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Laguerre-Volterra Observer-Controller Design and Its Applications ⋆

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Abstract: By expanding each kernel using the orthonormal Laguerre series, a Volterra functional series is used to represent the input-output relation of a nonlinear dynamic system. With the feedback of the modeling error, we give a novel nonlinear observer-controller design, based on which both the stabilization and tracking problems are solved. To illustrate the effectiveness of the design algorithm, we present the analysis of stability and steady-state performance. The algorithm is further applied on a chemical reactor temperature control system. The Laguerre-Volterra observer-controller design has shown its great potential for a large class of nonlinear dynamic systems frequently encountered in industrial applications.

1. INTRODUCTION

Although most industrial processes display nonlinear dynamics, in practice most control systems are based on linear control design methods. For a mild nonlinearity, a linear approximation of the process dynamics around the operating point is generally satisfactory. If large deviations from this operating point occur, then the model and control system have to be retuned (Dumont & Fu [1993]). If such retuning is frequently required, then automatic tuning or adaptive control becomes attractive. Obviously, an accurate knowledge of the process nonlinearity combined with a reliable nonlinear control design technique would eliminate the need for adaptive control under such circumstances, which is the approach taken here.

In this paper, we will use the Laguerre-filter-based nonlinear dynamic model (Dumont & Fu [1993], Wahlberg & Mäkilä [1996], Heuberger *et al.* [1995]) to represent a stable nonlinear system with fading memory (i.e., the effects of past inputs on the output are negligible after some finite time). Feeding back the modeling error, we will later design a nonlinear state observer and then derive an output feedback control law for both the stabilization and the tracking problems. The general Volterra series representation, whose kernels are assumed to be in the $\mathbb{L}_2$ space, is further approximated by Laguerre series. When higher-order Volterra kernels are neglected and a proper Laguerre filter pole is selected, the number of model parameters required to describe the plant will be small. In recent years, Laguerre filters have been successfully applied to design linear adaptive controllers (Adel *et al.* [1999], Dumont *et al.* [1990], Zhang *et al.* [2006]). As a result, there has been a renewed interest in using Laguerre filters to describe stable linear plants. Compared with the FIR (Finite Impulse) or ARMA (Auto-Regressive Moving Average) model, the advantages of using the Laguerre model include: good approximation of systems with varying time-delay; tolerance to unmodelled dynamics and reduced sensitivity to the estimated parameters; orthogonality of the regression vector under white excitation; no need for the model order and process delay assumption; good low frequency match between the estimated and true plants (Zervos & Dumont [1988]). For a given accuracy requirement, the truncation length of the Laguerre series approximation can be reduced by properly choosing the Laguerre filter pole. The optimum pole selection method has been discussed for the continuous-time linear case (Wang & Cluett [1994]), the discrete-time linear case (Fu & Dumont [1993]) and discrete-time nonlinear case (Campello *et al.* [2004]).

The Laguerre-Volterra model was first proposed and analyzed by Schetzen [1980]. Boyd & Chua [1985] proved the superiority of this model structure compared with some other empirical models such as the NARMAX (Nonlinear Auto-regressive Moving Average with eXogenous inputs) model, and NARX (Nonlinear Auto-regressive with eXogenous inputs) model (Henson [1997]), etc., when capturing the dynamics of FMNSs (Fading Memory Nonlinear Systems). Thereafter, more and more researchers recognize the potential of the Laguerre-Volterra model. One of the

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typical methods is the NMPC (Nonlinear Model Predictive Control) with single-step control horizon, which was successfully applied to a wood chip refiner motor load control system for mechanical pulping (Dumont *et al.* [1994]). Parker & Doyle [1998] extended these results from terminal point tracking to dynamic error analysis, extended the control horizon to double-step to handle more complex nonlinear dynamics, and validated its efficiency by results on a continuous bioreactor (Dumont *et al.* [1994]). However, due to the complex nature of moving horizon optimization of NMPC, two key theoretical issues, i.e. closed-loop stability and steady-state performance, have not been intensively investigated by the existing methods. The most relevant recent results are Lyapunov-based predictive controllers that guarantee feasibility and closed-loop stability form an explicitly characterized set of initial conditions for nonlinearity (Mhaskar *et al.* [2005]) and uncertainty (Mhaskar [2006]) of industrial processes. As to the steady-state performance analysis, although steady-state tracking has been demonstrated for reachable and unreachable set-points, no formal proof is provided so far (Parker & Doyle [1998, 2001]).

