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Formulating and Solving Robust Fault Diagnosis Problems Based on a H_∞ Setting

Jie Chen

School of Engineering and Design, Brunel University, Uxbridge, UB8 3PH,UK
`email:Jie.Chen@brunel.ac.uk`

Abstract: This paper studies the robust fault detection problem using the standard H_∞ filtering formulation. With this formulation, the minimization of the disturbance effect on the residual is formulated as a standard H_∞ filtering problem and the design is solved using standard H_∞ techniques. To facilitate the enhancement of the residual sensitivity to the fault, the difference between the residual and the fault (or filtered fault) is minimized against the disturbance and the fault. The residual generated in this way is a faithful replicate of the fault and the reliable detection can be achieved. The paper also incorporates the modeling error into the robust residual design using the standard H_∞ filtering formulation.

Keywords: Fault diagnosis, fault detection, robust estimation, H_∞ optimization

1. INTRODUCTION

There are many ways, such as the unknown input observer, eigenstructure assignment, optimally robust parity relations, for eliminating or minimizing disturbance and modeling error effects on residual and hence for achieving robustness in fault detection and isolation (FDI) Chen and Patton [1998], Patton et al. [2000]. H_∞ optimization is a robust design method with the original motivation firmly rooted in the consideration of various uncertainties, especially the modeling errors. It is reasonable to seek an application of this technique in the robust design of FDI systems. This paper studies the H_∞ optimization method for robust residual generation of FDI.

The early work of using H_∞ optimization techniques for robust FDI was based on the use of factorization approach Frank [1994], Frank and Ding [1993, 1994, 1997], Qiu and Gertler [1993]. The factorization-based H_∞-optimization technique is useful in solving FDI problems. However, the more elegant and advanced H_∞-optimization methods are based on the use of the Algebraic Riccati Equation (ARE) Doyle et al. [1989], Shaked and Theodor [1992], Zhou et al. [1996]. Mangoubi et al. [1992] first solved the robust FDI estimation problem using the ARE approach via the use of H_∞ and μ robust estimator synthesis methods developed by Appleby et al. [1991]. A direct formulation of the FDI problem as a robust H_∞ filter design problem with the solution of an ARE was given in Edelmayer, Bokor, and Keviczky (1994, 1996, 1997a). Mangoubi et al. [1995] combined H_∞ robust FDI design with statistical methods for FDI. To deal with modeling errors as well as disturbances in robust FDI design, Niemann and Stoustrup [1996] introduced modeling error blocks into the standard H_∞ observer design. The weighting factors are then introduced in the problem formulation for finding an optimal FDI solution. This is further extended to non-linear systems where the nonlinearity is treated in the same way as a modeling error block Stoustrup and Niemann [1998].

The majority of studies discussed so far involve the use of a slightly modified H_∞ filter for the residual generation, i.e. the design objective is to minimize the effect of disturbances and modeling errors on the estimation error and subsequently on the residual. However, robust residual generation is different from the robust estimation because it does not only require the disturbance attenuation. The residual has to be remain sensitive to faults whilst the effect of disturbance is minimized. Sauter et al. [1997] studied this problem where the fault sensitivity is enhanced by applying an optimal post-filter to the "primary residual". The problem of enhancing fault sensitivity while increasing robustness against disturbances and modeling errors was studied extensively by Sadrnia et al. [1997a, 1997b, 1997c]. The essential idea is to reach an acceptable compromise between disturbance robustness and fault sensitivity. In the beginning, an observer with very small disturbance sensitivity bound is designed via an ARE. Then, the fault sensitivity is checked. If the fault sensitivity is too small, the disturbance robustness requirement should be relaxed, i.e. to design another optimal observer with an increased disturbance sensitivity bound. This procedure is likely to be repeated several times. The final goal is to find a design which provides the maximum ratio between fault sensitivity and disturbance sensitivity.

