique et Automatique. 59508 Douai, France*
** *Laboratoire d'Automatique Génie Informatique et Signal. UMR CNRS 8146, 59655 Villeneuve d'Ascq, France.*
e-mail: {akhenak, bako, lecoeuche}@ensm-douai.fr

Abstract: In this paper we propose a strategy for fault detection and isolation without any fixed model of the system to be supervised. The proposed approach is based on the identification of the parameters characterizing the system without any *a priori* knowledge. Our contribution consists in developing a specific identification scheme that is insensitive to a certain type of faults. The identified parameters are then invariant to the presence of actuator or sensor faults. Thereafter, a fault estimation procedure is proposed in order to detect sensor or actuator faults. The paper ends with a simulation example which highlights the effectiveness of the proposed approach.

1. INTRODUCTION

Model based Fault Detection and Isolation (FDI) depends heavily on the presence of an analytical model of the process. Based on the concept of analytical redundancy, a residual signal is generated by comparing the measured output signal and the estimated one from the nominal system model. After being processed, this residual can be used as an indicator of the fault Frank [1990] Gertler [1998] Chen and Patton [1999] Akhenak et al. [2003]. The main disadvantage of this class of methods is that, being based on fixed nominal model of the system, it can be very sensitive to parameter variations, various operating conditions, disturbances. In that case, natural changes in the dynamics could wrongly be interpreted as faults, what is much penalizing in the case of switching systems.

This problem could be solved by introducing a continuous update of the model on which the monitoring and diagnosis functions rest on. Instead of exploiting the nominal model, the actual system could be approximated by an up-to-date model estimated through identification procedures Pekpe et al. [2004]Pekpe and Lecoeuche [2008]. The diagnosis decision criteria are then based on this up-to-date model. In a general way, a fault or drift acting on the system would be characterized by evolutions of the identified parameters and specific decision rules are built to discriminate the reasons of the evolutions. The benefit is that, in the case of non-stationary system, it could be possible to distinguish natural changes in the dynamics from faults. The decision thresholds are then more robust for the fault detection but the fault estimation is made more complicated Lecoeuche et al. [2006].

In order to improve the fault diagnosis for switching systems, the presented approach is based on a robust identification with respect to a certain type of faults. The key idea is to use, in a first stage, a specific identification scheme which makes the estimation of the parameters invariant to sensor or actuator faults. In this case, we have an up-to-date model where evolutions of its parameters characterize only changes in the system dynamics (new operating conditions, non-stationarities...). The proposed FDI method is dedicated to switched linear systems whose subsystems are described by state space models. The switching times, the orders (that are potentially different from a submodel to another) and the parameters of the submodels are all assumed to be unknown.

Each submodel is identified by using the Observer Kalman filter IDentification (OKID) algorithm through an estimation of the Markov parameters Juang [1994]. In order to guarantee that the identification algorithm is not affected by the sensor or actuator faults, we consider a projection of the regression equation onto the orthogonal complement of the space spanned by the faults considered. These faults are assumed to be constant but the method developed can also apply to slowly time-varying.

As the identified parameters are insensitive to sensor or actuator faults, a second stage is required to detect and to estimate these faults. The adopted approach consists in synthesizing sensitive residuals. One residual design procedure is dedicated to each type of fault. The sensor residual is computed by the comparison between the actual output of the system and its estimate which is obtained by aggregating the past inputs and the Markov parameters on a sliding window of data. The actuator residual is determined by designing an unknown input finite memory observer Nuninger et al. [1998] Alessandri et al. [2005]. The finite memory observer is more efficient because the estimation is based on a limited number of data Darouach et al. [1994].

The paper is organized as follows. Section 2 formulates the problem of identifying a switched system that is subject to a certain class of faults. The OKID algorithm is used to identify the orders and the parameters of the constituent submodels. In Section 3, two methods are proposed for sensor and actuator faults detection and isolation. The last section 4 gives a numerical example to illustrate the effectiveness of the proposed approach.

2. SWITCHING SYSTEM IDENTIFICATION

We consider a linear switched system whose global behavior results from switches among a number of linear subsystems. Each subsystem is described by the following linear, state space model

$$x_{s,k+1} = A_s x_{s,k} + B_s(u_k + \Delta u) + v_k \quad (1a)$$
$$y_k = C_s x_{s,k} + D_s(u_k + \Delta u) + \Delta y + w_k \quad (1b)$$

where the process noise $v_k \in \mathbb{R}^{n_s}$ and the measurement noise $w_k \in \mathbb{R}^l$ of the s-th local model are white noise and uncorrelated with the input $u_k \in \mathbb{R}^m$. The components Δu and Δy refer respectively to actuator and sensor faults that are assumed to be unknown constant additive numbers. $x_{s,k} \in \mathbb{R}^{n_s}$ and $y_k \in \mathbb{R}^l$ represent respectively the state and output vector of s-th model. Each local model is assumed to be stable and active during a minimal time τ.

Given input-output measurements from a system such as (1), our objective in this section is to work out an identification scheme that would be insensitive to a certain class of faults (sensor or actuator faults). By identification of the switched system, we mean that we estimate the order, the parameters of each local linear model and the number of the constituent submodels of (1). Then, without knowing the faults components Δu and Δy, the discrete state s, the dimension n_s of the state vector as well as the the system matrices (A_s, B_s, C_s, D_s) have to be estimated.