Aiming at resolving these serious problems, we bring a new Laguerre-Volterra observer-controller design algorithm. The main contributions of this paper are: **a)** By feeding back the error of the outputs of the plant and the Laguerre-Volterra model, we propose a new state observer, which can facilitate the later design of a linear control law. **b)** For the tracking problem, a pre-optimized offset is introduced into a state feedback control law, which minimizes the steady-state error provided that the Laguerre filter pole is chosen properly. **c)** The design is further applied on a chemical reactor temperature control system to illustrate its effectiveness. This method is expected to make full use of the advantages of the Laguerre-Volterra model. It is worth mentioning that, by extending the Laguerre state to a general system state, this nonlinear observer may be improved to a more general one called the Volterra observer, which has the potential of yielding a novel research method for a larger class of nonlinear dynamic systems.

The rest of this paper is organized as follows: the Laguerre-Volterra model is introduced in Section 2. In Section 3, a novel Laguerre-Volterra observer-controller design is proposed, based on which both the stabilization and tracking problems are solved. Two theorems addressing the closed-loop stability and steady-state performances are given, respectively. Section 4 presents the experimental control performances. Finally, the conclusion is given in Section 5.

2. LAGUERRE-VOLTERRA MODEL

For nonlinear systems, if their dependencies on past inputs decrease rapidly enough with time, their input-output relations can be precisely described by the Volterra series as follows:

$$y(t) = h_0(t) + \sum_{n=1}^{\infty} \int \cdots \int h_n(\tau_1, \cdots, \tau_n) \prod_{i=1}^{n} u(t-\tau_i) d\tau_i, \quad (1)$$

where the functions $h_n(\tau_1, \cdots, \tau_n)$ are the Volterra kernels representing the nonlinear dynamics. This kind of system is called a FMNS (Fading Memory Nonlinear System) (Boyd & Chua [1985]), which is well-behaved in the sense that it will not exhibit multiple steady-states or other related phenomena like chaotic responses. Fortunately, most industrial processes, such as pH neutralization process, heat exchange process, etc, are FMNSs. In practice, the Volterra series is usually truncated to some small finite value M.

Now, we denote the ith-order Laguerre time function by

$$l_i(t) = \int_0^{\infty} \phi_i(\tau) u(t-\tau) d\tau. \quad (2)$$

Since $\{\phi_i\}$ forms a complete orthonormal set in the $\mathbb{L}_2$ space and assume that the Volterra kernels are stable, we can write

$$h_n(\tau_1, \cdots, \tau_n) = \sum_{i_1=1}^{\infty} \cdots \sum_{i_n=1}^{\infty} c_{i_1,\cdots,i_n} \prod_{j=1}^{n} \phi_{i_j}(\tau_j), \quad (3)$$

where $c_{i_1,\cdots,i_n}$ $(n \geq 1)$ are constant coefficients. By using the orthonormal properties of Laguerre functions, the input/output model becomes

$$y_m(t) = c_0(t) + \sum_{i=1}^{N} c_i l_i(t) + \sum_{n=1}^{N} \sum_{m=1}^{N} c_{nm} l_n(\tau_1) l_m(\tau_2) + \cdots \quad (4)$$

with $c_{ij} = c_{ji}$. The expansion error approaches zero as N and M tend to infinity. However, to simplify the analysis in a finitely dimensional state apace, we make the following assumption in this paper.

A1. The input-output model (1) can be accurately represented by (4) with finite Volterra series truncation length M and finite Laguerre series truncation length N of each Volterra kernel.

