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Towards Automatic Model Based Controller Design for Reconfigurable Plants *

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Abstract: This paper introduces model-based Plug and Play Process Control, a novel concept for process control, which allows a model-based control system to be reconfigured when a sensor or an actuator is plugged into a controlled process. The work reported in this paper focuses on composing a monolithic model from models of a process to be controlled and the actuators and sensors connected to the process, and propagation of tuning criteria from these sub-models, thereby accommodating automatic controller synthesis using existing methods. The developed method is successfully tested on an industrial case study from Danfoss A/S, where the process to be controlled is the indoor temperature of a house and the actuators acting on the process are a floor heating system and an electric radiator.

1. INTRODUCTION

A complex process, such as a power plant or a water distribution system, might comprise hundreds or thousands of sensors and actuators. Adding or removing just one sensor or actuator, however, might in extreme cases require a complete re-design of the entire control system, with a tremendous cost involved. Currently, such changes are primarily implemented during a scheduled recommissioning of the process control system even though online reconfiguration would have yielded a more optimal performance. The lack of flexibility in such a system and the expenses involved with reconfiguration make the industry reluctant to implement advanced control technology in the first place or even upgrade the subsystems, for instance by adding sensors or actuators, in order to achieve optimal performance.

Traditionally, the high cost of controller design has been lowered by using PID controllers, and tuning these using heuristic tuning rules. see e.g. Ziegler and Nichols (1942), Skogestad and Postlethwaite (1996). This makes PID control the most commonly used controllers in industrial process control, because of the simple structure and ease of understanding it.

The reluctance towards using advanced control technology might in part be ascribed to the expenses involved with recommisioning, even though, once the advanced control system is operational it would yield a better performance.

2. PROBLEM FORMULATION

This research aims at lowering the cost associated with recommissioning a control system, by developing model based control algorithms that are flexible with regards to the sensor and actuator configurations of the controlled process.

The vision is that the control system of a plant shall become aware of the existence of a new component, e.g. an added actuator or sensor, in the plant. Furthermore, as a reaction to this awareness, the control system shall reconfigure itself such that the newly expanded plant will work in an optimal manner. One of these scenarios is depicted in Figure 1, where a new actuator is connected to the controlled process and the controller utilize this new actuator to control the process better. The presented concept should not be confused with the concept used within flexible manufacturing systems as described in Sethi and Sethi (1990) and in more detail in Lee (1997). The aim is not for the manufacturing system to be flexible towards different products or a changing market, but for the control algorithm to be flexible towards upgrades of the manufacturing system.
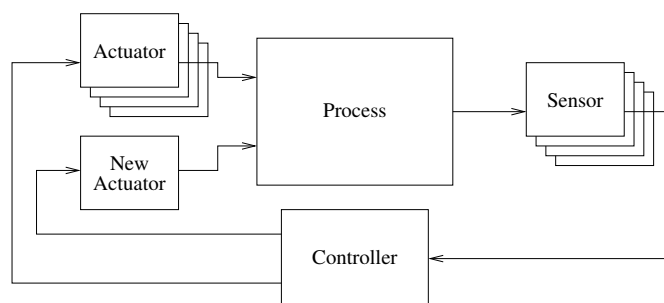

Fig. 1. The envisioned Plug and Play Process Control System; a new actuator is added to a controlled process.

A hypothetical algorithm that automates the task of reconfiguring a control system for a reconfigured plant shall have the following features:

- Model based, i.e., the model of the existing system and a model of the new piece of hardware shall be used by the algorithm for synthesising a controller for the perturbed plant.
- Automatic, i.e., no manpower shall be used on upgrading or tuning the controller when a new piece of hardware is added to the system, other than the manpower used to model the actuator.
- Incremental, i.e., the algorithm shall add new components to the controller without removing any, such that existing safety interlocks, that are part of the existing controller can be reused, e.g. safety code that assures a plant is shut down if a dangerous state is reached

The development of the above hypothesised algorithm is initiated by formulating a framework in which a monolithic model is composed of a model of the process to be controlled and the actuators connected to the process. The sub-models are augmented with tuning information, such that it is possible to automatically tune a monolithic controller for the composed system.