To begin with the identification procedure, assume that the s-th local model is active on a time window $[k-\alpha,\ \ k]$ of width $\alpha < \tau$. Then, an input-output description of the above system can be obtained from equation (1) as:

$$y_k = C_s A_s^{\alpha} x_{s,k-\alpha} + \sum_{i=k-\alpha}^{k-1} C_s A_s^{k-i-1} B_s \tilde{u}_i + D_s \tilde{u}_k + \Delta y + \sum_{i=k-\alpha}^{k-1} C_s A_s^{k-i-1} v_i + w_k, \quad (2)$$

where the notation $\tilde{u}_k \doteq u_k + \Delta u$ has been used. By assuming that the system is stable, the influence of the term $C_s A_s^{\alpha} x_{s,k-\alpha}$ on the output y_k at time instants $k > \alpha$ can be neglected. But, for this approximation to hold, α need to be chosen all the larger as the system dynamics are slow.

To make up for this difficulty, the OKID algorithm Phan et al. [1995] introduces, under the assumption that (1) is observable, an observer gain $G_s \in \mathbb{R}^{n_s \times l}$ in order to transform the system (1) into an observer structure whose eigenvalues are close to zero. This results in the following equation:

$$\begin{cases} x_{s,k+1} = \bar{A}_s x_{s,k} + \bar{B}_s z_k + v_k + G_s w_k \\ y_k \quad\ = C_s x_{s,k} + D_s \tilde{u}_k + \Delta y + w_k, \end{cases} \quad (3)$$

where

$$\bar{A}_s = A_s + G_s C_s \in \mathbb{R}^{n_s \times n_s},$$
$$\bar{B}_s = [\, B_s + G_s D_s \ \ -G_s \,] \in \mathbb{R}^{n_s \times (l+m)}, \quad (4)$$
$$z_k = \begin{bmatrix} \tilde{u}_k \\ y_k - \Delta y \end{bmatrix} \in \mathbb{R}^{(l+m)}.$$

The vector z_k is an extended input vector to the modified system expressed by (3).

2.1 Estimation of the Markov parameters of the modified system

In this section, we estimate the Markov parameters of the modified system (3). The s-th local model being active on a time window $[k-\alpha,\ \ k]$ of width $\alpha < \tau$, its output can be expressed (see relation (2)) as:

$$y_k = C_s \bar{A}_s^{\alpha} x_{s,k-\alpha} + \bar{H}_{s,\alpha} \bar{z}_{k,\alpha} + \bar{H}^v_{s,\alpha} v_{k,\alpha} + w_k + \Delta y, \quad (5)$$

where

$$\bar{H}_{s,\alpha} = \begin{bmatrix} D_s & C_s\bar{B}_s & \cdots & C_s\bar{A}_s^{\alpha-1}\bar{B}_s \end{bmatrix} \in \mathbb{R}^{l\times((m+l)\alpha+m)}, \quad (6a)$$
$$\bar{H}^v_{s,\alpha} = \begin{bmatrix} C_s & C_s\bar{A}_s & \cdots & C_s\bar{A}_s^{\alpha-1} \end{bmatrix} \in \mathbb{R}^{l\times n_s(\alpha)}, \quad (6b)$$
$$\bar{z}_{k,\alpha} = \begin{bmatrix} \tilde{u}_k^\top & z_{k-1}^\top & \cdots & z_{k-\alpha}^\top \end{bmatrix}^\top \in \mathbb{R}^{((m+l)\alpha+m)}, \quad (6c)$$
$$v_{k,\alpha} = \begin{bmatrix} v_{k-1}^\top & \cdots & v_{k-\alpha}^\top \end{bmatrix}^\top \in \mathbb{R}^{n_s(\alpha)}. \quad (6d)$$

Recall that the gain G_s (that need not be known) is designed to make the eigenvalues of the matrix $\bar{A}_s$ close to zero. As a consequence of that, the matrix $\bar{A}_s^p$, for some p satisfying $\alpha > p > n_s$, can be neglected. Therefore,

$$C_s \bar{A}_s^p x_{s,k-p} \simeq 0 \quad \text{and} \quad C_s \bar{A}_s^j \bar{B}_s \simeq 0, \ \forall j \geq p.$$

Hence, the Markov parameters matrix $\bar{H}_{s,\alpha}$ can be replaced by $\bar{H}_{s,p}$, so that the relation (5) becomes:

$$y_k = \bar{H}_{s,p} \bar{z}_{k,p} + \Delta y + e_k, \quad (7)$$

where all the terms related to the noise have been gathered in $e_k = \bar{H}^v_{s,p} v_{k,p} + w_k$.