Under ***A1***, we can define a state

$$x(t) = [l_1(t), \cdots, l_N(t)]^{\mathsf{T}},$$

and the discrete-time Laguerre-Volterra model becomes

$$x(t+1) = Ax(t) + Bu(t), \quad (5)$$

$$y_m(t) = c_0 + Cx(t) + \sigma(x(t)), \quad (6)$$

where the nonlinear function σ is the sum of 2-th to N-th order polynomials in the form of

$$\sigma(x) = x^{\mathsf{T}} Dx + \cdots.$$

From Wang [2004], the matrices in the above model are given as follows

$$A = \begin{bmatrix} p & 0 & 0 & \cdots & 0 \\ \beta & p & 0 & \cdots & 0 \\ -p\beta & \beta & p & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-p)^{N-2}\beta & (-p)^{N-3}\beta & \cdots & \beta & p \end{bmatrix},$$

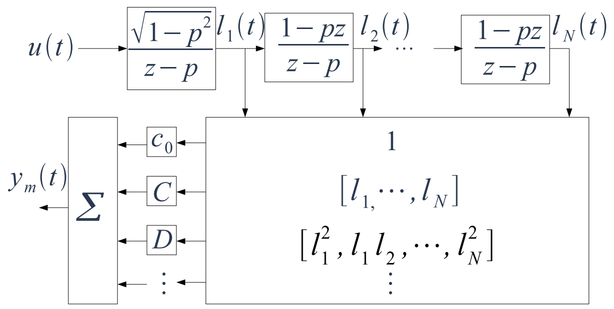

Fig. 1. Laguerre-Volterra nonlinear model

$$B = [\beta^{1/2}, (-p)\beta^{1/2}, \cdots, (-p)^{N-1}\beta^{1/2}]^{\mathsf{T}},$$

$$C = [c_1, \cdots, c_N],\ D = \begin{bmatrix} c_{11} & \cdots & c_{1N} \\ \vdots & \ddots & \vdots \\ c_{N1} & \cdots & c_{NN} \end{bmatrix},$$

where $\beta = \sqrt{1-p^2}$ and p is the Laguerre filter pole. The structure of the discrete-time Laguerre-Volterra model is shown in Fig. 1. In general, the initial value $x(0)$ can be pre-optimized as $x(0) = \sqrt{1-p^2}[1, -p, \cdots, (-p)^{N-1}]$. Here, a finite number of Laguere filters are used, indicating that the true plant is stable and observable in finite time. Eqs. (5) and (6) are an approximation of the Volterra functional series representation for a nonlinear system.

The model parameters $c_0,\ C,\ D, \cdots$ are in a linear regressive form and can be easily estimated by LSE (least square estimation) as follows (Dumont *et al.* [1994]). Assume

$$\theta = [c_0, \cdots, c_N, c_{11}, \cdots, c_{1N}, c_{21}, \cdots, c_{2N}, \cdots, c_{NN}, \cdots], \quad (7)$$

$$\begin{aligned} \Phi(t) = [1, l_1(t), \cdots, l_N(t), l_1^2(t), l_1(t)l_2(t), \cdots, \\ l_1(t)l_N(t), l_2(t)l_N(t), \cdots, l_2(t)l_N(t), \cdots, l_N^2(t), \cdots], \end{aligned} \quad (8)$$

one has

$$y(t) = \theta\Phi(t). \quad (9)$$

By Eq. (5), $\Phi(t)$ can be calculated with $u(t)$ at each sampling time, so the coefficients θ can be identified by RLSE (Recursive Least Square Estimation) (Ljung [1999]) with a forgetting factor λ .

Remark 1. In fact, each stable Volterra kernel in the $\mathbb{L}_2$ space can be accurately approximated by a more general type of models called orthonormal functional series (OFS). The pulse series, Laguerre series and Kautz series are the three typical OFS of 0-, 1st-, and 2nd-order, respectively. In addition, Heuberger *et al.* proposed a method to generate high order OFS. Along with the increase of the OFS order, the OFS model can handle more complex dynamics or higher order plant behaviors with higher speed of convergence. Thus, in order to obtain more efficient Volterra model for complex nonlinearities, one may turn to higher order OFS.