The tuning criteria that are to be propagated from the sub-models to the monolithic model are chosen to be cost functions, since this allows for tuning of a wide range of controller type, e.g. proportional integral/proportional integral derivative controllers, see He et al. (1998), linear quadratic regulators, see Franklin et al. (2002), model predictive controllers, see Qin and Badgwell (1997) and, if the models are extended to encompass hybrid systems, optimal model predictive control of hybrid systems, see Borrelli et al. (2005).

3. SOLUTION APPROACH

Let the system be divided into subsystems consisting of the process to be controlled, p, and the set of actuators working on the plant, $\mathcal{A}$. Let each of these subsystems model their own dynamic behavior, cost function, and range of validity, such that the system is given as the touple:

$$S = (p, \mathcal{A}). \tag{1}$$

Let the process, p, be modelled as the three-touple

$$p = (G_p, J_p, \mathcal{D}_p), \tag{2}$$

where G_p is an autonomous state space model

$$\dot{x}_p = A_p x_p, \tag{3}$$

J_p is the cost function of the process on the form

$$J_p = \int_0^\infty F_p(x_p) \, dt, \tag{4}$$

and $\mathcal{D}_p$ is the domain of validity of the cost function on the form

$$\mathcal{D}_p = \{x \in \mathcal{R}^{n_p} | \underline{x_n} \le x_n \le \overline{x_n}\}, \tag{5}$$

where n_p is the number of states in the model, and $\underline{x_n}$ and $\overline{x_n}$ are minimum and maximum bounds for the nth state.

Similarly, each actuator in the set of actuators working on the plant are modelled as the three touple

$$a = (G_a, J_a, \mathcal{D}_a), \tag{6}$$

where the dynamical behaviour is modelled as a state space model

$$G : \dot{x}_a = A_a x_a + B_a u_a \tag{7}$$

$$y_a = C_a x_a + D_a u_a, \tag{8}$$

the cost function of the actuator is a function of the state of the actuator and the input to the actuator given as

$$J_a(x_a, u_a) = \int_0^\infty F_a(x_a, u_a) \, dt, \tag{9}$$

and finally the domain of validity given by the rectangular space

$$\mathcal{D}_a = \{(x, u) \in \mathcal{R}^{n_a} \times \mathcal{R}^{m_a} \mid \underline{x_i} \le x_i \le \overline{x_i} \tag{10}$$
$$\wedge \underline{u_j} \le u_j \le \overline{u_j}\},$$

where n_a is the number of states in the model of the actuator, and m_a is the number of inputs.

The dynamical behaviour of the system, $S = (p, \mathcal{A})$, where the set of actuators working on the plant are given as $\mathcal{A} = \{1, \ldots, n\}$, is

$$\dot{x}_p = A_p x_p + \sum_{a \in \mathcal{A}} y_a, \tag{11}$$

or in state space form

$$\dot{x} = Ax + Bu, \tag{12}$$

where

$$A = \begin{bmatrix} A_p & C_1 & \cdots & C_n \\ & A_1 & & \\ & & \ddots & \\ & & & A_n \end{bmatrix}, \; B = \begin{bmatrix} D_1 & \cdots & D_n \\ B_1 & & \\ & \ddots & \\ & & B_n \end{bmatrix}, \tag{13}$$

$$x = \begin{bmatrix} x_p \\ x_1 \\ \vdots \\ x_n \end{bmatrix}, \; u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}. \tag{14}$$

The total cost given as

$$J(x, u) = J_p(x_p) + \sum_{a \in \mathcal{A}} J_a(x_a, u_a) \tag{15}$$

$$= \int_0^\infty F_p(x_p) + \sum_{a \in \mathcal{A}} F_a(x_a, u_a) \, dt \tag{16}$$

$$= \int_0^\infty F(x, u) \, dt, \tag{17}$$

and the domain of validity given as

$$\mathcal{D} = \mathcal{D}_p \times \mathcal{D}_1 \times \ldots \times \mathcal{D}_n. \tag{18}$$

