

# Online change detection and condition-based maintenance

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**Abstract:** The aim of this paper is to use the online change detection/ isolation methods in the framework of the condition-based maintenance. The purpose is to propose an adequate condition-based maintenance policy to a gradually deteriorating system with change of mode using on-line detection algorithms. The parameters defining the deterioration mode after the change can be unknown. The main purpose is to optimize a global cost criterion.

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## 1. INTRODUCTION

In this paper one consider a deteriorating system with two modes of degradation. The time of change of the degradation mode is unknown. On-line information is available from on-line monitoring about the change of mode. We propose to develop a maintenance model and to optimize the maintenance decision rule by using the available on-line informations. The deterioration parameters of the second mode are unknown but the set to which they belong is known. We deal with these parameters by using an adequate detection algorithm in the framework of the condition-based maintenance.

Throughout this paper we deal with a system which nominally deteriorates according to a given known mode and its mean deteriorating rate increases suddenly at an unknown time. The time of change of the degradation mode is called the change time. After the change time the system deteriorates according to an accelerated mode. An adaptive maintenance policy based on on-line change detection procedures is already proposed in Fouladirad et al. (2006, 2007a) when the parameters of the accelerated mode are known. In this paper we suppose that the parameters of the accelerated mode are unknown. They take values belonging to a known set. As these parameters are no longer known in advance, the methods proposed in Fouladirad et al. (2006, 2007a) are no-more appropriate. Hence, in this paper, a condition-based maintenance policy embedded with a detection and isolation procedure is proposed to take into account the presence of unknown parameters in the accelerated mode. The condition-based maintenance policy takes only into account the on-line observed degradation levels with respect to time-based decision parameters. The proposed parametric maintenance decision rule which uses an online detection procedure of the change of mode is optimized in order to minimize an average maintenance cost criterion. The on-line change detection/isolation algorithm should be adequate in the sense that it should minimize the detection delay and have a low false alarm rate and a low false isolation rate.

One consider a system with a gradually deterioration. A scalar aging variable can summarize the system condition Wang (2002). The ageing variable increases when

the system deteriorates and a failure occurs when the ageing variable (the system state) crosses a known fixed threshold  $L$  called failure threshold. The failure threshold corresponds to the limit value of wear beyond which the mission of the system is no longer fulfilled. Such systems are usually subject to non-obvious failures and informations about the system state are collected through inspections. Therefore, the system is known to be failed only if the failure is revealed by inspections. The system can be declared as "failed" as soon as a defect or an important deterioration is present, even if the system is still functioning. In this situation, its high level of deterioration is unacceptable either for economic reasons (poor products quality, high consumption of raw material, etc...) or for safety reasons (high risk of hazardous breakdowns). An example is given in van Noortwijk and van Gelder (1996), where the problem of maintaining coastal flood barriers is modeled. The sea gradually erodes the flood barrier and the barrier is deemed to have failed when it is no longer able to withstand the pressure of the sea.

If through inspections it is discovered that the system is failed, a corrective maintenance operation immediately replaces the failed system by a new one. In order to avoid a failure occurrence and the resulting period of inactivity of the system (i.e. time interval between the instant of failure and the following inspection) preventive maintenance actions are performed. The preventive maintenance operation is less costly than the corrective maintenance operation. The preventive maintenance action takes place when the system state exceeds a predetermined threshold known as the preventive threshold (case of a single-mode deteriorating system).

The global maintenance cost depends on the choice of the inter-inspection interval times and the value of the preventive threshold. For example, in the case of costly inspections it is not worthwhile to inspect often the system. But if the system is scarcely inspected, the risk of missing a failure occurrence increases. In Grall et al. (2002) or Dieulle et al. (2003) and Bérenguer et al. (2003) authors propose condition-based inspection/replacement and continuous monitoring replacement policies for a single-mode deteriorating system. In those previous works, a maintenance cost model is proposed which quantifies the

costs of the maintenance strategy and a method to find the optimal strategy leading to a balance between monitoring and maintenance efficiency is proposed. When the system undergoes a change of mode it seems reasonable to incorporate the on-line information available about the system in the maintenance decision rule. In Saassouh et al. (2005) authors propose an adaptive maintenance policy for a continuously monitored system provided that the instant of change of mode is immediately and perfectly detected. In Fouladirad et al. (2006, 2007a), authors studied the on-line change detection in the framework of the condition based maintenance in the case of known parameters after the change.