3. OBSERVER-CONTROLLER DESIGN

3.1 Stabilization Problem

Taking $x(t)$ and $\hat{x}(t)$ as the system state and estimated state, respectively, we extend the routine linear state observer

$$\hat{x}(t+1) = A\hat{x}(t) + Bu(t) + \Gamma[y_m(t) - c_0 - C\hat{x}(t)]$$

to a nonlinear observer as follows:

$$\begin{aligned} \hat{x}(t+1) = A\hat{x}(t) + Bu(t) \\ +\Gamma[y_m(t) - c_0 - C\hat{x}(t) - \sigma(\hat{x})] \end{aligned} \quad (10)$$

where $\Gamma \in \mathbb{R}^{N\times 1}$ is the observer gain. Then, we design the output feedback controller as

$$u(t) = K\hat{x}(t) \quad (11)$$

where $K \in \mathbb{R}^{1\times N}$ is the state-feedback gain. Let $e := x - \hat{x}$ be the state estimation error, and hence, $x_c = [x^{\mathsf{T}} \quad e^{\mathsf{T}}]^{\mathsf{T}}$ be the state vector of the closed-loop system (5), (6), (10) and (11). As a result, the closed-loop system can be put in a compact form of

$$x_c(t+1) = A_c x_c(t) + \Gamma_c \delta(x_c(t)) \quad (12)$$

where

$$A_c := \begin{bmatrix} A + BK & -BK \\ 0 & A - \Gamma C \end{bmatrix},\ \Gamma_c := \begin{bmatrix} 0 \\ -\Gamma \end{bmatrix},$$
$$\delta(x_c) := \sigma(x) - \sigma(x - e).$$

The following theorem provides a necessary stabilization condition and the attraction region of the closed-loop system. The advantage of the nonlinear observer (10) over the traditional linear one will be discussed in Section 4.

Theorem 2. Consider the system (5) and (6) under the following assumption:

A2. The pair (A, B) is controllable and the pair (A, C) is observable.

Then, there exist matrices K and Γ such that,

(i) $A + BK$ and $A - \Gamma C$ are Schur matrices (i.e., matrices with eigenvalues inside the unit circle in the complex plane) and hence the closed-loop system (12) is (locally) asymptotically stable.

(ii) Let $P = P^{\mathsf{T}} > 0$ and $Q = Q^{\mathsf{T}} > 0$ be any solution to the Lyapunov equation

$$A_c^{\mathsf{T}} P A_c - P = -Q.$$

Define a compact set

$$\mathbb{B}_\epsilon := \{x_c \in \mathbb{R}^{2N} \mid x_c^{\mathsf{T}} P x_c \le \epsilon\}.$$

Then, there exist $\epsilon, \varepsilon > 0$ such that

$$2x_c^{\mathsf{T}} A_c^{\mathsf{T}} P \Gamma_c \delta(x_c) + \Gamma_c^{\mathsf{T}} P \Gamma_c \delta^2(x_c) \le (1-\varepsilon) x_c^{\mathsf{T}} Q x_c, \quad \forall x_c \in \mathbb{B}_\epsilon. \quad (13)$$

(iii) Any set $\mathbb{B}_\epsilon$ given in (13) is a region of attraction for the closed-loop system (12), i.e., any trajectory of (12) starting from an initial state $x_c(0) \in \mathbb{B}_\epsilon$ converges to the equilibrium point $x_c = \mathbf{0}$.

Proof. See Appendix A.

3.2 Tracking Problem

For the tracking problem, following Chen's strategy (Chen [1999]) of tracking the set-point, one can set the control law as

$$u(t) = K\hat{x}(t) + \rho \quad (14)$$

where the offset ρ is determined by the set-point $r(t) = a$. Provided that Assumptions ***A1*** and ***A2*** are fulfilled, one has $\lim_{t\to\infty} \hat{x}(t) - x(t) = 0$. Accordingly, substituting Eq. (14) into Eq. (5) yields

$$\lim_{k\to\infty} \hat{x}(t) = \lim_{k\to\infty} x(t) = (I - A - BK)^{-1}B\rho, \qquad (15)$$

which can be substituted into Eq. (6) to yield the following polynomial equation in ρ:

$$\alpha_0 + \alpha_1\rho + \alpha_2\rho^2 + \alpha_3\rho^3 + \cdots = a \qquad (16)$$

with $\alpha_0 = c_0$, $\alpha_1 = C(I - A - BK)^{-1}B$, $\alpha_2 = B^{\mathsf{T}}((I - A - BK)^{-1})^{\mathsf{T}}D(I - A - BK)^{-1}B$, $\cdots$.

