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Entropy Based Optimization of Decentralized Supply Chain Networks

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Abstract: Supply chain is an organized combination of inbound logistics, production plants and multi-echelon distribution network. Decentralized distribution networks are common and prone to exogenous and endogenous uncertainties. Information uncertainty from the downstream customer and material uncertainty from the upstream supplier makes the supply chain behavior more complex. Previous attempts made to enhance the supply chain performance by optimizing the replenishment strategy do not pay attention to the issue of increasing uncertainty (and consequently operational complexity) in the system. Complexity generates unpredictability in supply chain behavior, affects customer satisfaction, and increases cost. This work aims to improve supply chain performance by quantifying and minimizing the complexity associated with the distribution system through entropy calculations in accordance with the business goal and demand pattern faced by the network.

Keywords: Complexity, Supply chain, Uncertainty, Entropy, Performance

1. INTRODUCTION

The operation of decentralized supply chains is complex due to exogenous uncertainties (e.g. customer demand) and endogenous uncertainties (e.g. problems with suppliers and manufacturers). The market demand is uncertain due to various aspects such as competition, advertisement, seasonality, economic growth and changes in product desirability. Endogenous uncertainties arise when the activities of supply-chain participants are not in harmony with one another. Uncertainty on the supplier side can arise due to unpredictable and uncontrollable factors in the supply of materials, frequency of changing suppliers, complexity of procurement technology, time specificity of materials, delivery frequency, delayed delivery and fluctuations in the selling price (Ho et al., [2005]). Manufacturing uncertainty is related to variations in manufacturing lead-time, product quality, changes in production technology and the complexity of manufacturing.

In decentralized supply chains (DSC), the diversity in management strategies at the distribution nodes makes supply chain management inefficient as well as more complex compared to a centrally managed supply chain (Jemal et al.,[2007]). In a DSC, all entities attempt to forecast downstream customer orders and accordingly decide the inventory they need to maintain for achieving a certain level of customer satisfaction. The available forecasting techniques are limited in accuracy - this leads to shortage or excess inventory at the distribution nodes and render them incapable of fully exploiting market opportunities. For a proficient operation, all entities in a supply chain should co-ordinate effectively among themselves to remain prompt and competent.

Traditional approaches trace the performance measures to optimize the supply chain performance. Optimizing replenishment rule parameters is a common and easily implementable choice. The limitation of this traditional approach is the possible adverse effects caused to other interacting nodes by transferring the uncertainty through information and material flows. For example, optimizing the network for customer satisfaction would build the inventory through aggressive replenishment transmitting uncertain information to the supplier. Optimizing the network for supply chain cost would optimize the inventory but can result in customer dissatisfaction and/or uncertain delivery to the customers.

To achieve good distribution logistics, all distribution nodes have to be operated at minimum complexity. The overall network must also be operated at minimum complexity. Therefore, complexity enumeration and targeted management are necessary to reduce the unpredictable nature of the system. The present work aims to create smarter supply chain with reduced uncertainty to achieve better supply chain operation. We consider minimizing the uncertainty (complexity) in information and material flows by manipulating the replenishment strategy and safety stock.

1.1 Literature Survey

Several researchers have modelled supply chain systems with a view to predict, analyze and optimize their performance. The source of cyclic disturbances in supply chain was first investigated by Forrester [1958]. Inventory control policies and production planning were found to cause uncertainty. Lin et al.[2004] modelled the decentralized distribution unit as a discrete system based on material and information flows. The derived model was utilized to analyze the behaviour of the distribution node under various replenishment strategies. These studies revealed that the PI and cascade heuristics absorb the uncertainty imported to the node and results in a "less backorder and excess inventory" situation. In contrast, the order-upto-policy results in "low inventory and high backorder" thereby generating and exporting uncertainty to the interacting nodes.