In this paper an adaptive maintenance policy based on an embedded optimal on-line change detection algorithm is proposed. The originality is due to the fact that the parameters after the change (i.e. the parameters of the second mode) can take unknown values, but these values belong to a known and finite set.

In section 2, the deteriorating system is described. Section 3 is devoted to the presentation of the on-line change detection algorithm. In section 4, an adequate maintenance decision rule is proposed. This algorithm takes into account the fact that the parameters after the change can take different values. A method for the evaluation of the maintenance cost is proposed in section 5. The theoretical results are analyzed by numerical implementations in section 6.

## 2. SYSTEM DESCRIPTION

One consider an observable system subject to accumulation of damage. The system state at time  $t$  can be summarized by a scalar random ageing variable  $X_t$ . In absence of repair or replacement action,  $(X_t)_{t \geq 0}$  is an increasing stochastic process, with initial state  $X_0 = 0$ . When the state of the process reaches a pre-determined threshold, say  $L$ , the system is said to be failed. The failure of the system occurs as soon as a defect or an important deterioration is present, even if the system is still functioning. In these conditions it is no longer able to fulfill its mission in acceptable conditions. The threshold  $L$  is chosen in respect with the properties of the considered system. It can be seen as a safety level which has not to be exceeded. The behavior of the deterioration process after a time  $t$  depends only on the amount of deterioration at this time.

The parameters of the deterioration process  $(X_t)_{t \geq 0}$  can suddenly change at time  $T_0$ . This means that the mean deterioration rate suddenly increases from a nominal value to an accelerated rate at time  $T_0$ . The first mode corresponds to a nominal mode denoted by  $M_1$  and the accelerated mode is denoted by  $M_2$ . In this paper, the case of two degradation mode is treated but this choice is not restrictif. The results exposed in this paper can be generalized to the case of multiple degradation modes which can be subject of further works.

In this paper, it is assumed that the deterioration process in mode  $M_i$  ( $i = 1, 2$ ), denoted by  $(X_t^i)_{t \geq 0}$ , is a gamma process i.e. for all  $0 \leq s \leq t$ , the increment of  $(X_t^i)_{t \geq 0}$  between  $s$  and  $t$ ,  $Y_{t-s}^i = X_t^i - X_s^i$ , follows a gamma prob-

ability distribution function with shape-parameter  $\alpha_i.(t-s)$  and scale parameter  $\beta_i$ . This probability distribution function can be written as follows:

$$f_{\alpha_i(t-s), \beta_i}(y) = \frac{1}{\Gamma(\alpha_i(t-s))} \cdot \frac{y^{\alpha_i(t-s)-1} e^{-\frac{y}{\beta_i}}}{\beta_i^{\alpha_i(t-s)}} \mathbf{1}_{\{y \geq 0\}}. \quad (1)$$

The average deterioration speed rate in mode  $M_i$  is  $\alpha_i(t-s) \cdot \beta_i$  and its variance is  $\alpha_i(t-s) \cdot \beta_i^2$ .

In difference with Fouladirad et al. (2006, 2007a) the second mode parameters are no longer known in advance. They Take their values in the following set:

$$S = \{(\alpha_{21}, \beta_{21}), \dots, (\alpha_{2K}, \beta_{2K})\} \quad (2)$$

As  $\text{card}(S) = K$ , there are  $K$  different possibilities of second degradation mode.