In order to identify the Markov parameters, we consider a sliding window of size F. We assume that F and p are such that $F + p < \tau$, making it thus possible to collect the input-output data involved in (7) from the same submodel s. Hence, by stacking Eq. (7) on the sliding window $[k-F+1,\ \ k]$, we have:

$$\bar{Y}_k^F = \bar{\mathrm{H}}_{s,p} \bar{Z}_{k,p}^F + \Delta\bar{Y}^F + \bar{E}_k^F, \quad (8)$$

with

$$\bar{Y}_k^F = [y_{k-F+1}\ y_{k-F+2}\ \cdots\ y_k] \in \mathbb{R}^{l\times F},$$
$$\bar{E}_k^F = [e_{k-F+1}\ e_{k-F+2}\ \cdots\ e_k] \in \mathbb{R}^{l\times F},$$
$$\bar{Z}_{k,p}^F = [\bar{z}_{k-F+1,p}\ \bar{z}_{k-F+2,p} \cdots\ \bar{z}_{k,p}] \in \mathbb{R}^{((m+l)p+m)\times F},$$
$$\Delta\bar{Y}^F = [\Delta y\ \cdots\ \Delta y] \in \mathbb{R}^{l\times F}.$$

Recall from the equation (4) that

$$z_k = \begin{bmatrix} u_k \\ y_k \end{bmatrix} + \begin{bmatrix} \Delta u \\ -\Delta y \end{bmatrix},$$

so that the matrix $\bar{Z}_{k,p}^F$ can be written as:

$$\bar{Z}_{k,p}^F = \underbrace{\begin{bmatrix} u_{k-F+1} & \cdots & u_k \\ \begin{bmatrix} u_{k-F} \\ y_{k-F} \end{bmatrix} & \cdots & \begin{bmatrix} u_{k-1} \\ y_{k-1} \end{bmatrix} \\ \vdots & \vdots & \vdots \\ \begin{bmatrix} u_{k-F-p+1} \\ y_{k-F-p+1} \end{bmatrix} & \cdots & \begin{bmatrix} u_{k-p} \\ y_{k-p} \end{bmatrix} \end{bmatrix}}_{\bar{\bar{Z}}_{k,p}^F} + \underbrace{\begin{bmatrix} \Delta u & \cdots & \Delta u \\ \begin{bmatrix} \Delta u \\ -\Delta y \end{bmatrix} & \cdots & \begin{bmatrix} \Delta u \\ -\Delta y \end{bmatrix} \\ \vdots & \vdots & \vdots \\ \begin{bmatrix} \Delta u \\ -\Delta y \end{bmatrix} & \cdots & \begin{bmatrix} \Delta u \\ -\Delta y \end{bmatrix} \end{bmatrix}}_{\Delta\Psi^F}$$

that is,

$$\bar{Z}_{k,p}^F = \bar{\bar{Z}}_{k,p}^F + \Delta\Psi^F.$$

Now, let us define

$$\Delta\bar{z} = \Big[(\Delta u)^\top \ \big[\, (\Delta u)^\top \ (-\Delta y)^\top \,\big] \ \cdots \ \cdots \ \big[\, (\Delta u)^\top \ (-\Delta y)^\top \,\big] \Big]^\top \in \mathbb{R}^{((m+l)p+m)}, \quad (9)$$

and

$$\varphi_F = [1 \cdots 1]^\top \in \mathbb{R}^F. \quad (10)$$

Then, by noting that

$$\Delta\Psi^F = \Delta\bar{z}\varphi_F^\top \quad \text{and} \quad \Delta\bar{Y}^F = \Delta y \varphi_F^\top,$$

Eq. (8) can be rewritten as

$$\bar{Y}_k^F = \bar{\mathrm{H}}_{s,p}\bar{\bar{Z}}_{k,p}^F + \left(\bar{\mathrm{H}}_{s,p}\Delta\bar{z} + \Delta y\right)\varphi_F^\top + \bar{E}_k^F. \quad (11)$$

In order to remove the unknown faults Δu and Δy from (11), we multiply it on the left by

$$\Pi_\varphi^\perp = I_F - \frac{\varphi_F\varphi_F^\top}{\varphi_F^\top\varphi_F} = I_F - \frac{1}{F}\varphi_F\varphi_F^\top.$$

This is equivalent to projecting (11) onto the orthogonal complement of the space spanned by the row vector $\varphi_F^\top$. The equation (11) becomes

$$\bar{Y}_k^F\Pi_\varphi^\perp = \bar{\mathrm{H}}_{s,p}\bar{\bar{Z}}_{k,p}^F\Pi_\varphi^\perp + \bar{E}_k^F\Pi_\varphi^\perp. \quad (12)$$

As the noises v_k and w_k are zero mean processes, an estimate of $\bar{\mathrm{H}}_{s,p}$ can be derived as

$$\bar{\mathrm{H}}_{s,p} = \mathbf{E}\left[\bar{Y}_k^F\Pi_\varphi^\perp\left(\bar{\bar{Z}}_{k,p}^F\Pi_\varphi^\perp\right)^\dagger\right], \quad (13)$$

where $\mathbf{E}[\cdot]$ refers to the mathematical expectation operator. Note that the existence of the pseudo-inverse of $\bar{\bar{Z}}_{k,p}^F\Pi_\varphi^\perp$ requires this latter matrix to be full row rank. This is related to the sufficiency of excitation of the submodel s within the data acquisition window that is of size F. Therefore, the input u_k must be rich enough and $\bar{\bar{Z}}_{k,p}^F\Pi_\varphi^\perp$ must have a much larger number of columns than rows, that is, $(m+l)\times p + m \ll F$.