In this way, the offset ρ can be obtained by solving Eq. (16), hence one can obtain the control law (14) or tracking the set-point $r(t) = a$. The structure of this controller is shown in Fig. 2.

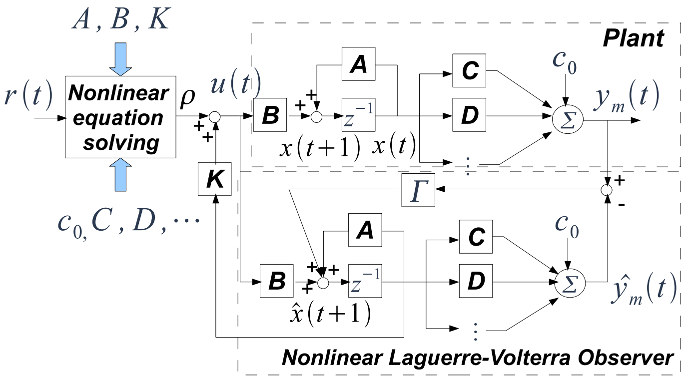

Fig. 2. Controller structure

Remark 3. If M is an odd number, Eq. (16) has at least one real root. Otherwise, the existence of roots cannot be ensured, in which case we must make compromise between approximation accuracy and feasibility, and decrease M by one. One feasible solution is returned for each intersection of the set point with the steady-state locus, and a single viable root is returned when the reference is not reachable (e.g., below the steady state locus in a minimization objective). Theoretically, the larger M (Volterra series truncation length), the higher approaching accuracy can be obtained. However, in practice, most nonlinear systems can be solved with satisfaction for $M \le 3$.

Remark 4. The Laguerre-Volterra model is suitable for FMNS with time-delay, especially variational time-delay. Experience with the Laguerre-Volterra model indicates that the Laguerre series truncation length N can be selected from 5 to 15 in general. For processes with long time-delay, N should be further increased. Furthermore, for fixed N and M, it is not difficult to find suitable p, K and Γ satisfying $\boldsymbol{A2}$.

Now, we will analyze the tracking accuracy of this algorithm. Recall the Laguerre-Volterra model (4) in continuous time domain, which can be written in discrete-time from as

$$y_m(t) = \sum_{m=1}^{M} \sum_{t_1=0}^{\infty} \cdots \sum_{t_m=0}^{\infty} h_m(t_1, \cdots, t_m) \prod_{j=1}^{m} u(t - t_j) \qquad (17)$$

with

$$h_m(t_1, \cdots, t_m) = \sum_{i_1=1}^{\infty} \cdots \sum_{i_m=0}^{\infty} c_{i_1,\cdots,i_m} \prod_{j=1}^{m} \phi_{i_j}(t_j).$$

Inspired by Fu's method (Fu & Dumont [1993]), Campello *et al.* [2004] have proven that a reasonable optimization index for the Laguerre filter pole p is

$$\min_{-1<p<1} J := \sum_{m=1}^{M} \frac{J_m}{m}$$

with $J_m := \sum_{i_1=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} (i_1 + \cdots + i_m) c^2_{i_1,\cdots,i_m}$. With the assistance of such optimization method, we can give a theorem to analyze the steady-state performance of the present control algorithm.