This paper starts with the formulation of the robust residual generation problem within the standard H_∞ filtering framework, i.e. to generate the residual whose sensitivity to disturbances is minimized. To facilitate reliable FDI, the residual sensitivity to faults has to be maintained (or maximized) in addition to the minimization of the disturbance sensitivity. This paper solves this problem via the minimization of the difference between the residual and the fault against the disturbance and the fault, i.e. the objective is to replicate the fault using the residual. In this way, the residual sensitivity to the fault is indirectly maximized. The residual sensitivity to the modeling error can be minimized if the modeling error is approximately represented by the disturbance vector with the estimated distribution matrix Patton and Chen [1992, 1993]. However, the modeling error can be handled directly using standard H_∞. This paper shows the way of including the modeling error in the robust residual design within the standard H_∞ framework.

2. RESIDUAL GENERATION

A system with faults and disturbances can be described by the state space model as:

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A & B & R_1 & E_1 \\ C & D & R_2 & E_2 \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \\ f(t) \\ d(t) \end{bmatrix} \quad (1)$$

where $x \in \mathbb{R}^n$: state vector, $y \in \mathbb{R}^m$: output vector, $u \in \mathbb{R}^r$: known input vector, $d \in \mathbb{R}^q$: unknown input (disturbance) vector, $f \in \mathbb{R}^g$ represents the fault vector which is considered as an unknown time function. A, B, C, D, E_1, E_2, R_1 and R_2 are known matrices. It is possible to use the above model to describe sensor faults, control actuator faults as well as component faults. The details of transforming all faulty situations into the above model can be find in Chen and Patton [1998]. Modeling errors can be treated as approximate disturbances Patton and Chen [1992, 1993]. The residual can be generated by using a full-order observer with K being the gain matrix and W being the residual weighting matrix:

$$\begin{bmatrix} \dot{\hat{x}} \\ r \end{bmatrix} = \begin{bmatrix} A-KC & K & B-KD \\ -WC & W & -WD \end{bmatrix} \begin{bmatrix} \hat{x} \\ y \\ u \end{bmatrix} \quad (2)$$

When the residual generator (2) is applied to the system (1), the resulting residual is given by:

$$\begin{bmatrix} \dot{e} \\ r \end{bmatrix} = \begin{bmatrix} A-KC & R_1-KR_2 & E_1 \\ WC & WR_2 & WE_2 \end{bmatrix} \begin{bmatrix} e \\ f \\ d \end{bmatrix} \quad (3)$$

It can be seen that the input $u(t)$ has no effect on the residual. Therefore, the input can be ignored in the residual generator design. To achieve perfect FDI, a residual signal should only be affected by faults, i.e.

$$\begin{cases} r(t) = 0 & \text{if } f(t) = 0 \\ r(t) \neq 0 & \text{if } f(t) \neq 0 \end{cases} \quad (4)$$

Note that this definition is defined after the transients of the residual generator have been settled down. When there are no disturbances and modeling errors, the perfect fault detection can easily be achieved by any residual generation method satisfying the detectability condition. For systems with disturbances and modeling errors, perfect fault detection cannot be always achieved. In this situation, the optimal (or approximately perfect) fault detection is tackled by robust residual generation methods.

3. ROBUST RESIDUAL GENERATION WITH DISTURBANCE ATTENUATION

Let us consider the disturbance attenuation problem, i.e. to design a residual generator where the effects of disturbances on the residual is minimized. To tackle this problem, the following system model is used:

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & E_1 \\ C & E_2 \end{bmatrix}}_{G_d(s)} \begin{bmatrix} x(t) \\ d(t) \end{bmatrix} \quad (5)$$

$$\text{or} \qquad y(s) = G_d(s)d(s) \quad (6)$$

To generate a residual, we need to estimate an auxiliary signal $z(t)$. The residual signal is the difference between the real and estimated values of this auxiliary signal. Since we need to compare the estimation with the real value of this auxiliary signal, the real value of $z(t)$ should be available. Therefore, we can define the weighted output as an auxiliary signal which is to be estimated by H_∞ filter.

$$z(t) = My(t) \quad (7)$$

The residual signal is thus:

$$r(t) = z(t) - \hat{z}(t) \quad (8)$$

The design requirement for robust residual generation is to minimize the following performance index.

$$J_d := \|G_{rd}(s)\|_\infty = \sup_{0<\|d\|_2<\infty} \frac{\|r\|_2}{\|d\|_2} \quad (9)$$

where $G_{rd}(s)$ is the transfer matrix from disturbance to residual. The idea of estimating the auxiliary $z(t)$ is illustrated by Fig. 1. The following evaluation signal is used to measure the estimation quality:

$$\tilde{z}(t) = z(t) - \hat{z}(t) \quad (10)$$

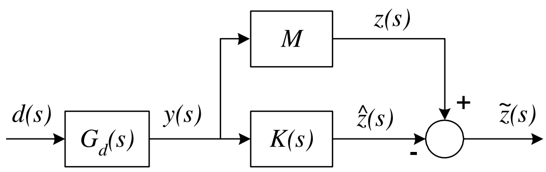

Fig. 1. Formulation of disturbance attenuation

The system illustrated by Fig. 1 can be reformulated into a standard H_∞ problem as given in Fig. 2.