2.2 Estimation of the Markov parameters of the system

This part consists in determining the Markov parameters $\mathrm{H}_{s,p}$ of the system (1) from those $\bar{\mathrm{H}}_{s,p}$ of the modified system (3). The parameters $\bar{\mathrm{H}}_{s,p}$ are written as follows:

$$\begin{aligned}\bar{\mathrm{H}}_{s,p} &= \left[D_s \; C_s\bar{B}_s \; \cdots \; C_s\bar{A}_s^{p-1}\bar{B}_s\right] \\ &= \left[\bar{h}_{s,0} \; \bar{h}_{s,1} \; \cdots \; \bar{h}_{s,p}\right],\end{aligned} \quad (14)$$

with

$$\begin{aligned}\bar{h}_{s,0} &= D_s, \\ \bar{h}_{s,j} &= C_s\bar{A}_s^{j-1}\bar{B}_s, \; j = 1, \cdots, p \\ &= \left[\bar{h}_{s,j}^{(1)} \; -\bar{h}_{s,j}^{(2)}\right], \\ \bar{h}_{s,j}^{(1)} &= C_s\left(A_s + G_sC_s\right)^{j-1}\left(B_s + G_sD_s\right) \\ \bar{h}_{s,j}^{(2)} &= C_s\left(A_s + G_sC_s\right)^{j-1}G_s.\end{aligned} \quad (15)$$

By using a similar decomposition as in (15), one can easily obtain that:

$$h_{s,0} = \bar{h}_{s,0} = D_s \quad (16a)$$

$$h_{s,1} = C_sB_s = \bar{h}_{s,1}^{(1)} - \bar{h}_{s,1}^{(2)}D_s$$

$$h_{s,2} = C_sA_sB_s = \bar{h}_{s,2}^{(1)} - \bar{h}_{s,1}^{(2)}h_{s,1} - \bar{h}_{s,2}^{(2)}D_s$$

and generally, one can establish:

$$h_{s,j} = \bar{h}_{s,j}^{(1)} - \sum_{i=1}^{j}\bar{h}_{s,j}^{(2)}h_{s,j-i}, \; j = 1, \cdots, p \quad (17a)$$

$$h_{s,j} = -\sum_{i=1}^{j}\bar{h}_{s,j}^{(2)}h_{s,j-i}, \; j = p+1, \cdots, \infty. \quad (17b)$$

As it is apparent from (17), only the first $p < F$ Markov parameters need to be computed. At any order $j \geq p+1$, $h_{s,j}$ appears to be a linear combination of these first p Markov parameters. Thus, OKID algorithm presents the advantage of reducing significantly the number of Markov parameters that are necessary for the whole identification procedure.

2.3 Identification of a realization A_s, B_s, C_s and D_s

The aim here is the estimation of the order n_s and a realization (A_s, B_s, C_s, D_s) of the system (1) from the Markov parameters. This is achieved by making use of the Eigenvalue Realization Algorithm (ERA) Juang [1994]. Given the Markov parameters estimated above, this algorithm determines the order together with a minimal realization of the submodel indexed by s.

(1) With the definitions

$$\mathcal{C}_s \doteq \left[B_s \; A_sB_s \; \cdots \; A_s^{p-1}B_s\right] \in \mathbb{R}^{n_s\times mp}$$

$$\Gamma_s \doteq \left[(C_s)^\top \; (C_sA_s)^\top \; \cdots \; (C_sA_s^{p-1})^\top\right]^\top \in \mathbb{R}^{lp\times n_s},$$

notice that

$$\mathcal{H}_s \doteq \begin{bmatrix} h_{s,1} & \cdots & h_{s,p} \\ \vdots & \vdots & \vdots \\ h_{s,p} & \cdots & h_{s,2p-1}\end{bmatrix} = \Gamma_s\mathcal{C}_s. \quad (18)$$

(2) Compute an SVD of the matrix $\mathcal{H}_s$ in (18) as

$$\begin{aligned}\mathcal{H}_s &= \left[\,U_{s,1} \; U_{s,2}\,\right]\begin{bmatrix} S_{s,1} & 0 \\ 0 & S_{s,2}\end{bmatrix}\begin{bmatrix} V_{s,1}^\top \\ V_{s,2}^\top\end{bmatrix} \\ &\simeq U_{s,1}S_{s,1}V_{s,1}^\top,\end{aligned} \quad (19)$$

where the singular values contained in $S_{s,2}$ are neglected.

(3) Then, the order n_s of the system can be obtained as the number of singular values in $S_{s,1}$. We can also retrieve the extended observability matrix Γ_s and the extended controllability matrix $\mathcal{C}_s$ as:

$$\Gamma_s = U_{s,1}S_{s,1}^{1/2} \quad \text{and} \quad \mathcal{C}_s = S_{s,1}^{1/2}V_{s,1}^\top. \quad (20)$$

(4) Now, by exploiting the A-invariance property of Γ_s, the system matrices can finally be computed as follows

$$\begin{cases} A_s = \left[\Gamma_s^\uparrow\right]^\dagger\Gamma_s^\downarrow, \\ B_s = \mathcal{C}_s(:, 1:m), \\ C_s = \Gamma_s(1:l, :), \\ D_s = h_{s,0}, \end{cases} \quad (21)$$

where $\Gamma_s^\uparrow = \Gamma_s(1:(p-1)l, :)$ and $\Gamma_s^\downarrow = \Gamma_s(l+1:pl, :)$.