In addition to the exogenous uncertainties, the replenishment strategy, demand forecaster, lead-times, batch orders, supply shortages and price variations are identified as major endogenous sources of uncertainties (Lee et al.[1997]). Information distortion (bullwhip) is not the only performance limiting factor. Conflicting objectives like resource minimization and output maximization must be resolved to achieve an efficient, optimal operation. The conflicting nature of these goals make the case for multi-objective optimization (Chen et al.[2003]). Generally, an intelligent distribution network attempts to achieve high customer satisfaction, minimal back order and minimal excess inventory. The most common approaches to solve multi-objective problems are: (i) combining the multiple objectives into a single objective function to obtain a single solution as in weighted sum method or utility functions, or (2) obtaining a set of non-dominated Pareto optimal solutions. For multiple-objective problems, it can be problematic to combine the objectives into a single objective. A slight perturbation in the parameters used to combine the objectives could result in very different optimal solutions. Therefore, evolutionary optimization methods that search the solution space not for one optimal solution but for a set of solutions that are Pareto-optimal are particularly useful. The decision maker has then the difficult task of picking out one solution for implementation.

An entropy based complexity optimization methodology overcomes the difficulties faced in the conventional methods. In the context of operational complexity, an ideal distribution node is one that either absorbs or (at the very least) does not add to the uncertainty imported to it. Reducing the variability in information and material flows is a crucial step to improve network performance. In this work, we show that it is possible to improve the performance of a supply chain by analyzing its time series data and employing an entropy-based complexity management methodology proposed here. Starting from a possibly well-established decentralized supply chain system, we evolve it into a more profitable decentralized supply chain system by following a systematic data analysis and optimization approach. A multi-echelon decentralized supply chain network is used to demonstrate the workability of the proposed framework.

2. PROBLEM DESCRIPTION

A simple supply chain system (see Figure 1) similar to that considered in Perea-Lopez et al.[2003] will be used to illustrate our methodology. However our method can be extended to larger and complex networks at the expense of more computational effort. The supply chain network (SCN) consists of three retailers ($i \in$ R1 to R3) connected with a distribution centre and services six different customers ($j \in$ C1 to C6). This network is fully decentralized pull-driven system where each distribution node belongs to a different company. The internal strategy practiced by the distribution nodes are decided by its management. The management may choose to set the inventory level at a constant (target) value or may make it responsive to the uncertain demand. In the proposed case study, all distribution entities are assumed to follow the responsive strategy (where the inventory target is changed in accordance with the uncertain demand pattern; see (7)) so as to increase customer satisfaction with less back order and minimal excess inventory. Our goal is to improve the performance of the retailer echelon of this SCN by minimizing the uncertainty present in it by revising the tactical decisions such as replenishment parameters and the safety stock level. First, we use time series data available from the SCN to quantify the uncertainty/complexity in it. This is followed by entropy based complexity optimization at the retailer echelon which essentially leads to a SCN with relatively less complexity in accordance with the business goals of the SCN.