Lemma 5. (Campello *et al.* [2004]) Assume that a nonlinear time-invariant stable system is represented by the Laguerre-Volterra model (17), where the kernels satisfy the stability and unit delay conditions, i.e.

$$\sum_{i_1=0}^{\infty} \cdots \sum_{i_m=0}^{\infty} |h_m(t_1, \cdots, t_m)| < \infty,$$
$$h_m(t_1, \cdots, t_m) = 0 \text{ (if } \exists l \in \{1, \cdots, m\} \text{ such that } t_l = 0)$$

and the Laguerre filter pole p (see Eq. (5)) is pre-optimized by

$$p = \frac{2\bar{Q}_1 - 1 - \bar{Q}_2}{2\bar{Q}_1 - 1 + \sqrt{4\bar{Q}_1\bar{Q}_2 - \bar{Q}_2^2 - 2\bar{Q}_2}} \qquad (18)$$

where the definitions of $\bar{Q}_1$ and $\bar{Q}_2$ are displayed in Campello *et al.* [2004]. Then it minimizes the upper bound of the squared norm of the error resulting from the finite Volterra series truncation length M and finite Laguerre series truncation length N of each Volterra kernel.

Theorem 6. (Steady-state Performance Theorem) Given a system satisfying the conditions in Lemma 5 and the control law is given by Eqs. (14)–(16), then the upper bound of the 2-norm of the closed-loop system's steady-state error $e = \lim_{t\to\infty}(y(t) - a)$ can be minimized if the filter pole p is pre-optimized by Eq. (18), where $r(t) = a$ is the set-point curve.

Proof. As $t \to \infty$, or $z \to 1$, substituting Eq. (15) into Eq. (6) yields

$$\lim_{t\to\infty} y_m(t) = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 + \alpha_3\delta^3 + \cdots.$$

Therefore, the steady-state error can be written as

$$\begin{aligned} e &= \lim_{t\to\infty}(y(t) - a) = \lim_{t\to\infty}(y(t) - y_m(t) + y_m(t) - a) \\ &= \lim_{t\to\infty}(y(t) - y_m(t)) + \lim_{t\to\infty}(y_m(t) - a) \\ &= \lim_{t\to\infty}(y(t) - y_m(t)). \end{aligned}$$

Thus, by Lemma 5, we have that if p is calculated by Eq. (18), the upper bound of the 2-norm of the steady error can be minimized. □

Remark 7. In real world systems, if all the Volterra kernels are expanded by using a single Laguerre basis, and is set according to Eq. (18), then the 2-norm of the steady-state error of the closed-loop system decreases quickly along with the increasing Laguerre Series truncation length N. Thus, if M and N are large enough, the error can be adjusted to be satisfactorily small or even eliminated in the absence of unmodelled dynamics. However, if these two parameters are too large, the online computational complexity would be increased remarkably, so we must trade-off between the steady-state performance and the real-time efficiency.

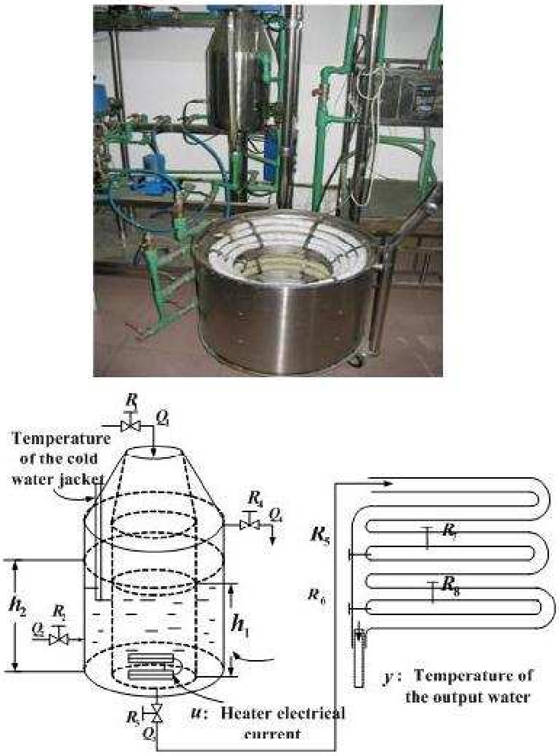

Fig. 3. Control system of a simulation chemical reactor embraced by a cold water jacket