3. FAULT DETECTION AND ISOLATION

From the previous section, we can correctly extract the parameters of the system in (1) although this latter is subject to unknown faults. Now, by using the estimates in (21), we will, in this section, focus on generating residuals that reflect the faults acting on the system (1). An ideal residual signal should remain zero in the fault-free case and nonzero when faults occur. But data collected in physical systems are generally corrupted by noise, and so, the residuals are different to zero even if there is no fault. Therefore, decision techniques such as the Geometric Moving Average (GMA) or the Cumulative

Sum (CUMSUM) Basseville and Nikiforov [1993] can be used to detect the apparition of faults. The decision task is not tackled here, but the fault detection procedure is limited to the generation of appropriate residuals.

Once a fault has been detected, it must be estimated. The estimate of the fault will provide information about the type of fault, its duration, its amplitude and even its probable evolution. We first discuss (Subsection 3.1) the case of sensor faults and then propose in Subsection 3.2 a method to deal with actuator faults.

3.1 Sensor fault detection and estimation

To restrict our attention on the estimation of sensor fault, we consider that the actuator is faultless, that is, $\Delta u = 0$. Then, the system (1) becomes:

$$\begin{cases} x_{s,k+1} = A_s x_{s,k} + B_s u_k + v_k \\ y_k = C_s x_{s,k} + D_s u_k + \Delta y + w_k. \end{cases} \quad (22)$$

The output is approximated by the Finite Impulsion Response model (FIR) defined by Eq. (2) where the term $C_s A_s^{\alpha} x_{s,k-\alpha}$ is considered to be negligible. Consequently, an estimate of the output is given by:

$$\hat{y}_k = \begin{bmatrix} D_s & C_s B_s & \cdots & C_s A_s^{f-1} B_s \end{bmatrix} \begin{bmatrix} u_k \\ u_{k-1} \\ \cdots \\ u_{k-f} \end{bmatrix}. \quad (23)$$

As the estimates of the Markov parameters are not influenced by the faults under consideration (see Eq. (12)), residuals for diagnostic can be generated by a comparison between the measured and the estimated outputs. Hence, the residual

$$r_k = y_k - \hat{y}_k \quad (24)$$

is expected to reflect the potential occurrence of a sensor fault Δy as represented in (22).

Remark 1. In the case of MIMO systems (l outputs), the residual is composed of l components. Each component $r_k(i)$ of r_k reflects the fault that potentially affects the i-th output of the system (1). □

3.2 Actuator fault detection and estimation

Let us now turn to the challenging problem of detecting and estimating an actuator fault in a switching context. Similarly to the previous case, we consider this time that the sensor is faultless, meaning that $\Delta y = 0$ in (1). To deal with this problem, an unknown input finite memory observer is proposed for the identification of the actuator fault. Based on the matrices (21), the observer structure is carried out by treating the unknown component Δu as part of an augmented state vector Hocine et al. [2005]. In this procedure, an assumption is made that the actuator fault is constant or slowly time varying on the sliding window of size F.

The augmented system is written as:

$$\begin{cases} x'_{s,k+1} = \tilde{A}_s x'_{s,k} + \tilde{B}_s u_k + v'_k \\ y_k = \tilde{C}_s x'_{s,k} + \tilde{D}_s u_k + w_k, \end{cases} \quad (25)$$

where

$$x'_{s,k} = \begin{bmatrix} x_{s,k} \\ \Delta u \end{bmatrix}, \quad v'_k = \begin{bmatrix} v_k \\ 0 \end{bmatrix},$$

$$\tilde{A}_s = \begin{bmatrix} A_s & B_s \\ 0 & I \end{bmatrix}, \; \tilde{B}_s = \begin{bmatrix} B_s \\ 0 \end{bmatrix},$$

$$\tilde{C}_s = [\, C_s \;\, D_s \,], \;\; \tilde{D}_s = D_s.$$

For future use, we define

$$L_{s,F} = \left[(\tilde{C}_s)^{\top} \cdots (\tilde{C}_s \tilde{A}_s^{F-1})^{\top} \right]^{\top} \quad \in \mathbb{R}^{lF \times (n_s+m)},$$

$$T_{s,F} = \left[\tilde{A}_s^{F-1} \tilde{B}_s \;\, \tilde{A}_s^{F-2} \tilde{B}_s \;\cdots\; \tilde{B}_s \right] \quad \in \mathbb{R}^{(n_s+m) \times mF},$$

$$\Lambda_{s,F} = \begin{bmatrix} \tilde{D}_s & 0 & \cdots & 0 \\ \tilde{C}_s \tilde{B}_s & \tilde{D}_s & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}_s \tilde{A}_s^{F-2} \tilde{B}_s & \tilde{C}_s \tilde{A}_s^{F-3} \tilde{B}_s & \cdots & \tilde{D}_s \end{bmatrix} \quad \in \mathbb{R}^{lF \times mF},$$

$$\Omega_{s,F} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \tilde{C}_s & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}_s \tilde{A}_s^{F-2} & \tilde{C}_s \tilde{A}_s^{F-3} & \cdots & 0 \end{bmatrix} \quad \in \mathbb{R}^{lF \times (n_s+m)F}.$$