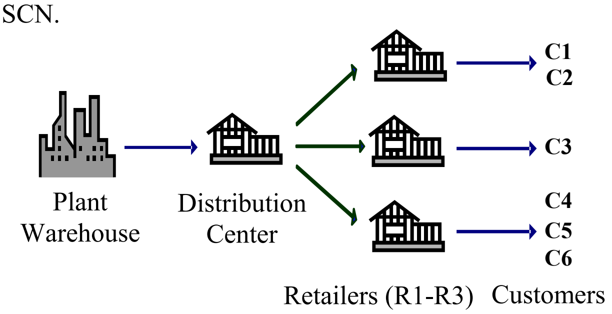


Figure 1: Schematic diagram of Decentralized Supply Chain

2.1 Material and Information Balances: The dynamic behaviour of the distribution nodes is modelled using material and information flows (see Figure 2). The discrete model proposed by Lin et al.[2004] is described next. The main objective of the distribution node 'i' is to organize its inventory position, at discrete time t, $I_{p,i}(t)$ at the desired target level. For node 'i', let Y_{pi} denote the material flow from its supplier and Y_{ij} denote the material flow from node 'i' to a downstream node 'j'. The inventory position at time t depends on the inventory position at time t-1, the materials received and the materials dispatched from node 'i'. The inventory position $I_{p,i}(t)$ is also the sum of the inventory at-hand $I_{H,i}(t)$ and the inventory on-road $I_{R,i}(t)$. Inventory on-road $I_{R,i}(t)$ is the sum of orders that have been despatched by the supplier, but has not been received by the distributor node due to the lead time equivalent to L time samples. The lead time (L) is the time taken by the supplier to satisfy the orders placed by the downstream nodes. It includes the time taken by the distributor node to place the order, the time taken by the supplier to process the order and the product transportation time. The order is assumed to be communicated instantaneously using advanced information technologies. Therefore, the lead time mainly corresponds to the time taken by the supplier to process the downstream order and the transportation time. The lead time depends on the geographical location of the supplier and customer, modes of transportation available, and the product availability. The lead time information can be obtained from the authorities of the distribution node or estimated from time-series data gathered from the supply chain.

Based on the above description, the following equations can be written for distribution node 'i':

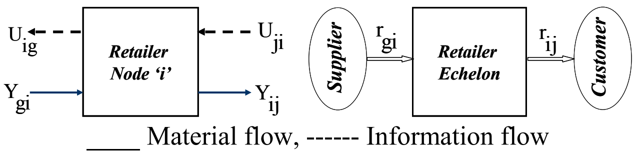


Figure 2: Schematic representation of Distribution Node

$$I_{p,i}(t) = I_{p,i}(t-1) + Y_{pi}(t) - Y_{ij}(t) \quad (1)$$

$$I_{p,i}(t) = I_{H,i}(t) + I_{R,i}(t) \quad (2)$$

Equation (1) can be rewritten using z-transform as:

$$I_{p,i}(z^{-1}) = \frac{1}{1-z^{-1}} Y_{pi}(z^{-1}) - \frac{1}{1-z^{-1}} Y_{ij}(z^{-1}) \quad (3)$$

The dynamics of the inventory at-hand is similar to that of the inventory position with the only change being that the material dispatched by the supplier at time t-L is used in the RHS of the material balance (4).

$$I_{H,i}(t) = I_{H,i}(t-1) + Y_{pi}(t-L) - Y_{ij}(t) \quad (4)$$

Equation (4) can be expressed using z-transform as

$$I_{H,i}(z^{-1}) = \frac{z^{-L}}{1-z^{-1}} Y_{pi}(z^{-1}) - \frac{1}{1-z^{-1}} Y_{ij}(z^{-1}) \quad (5)$$

Equations (1), (2) and (4) may be used to express inventory on-road $I_{R,i}(t)$ as:

$$I_{R,i}(z^{-1}) = \frac{1-z^{-L}}{1-z^{-1}} Y_{pi}(z^{-1}) \Leftrightarrow I_{R,i}(t) = \sum_{k=t-L}^{t} Y_{pi}(k) \quad (6)$$

Equation (6) notes that the inventory on-road at time t is the sum of the orders satisfied by the supplier during the past L time periods (but not received at node 'i' at time t due to the transportation delay). $Y_{pi}(k)$ is the material shipped by the supplier at time k against an order placed by the node 'i' $U_{ip}(k)$.

In general, the decentralized node prefers to become more responsive to the market demand by maintaining a flexible inventory position. The flexibility in inventory position is achieved by setting desired inventory position target $SI_{P,i}(t)$ in response to the forecasted demand for L time periods (7). An exponential forecaster with $\alpha = 0.111$ (8) was used in all distribution nodes practising responsive strategy to forecast the downstream demand as suggested in the literature (Lin et al.[2005]).

$$SI_{p,i}(z^{-1}) = (L+2)\sum_{j} \bar{d}_j(z^{-1}) \quad (7)$$

$$\bar{d}_j(z^{-1}) = \frac{\alpha}{1-(1-\alpha)z^{-1}} d_j(z^{-1}) \quad (8)$$

with d_j representing the actual demand at downstream node 'j'.