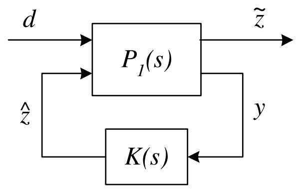

Fig. 2. H_∞ disturbance attenuation

The "equivalent" transfer matrix $P_1(s)$ for the standard problem of Fig. 2 is given by:

$$\begin{bmatrix} \tilde{z} \\ y \end{bmatrix} = \underbrace{\left[\begin{array}{c|cc} A & E_1 & 0 \\ \hline MC & ME_2 & -I \\ C & E_2 & 0 \end{array}\right]}_{P_1(s)} \begin{bmatrix} d \\ \hat{z} \end{bmatrix} \quad (11)$$

The sensitivity transfer matrix for this standard H_∞ formulation is given by:

$$G_{\tilde{z}d}(s) = LFT(P_1(s), K(s)) = MG_d(s) - K(s)G_d(s) \quad (12)$$

where LFT denotes the linear fractional transformation. The standard H_∞ filtering problem is to find a filter $K(s) \in RH_\infty$ such that:

$$\|G_{\tilde{z}d}(s)\|_\infty < \gamma \quad (13)$$

where γ (>0) is a design parameter named as the performance bound. The filtering problem can be regarded as a special H_∞ problem. Compared with the control problems there is no internal stability requirement in the filtering problem. Therefore, standard H_∞ control techniques can be used to solved the problem (13). A simplified version for the solution of H_∞ robust residual generation, developed by Edelmayer et al. [1994,1996,1997b], is introduced here.

Theorem 1. (Edelmayer et al., 1994, 1996,1997b). : When $E_2 = 0$ (i.e. no disturbance in the output equation), (A, E_1) is a stabilizable pair and (C, A) is a detectable pair, then the optimal filter $K(s)$ which satisfies (13) is given by:

$$K(s) = \left[\begin{array}{c|c} A - YC^TC & YC^T \\ MC & 0 \end{array}\right] \tag{14}$$

where Y is the positive definite solution of the ARE:

$$\begin{aligned} &AY + YA^T + E_1E_1^T \\ &- Y\left(C^TC - \frac{1}{\gamma^2}(MC)^T(MC)\right)Y = 0 \end{aligned} \tag{15}$$

The design parameter γ determines the disturbance attenuation performance. The smaller this design parameter, the better the performance of disturbance attenuation. However, if this parameter is too small, the solution of (15) may not exist. Therefore, the design procedure starts with sufficiently large γ_0. Then, the performance bound is gradually reduced until a solution for (15) can not be found. The minimum design parameter γ_{min} is achievable performance bound. This procedure is known as γ*–iteration*.

The optimal filter (14) can be implemented using the state space equations in which the control input can also be included.

$$\begin{bmatrix} \dot{\hat{x}} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} A - K_0C & K_0 & B - K_0D \\ MC & 0 & MD \end{bmatrix} \begin{bmatrix} \hat{x} \\ y \\ u \end{bmatrix} \tag{16}$$

$$r(t) = My(t) - \hat{z}(t) \tag{17}$$

where the observer gain matrix is $K_o = YC^T$. The design problem discussed here gives a solution of the observer gain matrix K_o for achieving optimal disturbance de-coupling. However, it does not give any indication as to how the residual weighting matrix M can be determined.

The problem of maximizing fault effects with the constraint of minimizing disturbance effects has been investigated by other researchers as an H_-/H_∞ filtering problem. For excample, Li *et al* solved this problem firstly using matrix factorization approach [Jaimoukha, Li, and Papakos, 2006] and later solved using the linear matrix inequality approach [Li, 2007, Mazars, Jaimoukha, and Li, 2008].