It is supposed that the mode  $M_2$  corresponds to an accelerated mode so in the second mode the parameters are such that  $\alpha_2 \cdot \beta_2 > \alpha_1 \cdot \beta_1$ . The case of a slower second mode is discarded because the systems we consider can not be stabilized and deteriorate slower than the past. In some particular cases which are not considered in this paper the system can deteriorate slower in the second mode, i.e. a crack growth which can be very fast in the beginning and for environmental reasons ( the material, humidity,...) the growth rate changes and the crack grows slower and the growth can even stops.

Note that the gamma process is a positive process with independent increments. It implies frequent occurrences of tiny increments which makes it relevant to describe gradual deterioration due to continuous use such as erosion, corrosion, concrete creep, crack growth, wear of structural components van Noortwijk (2007); Cooke et al. (1997); Çınlar et al. (1977); Blain et al. (2007); Frangopol et al. (2004). Another interest of the gamma process is the existence of an explicit probability distribution function which permits feasible mathematical developments. It has been widely applied to model condition-based maintenance (see van Noortwijk (2007)).

## 3. ON-LINE CHANGE DETECTION

The on line abrupt change detection, with low false alarm rate and a short detection delay, has many important applications, including industrial quality control, automated fault detection in controlled dynamical systems. Authors in Basseville and Nikiforov (1993) present a large literature on the detection algorithms in complex systems. The application of these methods to a maintenance policy is not often discussed. A first attempt to use an optimal on line abrupt change detection in the framework of maintenance policy is presented in Fouladirad et al. (2006), Fouladirad et al. (2007a).

The on-line change detection algorithms permit to use the on-line available informations on the deterioration rate to detect the occurred abrupt change time. These algorithms take into account the informations collected through inspections, so they treat with on-line discrete observations (i.e. system state at times  $(t_k)_{k \in \mathbf{N}}$ ). The quality of an on-line change detection algorithm depends on its mean delay to detection and its false alarm rate.

The authors in Fouladirad et al. (2006), Fouladirad et al. (2007a) considered the case of two deteriorating modes (one change time) and known parameters after the change. In this paper, the aim is to propose an adequate detection/isolation method when the accelerated mode parameters can take unknown values. These values belong to a known set defined in (2).

We collect observations  $(Y_k)_{k \in \mathbf{N}}$  at inspection times  $k \in \mathbf{N}$ . These observations are the increments of the degradation process which is a gamma process. Hence  $Y_k$  for  $k \in \mathbf{N}$  follows a gamma law with density  $f_{\theta_i} = f_{\alpha_i \Delta t, \beta_i}$  according to the degradation mode  $M_i$ ,  $i = 1, 2$ . We shall denote  $f_l = f_{\alpha_{2l} \Delta t, \beta_{2l}}$ ,  $l = 1, \dots, K$ , the density function associated to the accelerated mode when  $(\alpha_2, \beta_2) = (\alpha_{2l}, \beta_{2l})$ .

We shall denote by  $N$  the alarm time at which a  $\nu$ -type change is detected/isolated and  $\nu$ ,  $\nu = 1, \dots, K$ , is the final decision. A change detection/isolation algorithm should compute the couple  $(N, \nu)$  based on the observations  $Y_1, Y_2, \dots$ . We shall denote by  $\text{Pr}_0$  the probability knowing that no change of mode is occurred,  $\text{Pr}_{T_0}^l$  the probability knowing that the change of mode is occurred at  $T_0$ . Under  $\text{Pr}_{T_0}^l$  the observations  $Y_1, Y_2, \dots, Y_{T_0-1}$  have each the density function  $f_{\theta_1}$  and a change at  $T_0$  has occurred and  $Y_{T_0}$  is the first observation with distribution  $f_l$ ,  $l = 1, \dots, K$ .  $\mathbf{E}_0$  (resp.  $\mathbf{E}_{T_0}^l$ ) is the expectation corresponding to the probability  $\text{Pr}_0$  (resp.  $\text{Pr}_{T_0}^l$ ).