The rate at which downstream orders are satisfied by node 'i' depends on the inventory level at-hand. Whenever inventory at-hand is high, the distribution node can satisfy all downstream customer orders ($m_i = 1$); when it has limited inventory, the distributor has a policy of satisfying equal proportion of all downstream orders ($0 \le m_i \le 1$). This order processing is modelled by (9).

$$Y_{ij} = z^{-1} \sum m_i \times d_j \qquad i\in[1,3],\ j\in[1,6] \quad (9)$$

2.2 Market Demand: The distribution network is subjected to two patterns of market demand in order to analyze the workability of the proposed entropy based optimization framework. The first type represents a stationary demand pattern (stationary stochastic demand) and the second type represents non-stationary demand. Both stationary and non-stationary demand patterns are generated by a white noise sequence (ξ_j) passing through suitable filters (Lin et al.[2005]). See (10) and (11). For stationary demand, ξ_j has mean = 5, variance = 1 and for non-stationary demand, ξ_j has mean = 0, variance = 1. Specifically, the two demand patterns are represented in z-domain as:

Stationary Demand: $$d_j(z^{-1}) = \frac{1}{1-0.6z^{-1}} \xi_j(z^{-1}) \quad (10)$$

Non-Stationary Demand: $$d_j(z^{-1}) = \frac{1}{1-z^{-1}} \frac{1}{1-0.6z^{-1}} \xi_j(z^{-1}) \quad (11)$$

2.3 Replenishment Strategy: For large lead time systems, managing inventory position at all the distribution nodes is regarded as the key to supply chain performance and stability. In most situations, order-upto-policy (12) is used as the replenishment strategy to manage inventory position in the distribution system. The order-upto-policy that manages inventory at-hand (instead of inventory position) is a subset of above case when the lead time tends to zero.

$$U_{ip}(t) = K_i\left(SI_{p,i}(t) - I_{p,i}(t)\right) + SS_i \quad (12)$$

where $SI_{p,i}(t)$ is given by (7), K_i is the replenishment parameter which is unity in order-upto-policy, and SS_i is the safety stock of the retailer node 'i'.

A phenomenon known as bullwhip effect magnifies the uncertainty in information flow as one moves to the upstream nodes from the customer nodes. This demand amplification manifests as large swings in inventory level - huge build-up in inventory (excess inventory) followed by accumulation of back order (stock outs). Bullwhip (BW) is quantified as the ratio of variance of outgoing order (to the supplier) to the variance in incoming order (from the downstream nodes). Mestan et al. [2006] compute it as the average of distortion obtained at each time period. Constraining BW≤1 signifies no information distortion, but may lead to poor replenishment if BW<<1. So, high and low BW's can affect supply chain performance. Replenishment rules such as order-upto-policy are known to cause high information distortion. *Choosing the right replenishment parameters in relation to the demand pattern and business goals is a challenging task.*

Customer satisfaction (CS) is one of the important performance metrics in supply chains. High CS is required in order to remain competitive in the marketplace. CS can be quantified as the percentage of downstream customer orders satisfied by the distribution system. The CS metric is strongly related with cost metrics such as excess inventory (EI) and back order (BO). Allocating more inventory than required will increase CS and increase inventory holding costs, whereas allocating lesser inventory than required will decrease CS and leads to accumulation of back orders. An ideal distributor should have CS=1, EI & BO equal to zero and information distortion, BW=1.

3. PROPOSED METHODOLOGY

The entropy based performance improvement framework (figure 3) starts with modelling the actual representation of the network from the knowledge of network topology and the business strategy practiced at all the entities. The network topology includes the connectivity between customers, retailers, distribution centers and the plant warehouse. The distribution nodes may differ in the internal strategy in the aspects of demand forecasting, order processing, inventory allocation and product replenishment. The distribution system can be modelled by combining the information gathered about the supply chain and through the analysis of time series data generated by it. The time-series data is a valuable resource to compute the complexity and performance of the existing network.