By minimising the quadratic difference of the cost to go function of the entire plant, $F(x, u)$, and a quadtratic const to go function on the form

$$F_q(x, u) = x^T Q x + u^T R u + c, \tag{19}$$

over the rectangular domain $\mathcal{D}$ the weight matrices Q and R for an optimal controller with a cost function on the form

$$J_q = \int_0^\infty x^T Q x + u^T R u \, dt, \tag{20}$$

was found. The constant c was added to the quadratic cost to go function to remove constant cost from $F(x, u)$, so that constant costs would not affect the dynamics of the controlled system.

The minimization problem was formulated as

$$\min_{Q, R, c} I[(F_q(x, u) - F(x, u))^2]$$
$$s.t. \ Q = Q^T \qquad\qquad (21)$$
$$R = R^T,$$

where

$$I[\cdot] = \int_{\underline{u_m}}^{\overline{u_m}} \cdots \int_{\underline{u_1}}^{\overline{u_1}} \int_{\underline{x_n}}^{\overline{x_n}} \cdots \int_{\underline{x_1}}^{\overline{x_1}} \cdot dx_1 \cdots dx_n du_1 \cdots du_m. \qquad (22)$$

A stationary point was found by differentiating with respect to all entries $q_{i,j}$ in Q, where $i \le j$ (since $Q = Q^T$), giving the following $\frac{N^2+N}{2}$ equations, where N is the number of rows in Q:

$$0 = \frac{\partial}{\partial q_{i,j}} I\left[(F_q(x, u) - F(x, u))^2\right]$$
$$= I\left[\frac{\partial}{\partial q_{i,j}}(F_q(x, u))^2\right]$$
$$- 2I\left[\frac{\partial}{\partial q_{i,j}} F_q(x, u) F(x, u)\right]$$
$$= I\left[x_i x_j \left(x^T Q x + u^T R u + c\right)\right] - I\left[x_i x_j F(x, u)\right]. \qquad (23)$$

Similarly, for all entries $r_{i,j}$ in R, where $i \le j$ (since $R = R^T$), gives the $\frac{M^2+M}{2}$ equations, where M is the number of rows in R:

$$0 = \frac{\partial}{\partial r_{i,j}} I\left[(F_q(x, u) - F(x, u))^2\right]$$
$$= I\left[u_i u_j \left(x^T Q x + u^T R u + c\right)\right] - I\left[u_i u_j F(x, u)\right], \qquad (24)$$

and for c

$$0 = \frac{\partial}{\partial c} I\left[(F_q(x, u) - F(x, u))^2\right]$$
$$= I\left[x^T Q x + u^T R u + c\right] - I\left[F(x, u)\right]. \qquad (25)$$

Solving these $\frac{N^2+N}{2} + \frac{M^2+M}{2} + 1$ linear equations with as many unknowns yield exactly one stationary point that is the minimum.

4. EXAMPLE

The above was used to develop an optimal controller for a house model that initially has floor heating and later has an electric radiator added. The system that is to be controlled is sketched in Figure 5. If a new actuator is added to the system the following occurs:

(1) A new system model as given in (1) with the new actuator is comprised from the submodels given in the following.
(2) Based on the new system model a new quadratic approximation of the cost function is found by solving (23)-(25).
(3) A new optimal controller for the reconfigured plant is synthesised using the weight matrices Q and R from the quadratic cost function approximation and the system and input matrices from the system model, given in (13).