The mean time before the first false alarm of a  $j$  type is defined as follow:

$$\mathbf{E}_0(N^{\nu=j}) = \mathbf{E}_0 \left( \inf_{k \geq 1} \{N(k) : \nu(k) = j\} \right) \quad (3)$$

The mean time before the first false isolation of a  $j$  type is defined as follow:

$$\mathbf{E}_{T_0}^l(N^{\nu=j}) = \mathbf{E}_{T_0}^l \left( \inf_{k \geq 1} \{N(k) : \nu(k) = j\} \right) \quad (4)$$

As it is initially proposed by Lorden (1971), usually in on-line detection algorithms the aim is to minimize the mean delay for detection/isolation in the worst case:

$$\bar{\tau}^* = \max_{1 \leq l \leq K} \bar{\tau}_l^*, \quad (5)$$

$$\bar{\tau}_l^* = \sup_{T_0 \geq 1} \text{esssup} E_{T_0}^l(N - T_0 + 1 | N \geq T_0, X_1, \dots, X_{T_0-1}).$$

for a given minimum false alarm rate or false isolation rate:

$$\min_{0 \leq i \leq K} \min_{1 \leq j \neq i \leq K} \mathbf{E}_{T_0}^i(N^{\nu=j}) = a \quad (6)$$

where  $\mathbf{E}_{T_0}^0 = \mathbf{E}_0$ . Let us recall the detection /isolation algorithm initially proposed by Nikiforov (1995). We define the stopping time  $N^{l*}$  in the following manner:

$$N^{l*} = \inf_{k \geq 1} N^{l*}(k), \quad (7)$$

$$N^{l*}(k) = \inf \{t \geq k, \min_{0 \leq j \neq l \leq K} S_k^t(l, j) \geq h\} \quad (8)$$

where  $S_k^t(l, j) = \sum_{i=k}^t \log \frac{f_l(Y_i)}{f_j(Y_i)}$ , and  $f_0(\cdot) = f_{\theta_1}(\cdot)$ .

The stopping time and the final decision of the detection/isolation algorithm are presented as it follows:

$$N^* = \min\{N^{1*}, \dots, N^{K*}\} \quad (9)$$

$$\nu^* = \text{argmin}\{N^{1*}, \dots, N^{K*}\} \quad (10)$$

In Nikiforov (1995) author proved that the mean time to the detection in the worst case  $\bar{\tau}^*$  defined by (5) satisfies the following relations:

$$\bar{\tau}^* \leq \max_{1 \leq l \leq K} \mathbf{E}_{T_0}^l(N^*) \sim \frac{\ln(a)}{\rho^*} \text{ as } a \rightarrow 0 \quad (11)$$

$$\rho^* = \min_{1 \leq l \leq K} \min_{0 \leq j \neq l \leq K} \rho_{lj} \quad (12)$$

$$\rho_{lj} = \int p_j \ln \left( \frac{p_j}{p_l} \right) d\mu \quad 0 \leq j \neq l \leq K \quad (13)$$

where  $\rho_{lj}$  is the Kullback-Leibler distance. The detection/isolation algorithm presented in this section reaches asymptotically the lower asymptotic bound  $\frac{\ln(a)}{\rho^*}$  initially proposed by Lorden (1971).

#### 4. MAINTENANCE POLICY

It is supposed that the considered deteriorating systems cannot be continuously monitored. For this reason, the deterioration level can only be known at inspection times. We shall denote by  $(t_k)_{k \in \mathbf{N}}$  the sequence of the inspection times defined by  $t_{k+1} - t_k = \Delta t$  (for  $k \in \mathbf{N}$ ) where  $\Delta t$  is a fixed parameter. When the deterioration level exceeds the threshold  $L$  between two inspections, the system continues to deteriorate until the next inspection. In order to avoid the occurrence of a failure, a preventive maintenance action has to take place before the deterioration level exceeds the threshold  $L$ . Furthermore, the inter-inspection time  $\Delta t$  has to be carefully chosen in order to be able to replace the system before the failure. There are two possible maintenance actions at each inspection time  $t_k$ :

- the system is preventively replaced if the deterioration level is close to  $L$ ,
- if a failure has occurred ( $Y_{t_k} > L$ ) then the system is correctively replaced.