4. CASE STUDY

Fig. 3 shows a chemical reactor with cold water jacket. The input and output fluxes of the inner tanks Q_1 and Q_3 are controlled by an electromagnetic valve R_1 and a manual valve R_3 respectively, while the input and output fluxes of the cold water jacket Q_2 and Q_4 by an electromagnetic valve R_2 and a manual valve R_4 respectively. In the upper subfigure of Fig. 3, the lower part is the long time-delay pipeline device, of which the mechanism is shown in the lower subfigure of Fig. 3. The control objective is to make the temperature stable at this pipeline's exit. The pipeline consists of three sub-pipelines with identical time-delay τ_0. One can modulate the valves $R_5 \sim R_8$ to set the length of the system's time-delay. Note that this system is really a heated tank system rather than a CSTR in the sense that there is no chemical reaction inside.

There are two PT thermal resistance sensors (WZP-270S-typed), whose accuracy is $\pm 0.1°C$, to measure the temperatures of the hot water at the pipeline exit and the cold water at the jacket, respectively. The control signal is constrained by $4mA \leq u \leq 20mA$. The water levels of the inner tank and the jacket are h_1 and h_2 respectively. Owing to its intrinsic mechanism, this system can be seen as a FMNS with long time-delays and uncertainties.

Control performances of *Algorithm b* (our proposed algorithm) in contrast to *Algorithm a* (a typical NMPC (Henson [1997]) based on NAARX (Nonlinear Additive Auto-Regressive models with eXogeneous inputs) model with prediction horizon $H_p = 7$, control horizon $H_u = 7$, input memory $s = 5$ and out memories $q = 3$ are shown in Figs. 4 and 5. Q_2 and h_1 are initially set to be $60l/h$ and

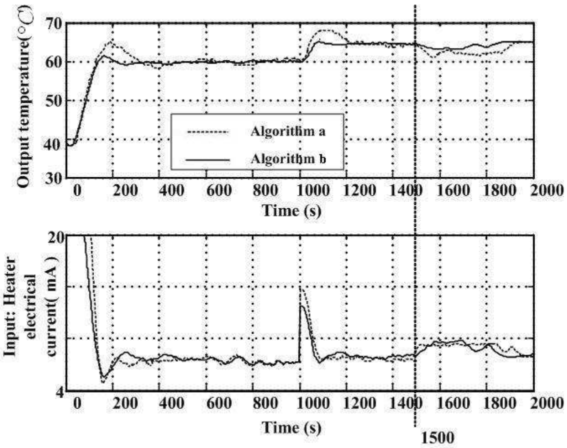

Fig. 4. Control performances of tracking $60°C - 65°C$

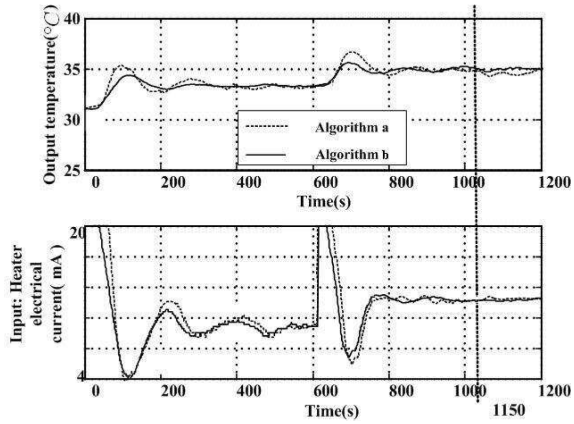

Fig. 5. Control performances of tracking $33.5°C - 35°C$

$200mm$, respectively. Control performances are shown in Fig. 4 (tracking the double-step $60°C - 65°C$) and Fig. 5 (tracking the double-step $30°C - 35°C$).