In the proposed methodology, key information and material flows are considered for analysis instead of performance measures like customer satisfaction, back order and excess inventory. Crucial flow variables are chosen in accordance with the business goal and the requirement of complexity handling strategy. The crucial variables are customer order (*information inflow*), order placed to the supplier (*information outflow*), delivery from the supplier (*material inflow*), delivery to the customer (*material outflow*), supplier reliability (trustfulness r_{gi}, i.e. *material/information ratio*) and node reliability (r_{ij}) to the customer. Each variable are categorized into two states/bins namely *desired state* and *undesired state* depending on whether the uncertain variables are in the affordable range or not. From time-series data, the probability of any variable residing in the desired (in-control) state is evaluated and transformed into a complexity measure called Shannon Entropy (SE). For example, at each discrete time period, the order placed to the supplier is considered to be in the desired state whenever it lies between (μ_{order}-2σ_{demand}) and (μ_{order} +2σ_{demand}). By categorizing the order data into desired (in-control) and undesired (not-in-control) states, the probability (P_i) is estimated as the fraction of times the order remains in the desired state. The observed probability (P_i) is converted into a complexity measure through (14) – SE is based on the probability of an uncertain variable residing in desired state (probability = P_i) and the probability that it is in the undesired state (probability = 1-P_i) (Sivadasan et al.[2002]). SE is zero if the probability of the desired state (P_i) is either zero or unity (13 and 14). Here, for the retailer node 'i', the order and demand are represented by the symbols U_{ip} and U_{ji} respectively. The entropy calculation can be extended to other relevant variables such as material delivery (Y_{ij}) to the customer by defining the in-control and not-in-control state with respect to the incoming material flow (Y_{pi}) from the suppliers.

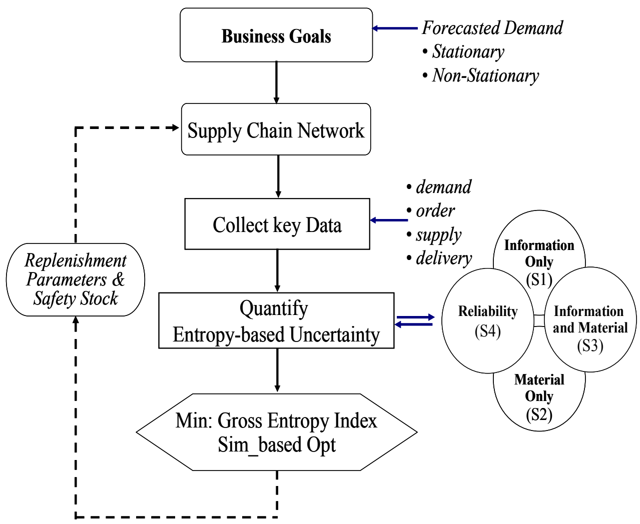


Figure 3: Complexity Management Framework

$$E_T = E(\text{desired}) + E(\text{undesired}) \tag{13}$$

$$E_T = P_i \log P_i + (1 - P_i)\log(1 - P_i) \tag{14}$$

$$E_{r,i} = \frac{E(\text{desired})}{E(\text{undesired})} = \frac{P_i \log P_i}{(1-P_i)\log(1-P_i)} \quad (15)$$

$$GEI = \sum_i E_{r,i}(\text{import}) - E_{r,i}(\text{export}) \quad (16)$$

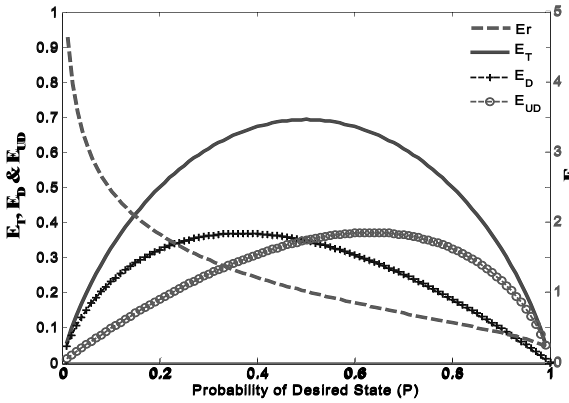


Figure 4: Entropy measure for the desired state (P_i)