The preventive replacement leads to restore the system in a "good as new" state. The decision about a possible replacement has to be taken according to the current state of the system and also with respect to the available information on the current mode of deterioration.

The time of change of degradation mode  $T_0$  is always unknown. Hence, a detection method can be used to identify  $T_0$ . The on-line change detection algorithm presented in section 3 is used to detect the change mode time  $T_0$  when the parameters after the change belong to the set presented in (2). The maintenance decision is based on a parametric decision rule according to the change detection result. As in Saassouh et al. (2005), Fouladirad et al. (2006) and Fouladirad et al. (2007a), the preventive maintenance decision is based on different preventive thresholds corresponding to each of the possible deterioration modes of the system. Such maintenance policies are extensions of inspection/replacement structures for single mode deteriorating systems. Let  $M_{\text{nom}}$  and  $M_{\text{ac}}$  be the decision thresholds associated to the "limit" cases corresponding to the single-mode deterioration system. The decision threshold  $M_{\text{nom}}$  (respectively  $M_{\text{ac}}$ ) is chosen in order to minimize the cost criterion of the nominal single-mode deteriorating system. In the nominal mode the threshold

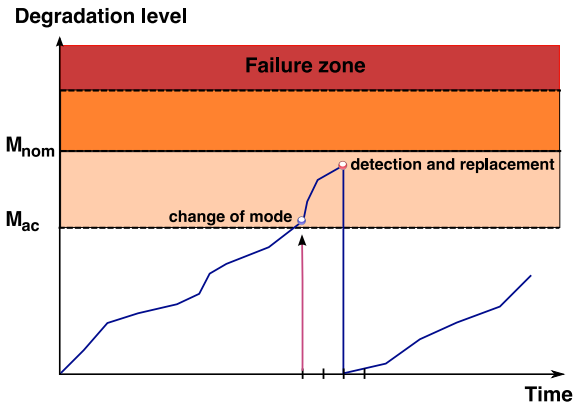


Fig. 1. Maintenance /detection policy.

$M_{nom}$  is effective and as soon as the system is supposed to have switched in the second mode (accelerated mode) then threshold is adapted from  $M_{nom}$  to  $M_{ac}$ .

The possible decisions which can arise at each inspection time  $t_k$  are as follows:

- If  $Y_{t_k} \geq L$  the system has failed then it is correctively replaced.
- If no change of mode has been detected in the current cycle (the system is supposed to be in nominal degradation mode) and  $Y_{t_k} \geq M_{nom}$  the system is preventively replaced.
- If a change of mode is detected at time  $t_k$  or has been detected earlier in the current cycle and if  $Y_{t_k} \geq M_{ac}$  then the system is preventively replaced. It is still functioning but too badly deteriorated according to the accelerated degradation mode.
- In all the other cases, the decision is reported to time  $t_{k+1}$ .

As a consequence of the previous decision rule, if the system is deteriorating according to the mode  $i$ ,  $i = 1, 2$ , and if a change of mode is detected at time  $t_{detect} = k\Delta t$ ,  $k \in \mathbf{N}$ , where  $t_{detect} \geq T_0$ , the two following scenarios can arise :

- If  $Y_{t_{detect}} < M_{ac}$  then the system is left unchanged and a replacement is performed at time  $t_n > t_{detect}$  such that  $Y_{t_{n-1}} < M_{ac} \leq Y_{t_n}$ .
- If  $Y_{t_{detect}} \geq M_{ac}$  then the system is immediately replaced.

These situations are depicted in 1 and 2. The parameters of the maintenance decision rule are respectively the thresholds  $M_{ac}$ ,  $M_{nom}$  and the parameters of the detection algorithm.