The evolution of the augmented system (25) on the sliding window $[k-F, \;\; k]$ can be summarized in the equation

$$\bar{y}_{k,F} = L_{s,F} x'_{s,k-F} + \Lambda_{s,F} \bar{u}_{k,F} + \bar{\eta}_{k,F}, \quad (26)$$

where

$$\bar{y}_{k,F} = \left[y_{k-F}^{\top} \cdots y_{k-1}^{\top} \right]^{\top},$$

and the vectors $\bar{u}_{k,F}$, $\bar{w}_{k,F}$ and $\bar{v}'_{k,F}$ are defined in a similar way. Here, the noise term $\eta_{k,F}$ is defined as $\bar{\eta}_{k,F} \doteq \bar{w}_{k,F} + \Omega_{s,F} \bar{v}'_{k,F}$.

As the noises w_k and v_k are zero mean processes, an estimate $\hat{x}'_{s,k-F}$ of the state at time step $k-F$, can easily be obtained as the solution of the following Least Squares criterion Hocine et al. [2005]:

$$J_k = \left\| L_{s,F} x'_{s,k-F} + \Lambda_{s,F} \bar{u}_{k,F} - \bar{y}_{k,F} \right\|_2^2,$$

subject to $x'_{s,k-F}$. This results in

$$\hat{x}'_{s,k-F} = L_{s,F}^{\dagger} \left(\bar{y}_{k,F} - \Lambda_{s,F} \bar{u}_{k,F} \right), \quad (27)$$

and finally, an estimate of the state at time k can be computed as

$$\hat{x}'_{s,k} = \tilde{A}_s^F \hat{x}'_{s,k-F} + T_{s,F} \bar{u}_{k,F}. \quad (28)$$

The actuator fault $\widehat{\Delta u_k}$ is estimated directly from $\hat{x}'_{s,k}$ since, as defined in (25), $\hat{x}'_{s,k} = \left[\hat{x}_{s,k}^{\top} \;\; \widehat{\Delta u_k}^{\top} \right]^{\top}$.

Remark 2. For the solution (27) to be consistent, the observability matrix $L_{s,F}$ must be full column rank. □

Summary. The whole identification based fault detection and isolation technique is implemented online. On any time window $[k-F, \, k]$, $k = F+1, \ldots, \infty$, the steps of the eventual algorithm may be as follows

(1) Estimate $\bar{H}_{s,p}$ by Recursive Least Squares using (12) and deduce the impulse responses $h_{s,j}$ for $j = 0, \ldots, 2p-1$.

(2) Obtain the matrices (A_s, B_s, C_s, D_s) from (21) as described in Subsection 2.3.
(3) Recognize switches as the events that originate abrupt changes in the estimates of the Markov parameters.
(4) Fault detection and isolation:
4-a Detect sensor faults by inspecting the residual $\widehat{\Delta y} = y_k - \hat{y}_k$ given in (24).
or
4-b Detect actuator faults by inspecting $\widehat{\Delta u}$ obtained through (27) and (28).

4. SIMULATION EXAMPLE

In order to demonstrate the performance of the proposed method, let us consider an example of MIMO (two inputs and two outputs) switched system, composed of three submodels, subject to sensor or actuator faults as described by (1). The switches that drive the system from a submodel to another are such that: the first, second and third submodels are active respectively on the time intervals [1, 499], [500, 1799], and [1800, 2499]. On [2500, 4000], we let the submodel 3 vary slowly in parameters. The variation concerns the dynamics matrix A_3 and the variation rate is chosen to be linear with respect to time and is such that $A(t_o) = A_3$ and $A(t_1) = A_1$ for $t_o = 2500$ and $t_1 = 4000$.

We also let the submodels be of different orders, that is, $n_1 = 3$, $n_2 = 2$ and $n_3 = 3$, where in the notation n_j, j represents the discrete state of the system. This requires that the transition matrices (the A-matrices) in (1) to be rectangular at the switching times. The matrices A_i, B_i, C_i, D_i, $i = 1, 2, 3$ are given by:

$$A_1 = \begin{bmatrix} 0.64 & 0.62 & 0 \\ -0.64 & 0.62 & 0 \\ 0 & 0 & -0.36 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.2 & -0.8 \\ 1 & -0.8 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.1 & 0.4 & 0 \\ -0.5 & 0.4 & 0 \\ 0 & 0 & -0.26 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.90 & -0.70 \\ 0.71 & -0.50 \\ 0.80 & 0.47 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 & -0.6 \\ 0.32 & -0.66 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.1 & -0.6 \\ 0.32 & -0.66 \\ 0.3 & 0.82 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.55 & 0.2 & 0.8 \\ 0.45 & 0.3 & 0.58 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.8 & -0.1 \\ 0.3 & 0.48 \end{bmatrix}, \quad C_3 = \begin{bmatrix} -0.8 & -0.1 & 0.7 \\ 0.3 & 0.48 & 0.9 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.97 & 0.63 \\ -0.32 & 0.95 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.5 & 0.3 \\ -0.2 & -0.5 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 0.5 & 0.3 \\ -0.2 & -0.5 \end{bmatrix}.$$

The excitation input u_k applied to the system is chosen as a white Gaussian zero-mean noise of variance unity. The simulation is run with an additive output white noise in the proportion of SNR = 35 dB.