The total entropy is a maximum when the occurrence of any state is equally probable. For example, in figure 4, the maximum entropy occurs when P is 0.5. Therefore, to minimize the network complexity, the system is driven towards the desired state by minimizing the ratio of the entropies of the desired and undesired states (i.e. minimize $E_{r,i}$) (15). The use of entropy makes it possible to combine the relevant variables into a common platform. Gross Entropy index (GEI) is the measure of overall complexity resident in the system due to the uncertain flows (16).

There are several complexity management strategies (17-20) available to handle uncertainties with respect to the business goals and demand pattern. Strategy-I (S-I) attempts to minimize the complexity of the distribution system by exporting (to the upstream nodes) the information uncertainty received by it. Strategy-II (S-II) attempts to equalize the uncertainty in the material flow imported from the supplier to the customer. Strategy-III (S-III) accounts for both information and material complexity in an additive manner to provide better information flow to the supplier and the material flow to the customers. Strategy-IV (S-IV) reflects the supplier trustfulness to its customer. The predictability and the performance of the distribution network can be improved by reducing the GEI through revising the tactical decisions such as replenishment rule parameters and safety stock. Pattern search (a direct search optimization tool in Matlab) is used to reduce the complexity at the retailer echelon through simulation based optimization technique.

Strategy I: $$\text{Min GEI} = \sum_i \left| E_{r,U_{ji}} - E_{r,U_{ip}} \right| \quad (17)$$

Strategy II: $$\text{Min GEI} = \sum_i \left| E_{r,Y_{pi}} - E_{r,Y_{ij}} \right| \quad (18)$$

Strategy III: $$\text{Min GEI} = \sum_i \left| E_{r,U_{ji}} - E_{r,U_{ip}} \right| + \left| E_{r,Y_{pi}} - E_{r,Y_{ij}} \right| \quad (19)$$

Strategy IV: $$\text{Min GEI} = \sum_i \left| E_{r,U_{ip}/Y_{pi}} - E_{r,U_{ji}/Y_{ij}} \right| \quad (20)$$

It must be noted that it is not a good idea for any node to absorb or export all the imported uncertainties. Rather, the uncertainty must be absorbed (and the rest be exported) in an intelligent way so that desired customer satisfaction and optimal inventory allocation is achieved.

4. RESULTS AND DISCUSSIONS

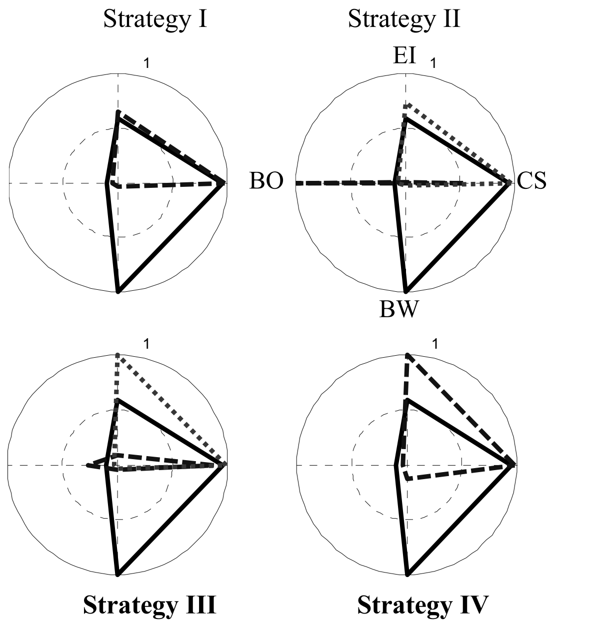
We now present the results of applying complexity management strategies (S-I to S-IV) to the distribution system described by Figure 1. The performance metrics are normalized and plotted in the polar plot (Figure 5) for various complexity management strategies. Each Cartesian co-ordinate of the polar plot symbolizes the performance metrics. Corresponding details about uncertainty ratio (export to import) are provided in Table 1 where S-I to S-IV indicate strategies, RP stands for order-upto-policy, S stands for Stationary demand pattern, NS stands for non-stationary demand pattern and (C) denotes inclusion of the customer satisfaction constraint. The export-import uncertainty ratio helps to describe the distribution system behaviour in terms of the generation or attenuation of uncertainty. Ratio values greater than one signifies uncertainty generation (i.e. more uncertainty is exported than what is imported) while the reverse holds for ratio values less than one. For complete uncertainty transfer (without any generation or absorption) this ratio is unity. From Table 1, it is seen that the complexity obtained by practicing any of the four strategies is significantly less in comparison with the complexity associated with order-upto-policy.