## 5. EVALUATION OF MAINTENANCE POLICY

Each time that a maintenance action is performed on the system, a maintenance cost is incurred. Each corrective (respectively preventive) replacement entails a cost  $C_c$  (respectively  $C_p$ ). Since a corrective maintenance operation is performed on a more deteriorated system, it is generally more complex and consequently more expensive than a preventive one. Hence it is supposed that  $C_p < C_c$ . The cost incurred by any inspection is  $C_i$ . In the period of

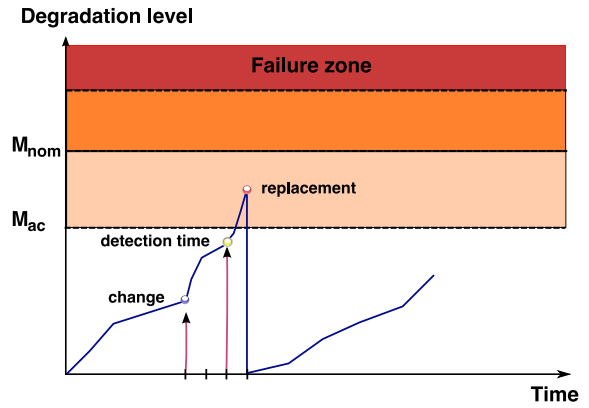


Fig. 2. Maintenance /detection policy.

unavailability of the system (i.e the time spent by the system in a failed state) an additional cost per unit of time  $C_u$  is incurred. All direct and indirect costs are already included in the unit costs  $C_i$ ,  $C_c$ ,  $C_p$ ,  $C_u$ . The maintenance policy is evaluated using an average long run cost rate taking into account the cost of each type of maintenance actions. Let us denote by  $N_p(t)$  the number of preventive replacements before  $t$ ,  $N_c(t)$  the number of corrective replacements before  $t$ ,  $d_u(t)$  the cumulative unavailability duration of the system before  $t$  and  $N_i(t)$  the number of inspections before  $t$ . We know that  $N_i(t) = \lfloor \frac{t}{\Delta t} \rfloor$  where  $\lfloor x \rfloor$  denotes the integer part of the real number  $x$ . Let us denote by  $T$  the length of a life-time cycle and  $T_L$  the random time at which the system state exceeds threshold  $L$ . The property of the regeneration process  $(X_t)_{t \geq 0}$  allows us to write:

$$C_\infty = \lim_{t \rightarrow \infty} \frac{\mathbf{E}(C(t))}{t} = \frac{\mathbf{E}(C(T))}{\mathbf{E}(T)} \quad (14)$$

where

$$C(t) = C_i N_i(t) + C_p N_p(t) + C_c N_c(t) + C_u d_u(t).$$

We know that

$$\mathbf{E}(N_p(T)) = \Pr(\text{cycle ends by a preventive repl.}),$$

$$\mathbf{E}(N_c(T)) = \Pr(\text{cycle ends by a corrective repl.})$$

$$\mathbf{E}(N_i(T)) = \mathbf{E}\left(\left\lfloor \frac{T}{\Delta t} \right\rfloor\right), \mathbf{E}(d_u(T)) = \mathbf{E}(T - T_L) \mathbf{1}_{\{T_L < T\}},$$

Let us set the ‘‘inspection scheduling function’’ introduced in Grall et al. (2002) be constant then the threshold  $M_{nom}$  and the inter-inspection time  $\Delta t$  can be obtained by numerical minimization of the cost criterion of the single-mode deteriorating system in mode  $M_1$ . The threshold  $M_{ac}$  corresponds to the optimal threshold for the single-mode deteriorating system in mode  $M_2$ .

In this work, the cost criterion is optimized as a function of the parameter of the considered maintenance policy : the detection threshold  $h$  defined in (7).

## 6. NUMERICAL IMPLEMENTATIONS

In this section we apply the maintenance policy presented in this paper to the case of a system with two degradation modes and four possible accelerated modes ( $K = 4$ ).