In Fig. 4, initially, R_5, R_6 are open and R_7, R_8 are closed to set time-delay as τ_0. In order to examine the present method's robustness to variational time-dalay, in the 1500th period, we open R_7, R_8 and close R_5, R_6 such that the time-delay is switched into $3\tau_0$. Furthermore, in Fig. 5, in order to examine the present method's robustness to external disturbance, we increase the flow passing through the cold water jacket from $60l/h$ to $80l/h$ in the 1150th sampling period. The statistical results show that the steady-state control performance is improved remarkably by *Algorithm b* with almost no loss of transient performances such as settling time and overshootings. These merits are due to the novel nonlinear observer based on Laguerre-Volterra model.

5. CONCLUSION

By using the Laguerre-Volterra model, we extend the routine linear observer to a more generalized nonlinear observer. In this way, the estimated Laguerre state combined with a state feedback gain is applied directly to yield a linear control law for the nonlinear stabilization problem. In addition, with the assistance of a pre-optimized offset, this control law can deal with the tracking problem. Finally, the experimental control performance on a chemical reactor temperature control system shows the feasibility and superiority of this novel NMPC method.

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Appendix A. PROOF OF THEOREM 2

(i) The Jacobian matrix of (12) is A_c which is a Schur matrix because $A + BK$ and $A - \Gamma C$ are. The Lyapunov liberalization method implies (12) is asymptotically stable.

(ii) For any $\alpha > 0$, there exists an ϵ such that

$$|\delta(x_c)| \leq \alpha \|x_c\|, \quad \forall x_c \in \mathbb{B}_\epsilon$$

since $\delta(x_c)$ is a sum of quadratic to N-th order polynomials. Then,

$$\begin{aligned} &2x_c^\mathsf{T} A_c^\mathsf{T} P \Gamma_c \delta(x_c) + \Gamma_c^\mathsf{T} P \Gamma_c \delta^2(x_c) \\ &\leq \left(2\|A_c^\mathsf{T} P \Gamma_c\|\alpha + \|\Gamma_c^\mathsf{T} P \Gamma_c\|\alpha^2\right)\|x_c\|^2. \end{aligned}$$

On the other hand

$$x_c^\mathsf{T} Q x_c \geq \lambda_{\min}(Q)\|x_c\|^2$$

where $\lambda_{\min}$ represents the minimal eigenvalue. So, it suffices to pick a sufficiently small α such that

$$2\|A_c^\mathsf{T} P \Gamma_c\|\alpha + \|\Gamma_c^\mathsf{T} P \Gamma_c\|\alpha^2 \leq (1-\varepsilon)\lambda_{\min}(Q).$$

(iii) Let

$$V(t) := x_c^\mathsf{T}(t) P x_c(t).$$

We will first show that any trajectory starting from inside $\mathbb{B}_\epsilon$ remains in $\mathbb{B}_\epsilon$. Otherwise, let the time $t+1$ be the first time the trajectory going outside of $\mathbb{B}_\epsilon$, i.e.,

$$x_c(t) \in \mathbb{B}_\epsilon, \;\; x_c(t+1) \notin \mathbb{B}_\epsilon.$$

As a result, we have

$$\begin{aligned} &V(t+1) - V(t) \\ &= [A_c x_c(t) + \Gamma_c \delta(x_c(t))]^\mathsf{T} P [A_c x_c(t) + \Gamma_c \delta(x_c(t))] \\ &\quad - x_c^\mathsf{T}(t) P x_c(t) \\ &= 2x_c^\mathsf{T}(t) A_c^\mathsf{T} P \Gamma_c \delta(x_c(t)) + \Gamma_c^\mathsf{T} P \Gamma_c \delta^2(x_c(t)) \\ &\quad - x_c^\mathsf{T}(t) Q x_c(t) \\ &\leq -\varepsilon x_c^\mathsf{T}(t) Q x_c(t). \qquad \text{(A.1)} \end{aligned}$$

In the above derivation, we use (13) due to $x_c(t) \in \mathbb{B}_\epsilon$. But, $V(t+1) \leq V(t)$ implies a contradiction of $x_c(t+1) \in \mathbb{B}_\epsilon$.

Now, we have shown that $x_c(t) \in \mathbb{B}_\epsilon$ holds for all t, so does the inequality (A.1). The proof is complete by using Lyapunov's Theorem. □