Table 1: Gross Entropy Import - Export Information

Demand	Export/Import Uncertainty Ratio				
Strategy	***RP***	***S-I***	***S-II***	***S-III***	***S-IV***
S	1.225	0.964	0.493	0.987	1.022
NS	1.495	1.00	1.157	1.00	1.416
Strategy (with C)	***RP***	***S-I***	***S-II***	***S-III***	***S-IV***
S	1.225	-	0.965	0.987	-
NS	1.495	1.298	1.393	1.364	1.41

4.1 Stationary Demand

The proposed framework minimizes the complexity at the retailer echelon and performed significantly better than the well established order-upto-policy. For stationary demand, the order-upto-policy generates 22.5% uncertainty in information flow which causes a large swing in inventory level thereby affecting the material flow to the customers. Use of S-I strategy resulted in CS > 97%, no information uncertainty generation but 3.5% attenuation in material uncertainty. This would allow the upstream nodes to operate (forecast and replenish) efficiently to allocate right quantity of inventory to satisfy the customer order. Otherwise the supplier would have suffered due to information distortion. Strategy II absorbs 50.6% information uncertainty, and minimizes the inventory cost but only gives CS = 53%. The CS obtained with strategies S-III and S-IV are 87.6% and 98.4% respectively. Figure 5 shows that by adopting the complexity management strategies, we can have less information distortion (BW) compared to the order-upto-policy. In addition, strategies S-II and S-III were also extended to minimize the complexity with a constraint of attaining the desired output (CS $\geq$ 95%). This results in CS values of 96.99% and 98.43% respectively. 3.5% information uncertainty is absorbed in S-II and 1.3% material uncertainty is absorbed in S-III.



___ Order-upto-policy, --- w/o and .....with CS Constraint

Figure 5: Polar representation of the Strategies I-IV

4.2 Non-Stationary Demand

The entropy based complexity management strategies (I to IV) is attempted for the non-stationary demand patterns also. Due to space limitation, we do not present the results for this case using polar plot. The CS attained by adopting strategies I to IV are 81.71%, 86.04%, 79.42% and 95.17% respectively. Interestingly, S-II provides better CS by adding 15.7% uncertainty in the replenishing order. Strategies I and III provide reasonable CS without adding uncertainty in either information or material flow. Strategy IV provided high CS by reflecting the operational complexity faced from the supplier to its customer. If the CS $\geq$ 95% constraint is placed, all strategies achieve this while trying to minimize complexity. Strategies I to IV generates and exports 29.8%, 39.3%, 36.4% and 41% of uncertainty respectively.

5. CONCLUSIONS

The proposed entropy based complexity minimization method is able to improve the performance of the distribution system significantly compared to the initial performance of the supply chain. This complexity management strategy can be extended to the overall network and for systems with more states of interest. For the system facing stationary and non-stationary demand, strategy I and IV performed relatively superior. When the customer satisfaction constraint is included, all strategies performed equally well for the non-stationary demand case. However, for stationary demand, strategy I and II still offered better improvements than strategies S-III and S-IV.

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