4.1 Identification of the system subject to the sensor and actuator faults

As argued in Section 2, we can correctly estimate the parameters of the switched system despite the possible faults that could affect it. This is illustrated in this subsection. Under the sensor and actuator faults plotted on Figure 1, data are collected from the switched system defined and the method of Section 2 is used for the identification.

The results obtained then are presented on Fig. 2 and Fig. 3. Figure 2 represents the magnitude of the system poles while the estimate of the order is depicted on Figure 3. These figures show that our algorithm (See Section 2) is robust with respect to the faults considered. Indeed, the presence of sensor or actuator faults in the form given in (1) does not affect the estimation of the order n_s and the parameters that characterize the switched system.

At each switching time we can notice that the estimated order increases suddenly up to ($l \times p$: number of rows of the Markov parameters matrix H_s, (see Eq.(18)) even if there is no change in the order. This phenomenon can be attributed to the presence of mixed data coming from two different local models.

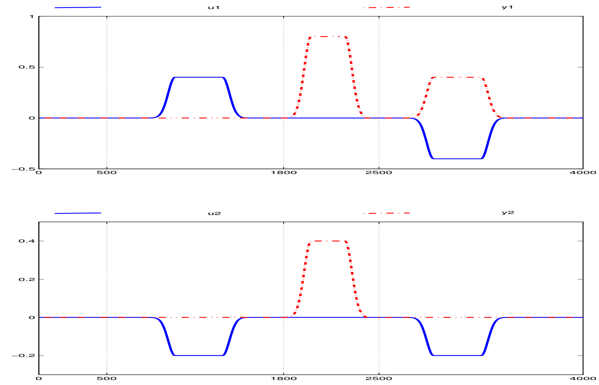

Figure 1. Simulation faults

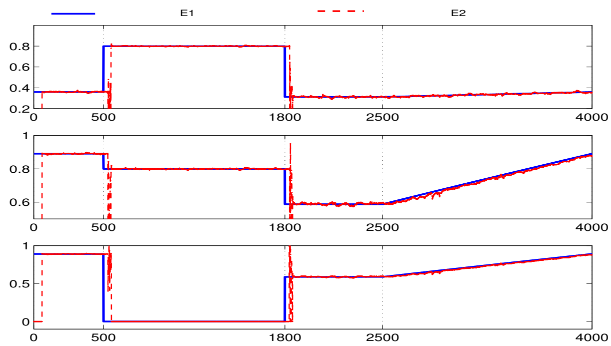

Figure 2. Eigenvalues in absolute value

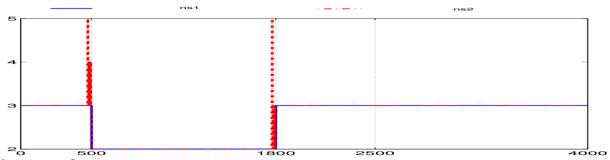

Figure 3. n_s and its estimate

4.2 Actuator fault detection and estimation

We consider in this subsection only actuator faults (as in Subsection 3.2). The faults occur on two different intervals: $\Delta T_1 = [800, 1500]$ (during the activation of the local model 2) and $\Delta T_2 = [2600, 3500]$. The width F of the sliding window is set to be $F = 30$.

We use, as described in Subsection 3.2, an unknown input finite memory observer to estimate the actuator fault. The results obtained for $\widehat{\Delta u}_k$ are given in Figure 4 and Figure 5. The actuator fault that was injected at time-steps t (t = 800 and t = 2600) respectively, are detected with a delay that is approximately equal to F.

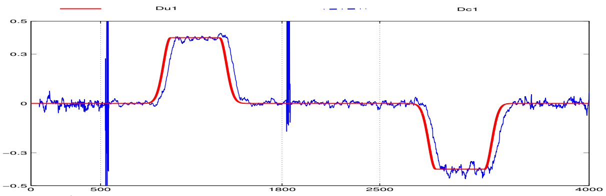

Figure 4. Δu_{1k} and $\Delta \hat{u}_{1k}$

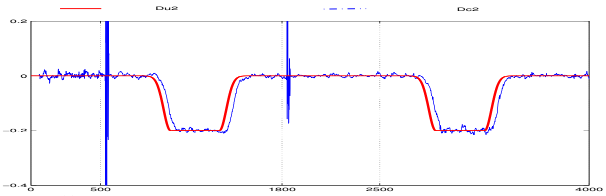

Figure 5. Δu_{2k} and $\Delta \hat{u}_{2k}$

4.3 Sensor fault detection and estimation

Now, we consider only the sensor fault (see Eq. (22)), the actuator being fault-free, i.e., $\Delta u = 0$. The sensor faults occur on the same intervals of time ΔT_1 and ΔT_2 defined previously.