The proposed maintenance policies are analyzed by numerical implementations. Throughout this section, the

real second mode cases	1	2	3	4
$\alpha_2$	2	1	2	1
$\beta_2$	1	3	2	7
$M_{ac}$	85.6	74.6	73.7	51.6

Table 1. Characteristic data of the second degradation mode.

values of the maintenance costs are respectively  $C_i = 5$ ,  $C_p = 50$ ,  $C_c = 100$  and  $C_u = 250$ . For the numerical calculations it is supposed that in the nominal mode  $M_1$ ,  $\alpha_1 = 1$  and  $\beta_1 = 1$ . Hence, the maintenance threshold  $M_{nom}$  is equal to 90.2. The previous value is the optimal value which minimizes the long run maintenance cost for a single mode deteriorating system in mode  $M_1$ . For this optimization (from Monte Carlo simulations), we use a single degradation mode results with  $\Delta t = 4$ . The couple  $(M_{nom}, \Delta t) = (90.2, 4)$  is the optimal couple which minimizes the long run maintenance cost for a single mode deteriorating system in mode  $M_1$ .  $T_0$  is simulated by a uniform law from Monte Carlo method. To evaluate each maintenance policy, four different accelerated modes are considered. So the parameters of the accelerated mode belong to the following set:

$$(\alpha_2, \beta_2) \in \{(2, 1), (1, 3), (2, 2), (1, 7)\} \quad (15)$$

The properties of the different second modes are presented table 1. The threshold  $M_{ac}$  corresponds to the optimal value which minimizes the long run maintenance cost for a single mode deteriorating system in mode  $M_2$ .

### 6.1 Parameter optimization

In this section, the problem of parameter optimization for the considered maintenance policies is investigated. The parameter of interest is the detection threshold  $h$  defined in (7).

The ‘‘optimal’’ value of  $h$  which leads to a minimal maintenance cost is numerically calculated. To define the optimal value of  $h$ , the maintenance cost, the false alarm rate and the isolation rate are obtained for different values of  $h$  in the interval  $[0, 15]$ . This interval is chosen because the cost remains stable around the same values after  $h = 15$ . The values of  $h$  corresponding to the lowest maintenance cost, lowest false alarm rate and highest correct isolation are defined. To study the impact of the variation of the threshold  $h$  on the properties of the maintenance policy, in addition to the maintenance cost, the probability of preventive and corrective maintenance for different values of  $h$  in the interval  $[0, 15]$  are calculated. The choice of  $h$  it is not always based on the value which minimizes the maintenance cost or which can optimizes the properties of the detection algorithm. For example, it is not sensible to take a value of  $h$  leading to the lowest maintenance cost if it corresponds to a false isolation rate close to 1.

We present in table 2 the properties of the maintenance versus detection/isolation algorithm corresponding to the value of  $h$  which leads to the lowest maintenance cost. It can be noted that except the case 1 ( $\alpha_2 = 2$  and  $\beta_2 = 1$ ) the maintenance costs are very low in comparison with the conservative case when only one threshold is used presented in table 5.

real second mode cases	1	2	3	4
Maintenance Cost	1.98	1.99	1.99	2.00
detection threshold	1	1	1	1
false alarm rate	0.9	0.89	0.89	0.88
correct isolation rate	0.87	0.03	0.04	0.05

Table 2. Optimal maintenance policy corresponding to the lowest maintenance cost.

real second mode cases	1	2	3	4
Maintenance Cost	1.99	2.22	2.37	2.67
detection threshold	5	7	6	15
false alarm rate	0.016	0.02	0.014	0.24
correct isolation rate 1	1	0	0	0

Table 3. Optimal maintenance policy corresponding to the lowest false alarm rate.

real second mode cases	1	2	3	4
Maintenance Cost	1.98	2.09	2.17	2.34
detection threshold	12	2	2	2
false alarm rate	0.018	0.3	0.31	0.27
correct isolation rate	1	0.19	0.2	0.3

Table 4. Optimal maintenance policy corresponding to the highest correct isolation rate.