The house is decomposed as shown in Figure 3, such that the dynamics of the floor, the dynamics of the room, and
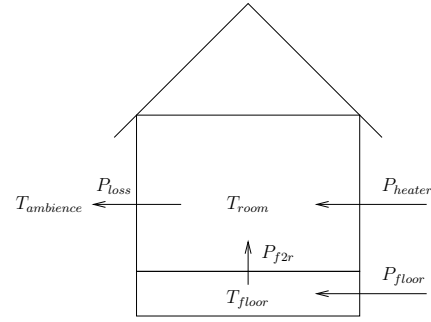

Fig. 2. House with floor heating and electric radiator.

the dynamics of the ambiance comprises the process, and the floor heating and the electric radiator are modelled individually as actuators.
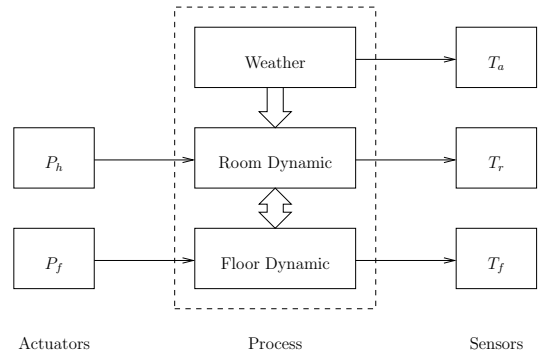

Fig. 3. Decomposed model of a house with floor heating and electric radiator.

4.1 Process: House Heat Transfer

The dynamics of the room temperature and the floor temperature was modelled using Newton's law of cooling

$$Q = hA(T - T_a), \qquad (26)$$

where Q is the heat transfer in $[W]$, h is the heat transfer coefficient, A is the surface area of the heat being transferred, T is the temperature of the object's surface and T_a is the temperature of the surroundings, yielding the following model:

$$\begin{bmatrix} \dot{T_r} \\ \dot{T_f} \end{bmatrix} = \begin{bmatrix} -\dfrac{k_1 A_f}{H_r} - \dfrac{k_2 A_w}{H_r} & \dfrac{k_1 A_f}{H_r} \\ \dfrac{k_1 A_f}{H_f} & -\dfrac{k_1 A_f}{H_f} \end{bmatrix} \begin{bmatrix} T_r \\ T_f \end{bmatrix} \qquad (27)$$

$$+ \begin{bmatrix} \dfrac{k_2 A_w}{H_r} \\ 0 \end{bmatrix} T_a, \qquad (28)$$

where T_r and T_f are the temperature of the room and the floor, T_a is the temperature of the ambiance, H_r and H_f are the specific heat capacity of the air in the room and the floor in $\frac{J}{°C}$, A_f and A_w are the areas of the floor and the walls in m^2, and k_1 is the heat transfer coefficients between the floor and the air in the room in $\frac{W}{m^2 \, °C}$, and k_2 is the heat transfer coefficients between the air in the room and the ambiance in $\frac{W}{m^2 \, °C}$.

The outdoor temperature was modelled as

$$T_a(t) = 5\cos(at), \qquad (29)$$

where a is a day in seconds, or in state space

$$\begin{bmatrix} \dot{T}_a \\ \ddot{T}_a \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a^2 & 0 \end{bmatrix} \begin{bmatrix} T_a \\ \dot{T}_a \end{bmatrix}, \begin{bmatrix} T_a(0) \\ \dot{T}_a(0) \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}. \quad (30)$$

Finally the process was augmented with two more states modelling constant references, one for the temperature of the room, r_r, and one for the temperature of the floor, r_f,

$$\begin{bmatrix} \dot{r}_r \\ \dot{r}_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (31)$$

Combining the above models gives the following model of the dynamics for the process:

$$G_p : \begin{bmatrix} \dot{T}_r \\ \dot{T}_f \\ \dot{T}_a \\ \ddot{T}_a \\ \dot{r}_r \\ \dot{r}_f \end{bmatrix} = A_p \begin{bmatrix} T_r \\ T_f \\ T_a \\ \dot{T}_a \\ r_r \\ r_f \end{bmatrix}, \quad (32)$$

where

$$A_p = \begin{bmatrix} -\dfrac{k_1 A_f}{H_r} - \dfrac{k_2 A_w}{H_r} & \dfrac{k_1 A_f}{H_r} & \dfrac{k_2 A_w}{H_r} & 0 & 0 & 0 \\ \dfrac{k_1 A_f}{H_f} & -\dfrac{k_1 A_f}{H_f} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (33)$$