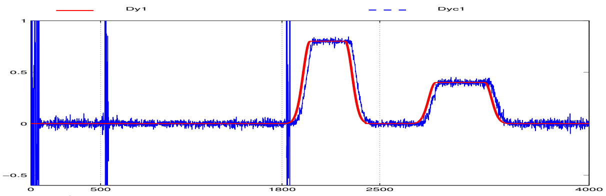

Figure 6. $r_{1k} = y_{1k} - \hat{y}_{1k}$

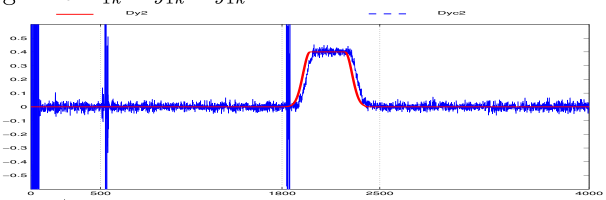

Figure 7. $r_{2k} = y_{2k} - \hat{y}_{2k}$

To identify the sensor fault, we compare the output vector of the system (22) and its estimate by the Markov parameters (23). The simulation results are shown in Figures 6 and 7. The residual vector $r_k = y_k - \hat{y}_k$ is used to detect and estimate the sensor faults due to the fact that the proposed identification algorithm is robust with respect to the faults injected on the system.

In this simulation, we have shown how to identify, from the input-output data, the order and the parameters of the switched system. Thereafter, from these identified parameters, we have implemented a strategy for sensor and actuator fault detection and isolation. In order to detect and estimate these faults, we use the diagnosis procedures developed separately in Sections 3.1 and 3.2.

However, each of these two procedures is valid only if one type of faults is injected on the system. When the system is simultaneously subject to sensor and actuator faults, the presented strategy permits to detect the fault on the system without giving anymore indication about the source of the fault.

5. CONCLUSION

In this work, we have proposed a strategy for fault detection and isolation without any a priori knowledge of the system to be supervised. The key idea is to use, as a first stage, a specific OKID identification algorithm which makes the estimation of the parameters insensitive to sensor or actuator faults. Therefore, the parameters identified characterize only changes in the system dynamics. As the identified parameters are insensitive to sensor or actuator faults, a second stage is required to detect and to estimate these faults. The approach followed consists in synthesizing sensitive residuals. The sensor residual is computed by the comparison between the actual output of the system and its estimate which is obtained by aggregating the past inputs and the estimated Markov parameters on a sliding window of data. The actuator residual is determined by designing an unknown input finite memory observer. The simulation results show good performance of this approach.

REFERENCES

A. Akhenak, M. Chadli, J. Ragot, and D. Maquin. Sliding mode multiple observer for fault detection and isolation. *42th IEEE Conference on Decision and Control*, 1:953 – 958, 2003.

A. Alessandri, M. Baglietto, and G. Battistelli. Receding-horizon estimation for switching discrete-time linear systems. *IEEE Transactions on Automatic Control*, 50(11):1736 – 1748, 2005.

M. Basseville and I. V. Nikiforov. Detection of abrupt changes. *New Jersey: Prentice Hall*, 1993.

J. Chen and R. J. Patton. Robust model-based fault diagnosis for dynamic systems. *Kluwer Academic Press, Dordrecht.*, 1999.

M. Darouach, M. Zasadzinski, and S. T. Xu. Full-order observers for linear systems with unknown input. *IEEE Transactions on Automatic Control*, 39 (3):606 – 609, 1994.

P. M. Frank. Fault detection diagnosis in dynamic systems using analytical and knowledge base redundancy - a survey and some new results. *Automatica*, 26(3):459 – 474, 1990.

J. Gertler. *Fault Detection and Diagnosis in Engineering Systems.* Marcel Dekker, 1998.

A. Hocine, D. Maquin, and J. Ragot. Finite memory observer for switching systems: Application to diagnosis. *16th IFAC World Congress, Prague, Czech Republic,*, 2005.

C. Juang. Applied system identification. *Prentice-Hall, Englewood Cliffs ; NJ*, 1994.

S. Lecoeuche, G. Mercère, and H. Amadou-Boubacar. Modelling of non stationary systems based on a dynamical decision space. *14th IFAC Symposium on System Identification, Newcastle*, pages 1 – 6, 2006.

W. Nuninger, F. Kratz, and J. Ragot. Finite memory generalised state observer for failure detection indynamic systems. *Proc. of the 37th IEEE Conference on Decision and Control*, 1:581 – 585, 1998.

K. M. Pekpe, G. Mourot, K. Gasso, and J. Ragot. Identification of switching systems using change detection technique in the subspace framework. *43rd IEEE Conference on Decision and Control*, pages 3720 – 3725, 2004.

K.M. Pekpe and S. Lecoeuche. Online clustering of switching models based on a subspace framework. *Nonlinear Analysis: Hybrid Systems*, in press, 2008.

M. Phan, L. G. Horta, J-N. Juang, and R. W. Longman. Improvement of observer/kalman filter identification (OKID) by residual whitening. *Transactions of the ASME*, 117:232 – 239, 1995.