In table 3 the properties of the maintenance versus detection/isolation algorithm corresponding to the value of  $h$  which leads to the lowest false alarm rate are exposed. It can be noticed that maintenance costs are very close to costs when only one threshold is used (without detection procedure). The use of the detection algorithms when a low false alarm is requested doesn’t improve the quality of the maintenance policy. In this configuration, a maintenance policy without detection procedure seems to be adequate.

The results in table 4 corresponds to the properties of the maintenance versus detection/isolation algorithm corresponding to the value of  $h$  which leads to the highest correct isolation. In this table except the case 1 ( $\alpha_2 = 2$  and  $\beta_2 = 1$ ) the highest correct isolation is not very high but the corresponding false alarm rate and maintenance costs are acceptable. The maintenance cost is still lower than the maintenance policy without detection procedure.

In the first case ( $\alpha_2 = 2$  and  $\beta_2 = 1$ ) the correct isolation rate is always very high. This should be due to the global optimization of the detection threshold  $h$ . This optimization is more sensitive to the properties of the first case where the two modes are very close. It is possible that if in the optimization procedure, for each second mode  $l = 1, \dots, K$ , a detection threshold  $h_l$  in equation (7) is used the result of the correct isolation could be different. But this method requests a complex optimization procedure and the feasibility is arguable.

If the only criteria is the result of the maintenance (low maintenance cost) we can neglect the value of false alarm rate and false isolation. But if in the maintenance procedure the properties of the detection algorithms are of great importance we can not base our choice only on the maintenance cost and we should take into account the properties of the detection algorithm.

case	1	2	3	4
costs	1.97	2.21	2.36	2.66

Table 5. Optimal costs corresponding to the maintenance with one preventive threshold.

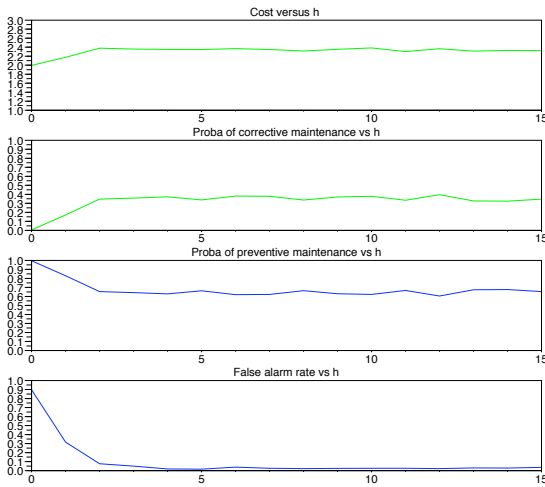


Fig. 3. Maintenance properties when  $\alpha_2 = 2, \beta_2 = 2$ .

In figure 3 the maintenance properties corresponding to the accelerated mode ( $\alpha_2 = 2, \beta_2 = 2$ ) are depicted. To illustrate the results the threshold  $h$  varies in  $[0, 15]$ . The maintenance cost is stable around 2.4 and reaches its minimum value 1.99 for  $h = 1$ . The probability of corrective maintenance is very low and the probability of preventive maintenance is very high. In this optimization it seems that the choice of the optimal value of the preventive threshold doesn't permit the degradation to be in the zone between  $M_{ac}$  and  $M_{nom}$ .

## 7. CONCLUSION

In this paper we have proposed a maintenance policy combined with an on-line detection method. This policy leads to a low maintenance cost. We took into account the possibility that the system could switch on to different accelerated modes. By considering this possibility the proposed detection algorithm is also an isolation algorithm. We have noticed that the lowest false alarm rate and false isolation rate does not always necessarily corresponds to the lowest maintenance cost. The proposed algorithm has generally low correct isolation rate and some times high false alarm rate. The aim in the future works is to improve these properties in order to obtain a low cost maintenance versus detection policy which can easily isolate the real accelerated mode.

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