Modelling the discomfort that arises from the tracking errors as quadratic, and assigning a cost for them gave the following performance function for the house:

$$J_p = \int_0^\infty 0.1(T_r - r_r)^2 + 0.05(T_f - r_f)^2 dt. \quad (34)$$

The domain of validity for the process was given as

$$\mathcal{D}_p = \{ (T_r, T_f, T_a, \dot{T}_a, r_r, r_f) \in \mathcal{R}^6| \quad (35)$$
$$0 \le T_r \le 30 \wedge 0 \le T_f \le 30 \wedge$$
$$-25 \le T_a \le 50 \wedge -25 \le \dot{T}_a \le 50 \wedge$$
$$10 \le r_r \le 30 \wedge 10 \le r_f \le 30\}$$

The model of the process to be controlled is then given by the three-touple

$$p = (G_p, J_p, \mathcal{D}_p). \quad (36)$$

4.2 Actuator: Floor Heater

The floor heating actuator dynamics was modelled as

$$G_f : y_f = \begin{bmatrix} 0 \\ \dfrac{10^3}{H_f} \end{bmatrix} P_f, \quad (37)$$

where P_f is the power put into the floor by the floor heating in kW. The corresponding cost function is given by the prices for district heating, $\$_{dh}$, as

$$J_f = \int_0^\infty \$_{dh} P_f dt. \quad (38)$$

The range of validity for the floor heating actuator was given as

$$\mathcal{D}_f = \{P_f \in \mathcal{R}|0 \le P_f \le 2\}. \quad (39)$$

The model of the floor heating actuator was then given by

$$f = (G_f, J_f, \mathcal{D}_f). \quad (40)$$

4.3 Actuator: Electric Radiator

The dynamics of the electric radiator was modelled as

$$G_h : y_r = \begin{bmatrix} \dfrac{10^3}{H_r} \\ 0 \end{bmatrix} P_h, \quad (41)$$

where P_h is the power put into the air in the room by the electric heater in kW. The cost function for the electric radiator is given by the price for electric power,

$$J_h = \int_0^\infty \$_{ep} P_h dt, \quad (42)$$

and the domain of validity is given as

$$\mathcal{D}_h = \{P_h \in \mathcal{R}|0 \le P_h \le 2\}, \quad (43)$$

given the model of the electric heater as

$$h = (G_h, J_h, \mathcal{D}_h). \quad (44)$$

4.4 Plant Reconfiguration and Simulation

Using the sub-models from above the following scenario is constructed; at time $t = 0$ a house heating system with floor heating on the form

$$S = (p, \{f\}) \quad (45)$$

is commisioned, by synthesising a linear quadratic regulator using the model and a quadratic approximation of the cost function from S as described previously. References to the system were set at $r_r = 22°C$ and $r_f = 25°C$, and the disturbance, i.e. the ambient temperature, as shown in Figure 4.
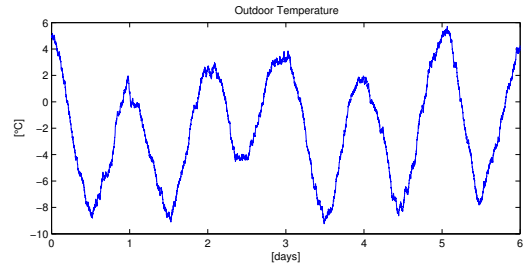

Fig. 4. Ambient temperature.

To improve the reference tracking abilities of the system a new actuator, an electric radiator, is added to the system at time $t = 3$, resulting in the augmented system

$$S' = (p, \{f, h\}). \quad (46)$$

As a consequence, a new linear quadratic controller that utilizes the new actuator is automatically synthesised, based on the new dynamic model and and a quadratic approximation of the cost function of S'. A simulation of this scenario is shown in Figure 5.

5. CONCLUSION AND FUTURE WORK

This paper presented a framework in which it was possible to compose a monolithic model of a system consisting of a set of actuators and a process to be controlled. Using this framework tuning information in the form of local
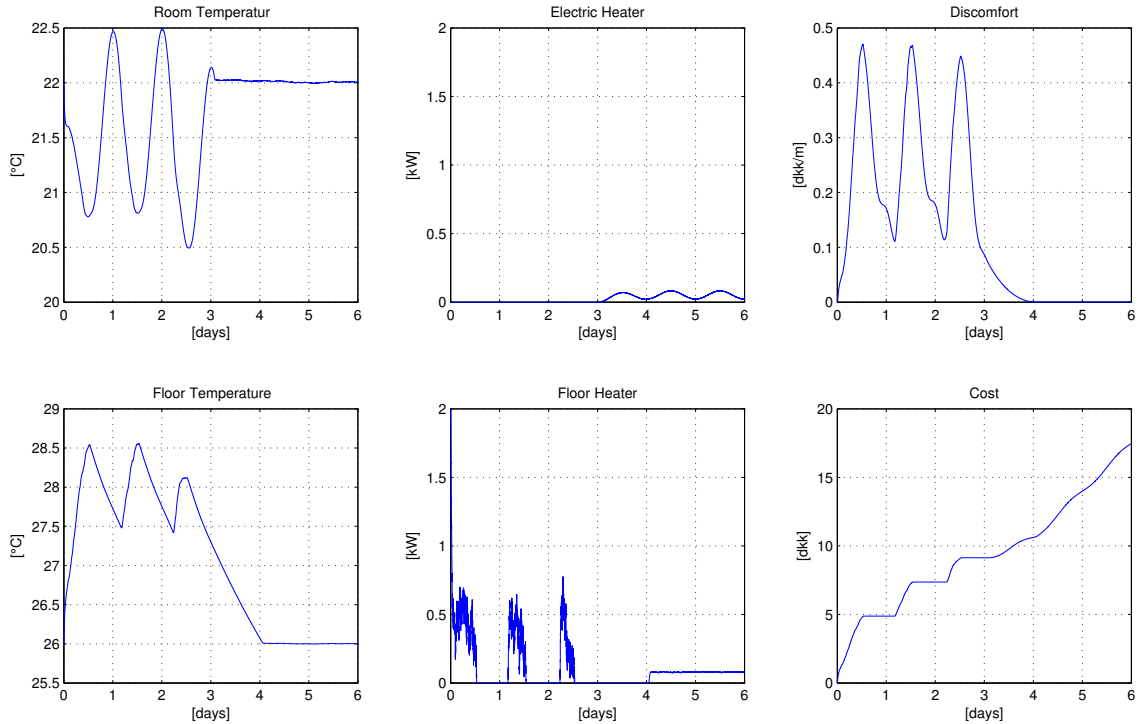
Fig. 5. Simulation results of S and S', a house heating system that at time $t = 3$ is augmented with an electric radiator to improve the reference tracking abilities.

cost functions for the individual elements of the system was propagated from the local sub-model to the global monolithic model and, based on the cost function for the monolithic model, a quadratic approximation usable for controller synthesis was found.

The developed framework was successfully tested on a case study provided by Danfoss A/S, where an actuator was plugged into an operating system and a new controller was synthesised on the fly.

The developed algorithm does not add components or change the components of the controller in an incremental manner. Instead, the whole controller is re-computed when a new actuator is added to the controlled process. A possible way to remedy this could be to constraint the structure of the controller such that it is e.g. distributed. Such a structure might lend itself to incremental changes of the controller when a new actuator or sensor is added to the controlled process.

The matrices in the approximation of the global cost function, Q and R, are not necessarily positive definite. In order for the developed framework to be generally applicable, future work should include either a specification of the family of cost functions for the actuators and the process, such that the approximation is positive definite, or add a further constrain on the minimization problem such that the matrices are positive definite by construction.

In future research two directions are considered. First is to expand the developed framework to encompass safety interlock, such that these can be propogated from the

sub-models to the composed model. Second is to let the sub-models be non-linear and investigate linearization strategies that exploit the structure the framework.

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