4. FAULT ESTIMATION WITH DISTURBANCE ATTENUATION

To detect faults reliably, the residual should be designed to have maximum sensitivity against faults. Reliable detection can be ensured if we can solve the optimization problem:

$$\max\left\{\frac{\|r\|_2}{\|f\|_2}\right\} \quad s.t. \;\; \min\left\{\frac{\|r\|_2}{\|d\|_2}\right\} \tag{18}$$

The maximization problem (18) cannot be easily formulated in an H_∞ setting. To solve this problem, a new performance index should be introduced. If we can make the residual as close to the fault as possible, then the residual can provide all information about the fault. That is to say the fault sensitivity has been maximized indirectly. The optimization problem is thus:

$$\min\left\{\frac{\|r - f\|_2}{\|f\|_2}\right\} \quad s.t. \;\; \min\left\{\frac{\|r\|_2}{\|d\|_2}\right\} \tag{19}$$

The solution of the problem in Eq. (19) actually solved the fault estimation problem. Sometimes, it is *not practical* to estimate the fault itself. We can then consider the estimation of a filtered version of the fault which can give us some indication about the fault itself. The problem is thus changed into the estimation of $\bar{f}(t)$ which is the filtered version of the fault, i.e

$$\bar{f}(s) = T(s)f(s) \tag{20}$$

where $T(s)$ is a RH_∞ transfer matrix and can be set as diagonal. The filtered fault estimation can then be defined as the following optimization problem.

$$\min\left\{\frac{\|r - \bar{f}\|_2}{\|f\|_2}\right\} \quad s.t. \;\; \min\left\{\frac{\|r\|_2}{\|d\|_2}\right\} \tag{21}$$

To formulate the fault estimation problem, let us consider a system with both fault and disturbance terms:

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A & R_1 & E_1 \\ C & R_2 & E_2 \end{bmatrix} \begin{bmatrix} x(t) \\ f(t) \\ d(t) \end{bmatrix} \tag{22}$$

or alternatively by the input-output model:

$$y(s) = G_f(s)f(s) + G_d(s)d(s) \tag{23}$$

where

$$G_f(s) = \left[\begin{array}{c|c} A & R_1 \\ \hline C & R_2 \end{array}\right] \;\; ; \;\; G_d(s) = \left[\begin{array}{c|c} A & E_1 \\ \hline C & E_2 \end{array}\right] \tag{24}$$

The control input does not affect the residual if there are no modelling errors in the transfer matrix between control input and the system output. Therefore, the control input can be ignored here. Our task here is to find an optimal estimation of the filtered fault when the system has both the fault and disturbance. This can be formulated according to the scheme given in Fig. 3.

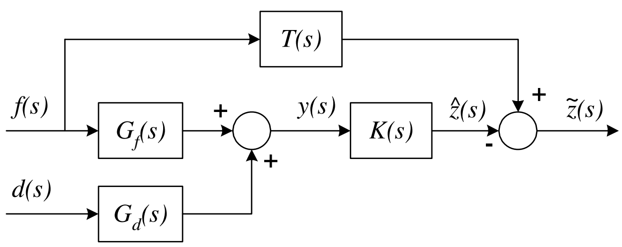

Fig. 3. Formulation of filtered fault estimation with disturbance attenuation

The objective of the problem is to minimize the following performance index

$$J_f := \|G_{rd_1}(s) - T(s)\|_\infty = \sup_{0<\|d_1\|_2<\infty} \frac{\|r - \bar{f}\|_2}{\|d_1\|_2} \tag{25}$$

where d_1 is the generalized disturbance vector which is defined as:

$$d_1 = \begin{bmatrix} f \\ d \end{bmatrix} \tag{26}$$

The problem of estimating the fault with the disturbance attenuation property can be reformulated in a standard H_∞ setting as illustrated in Fig. 4.

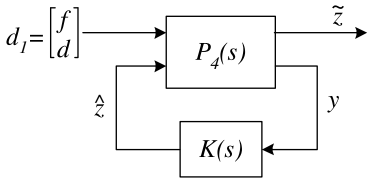

Fig. 4. Standard H_∞ formulation of filtered fault estimation with disturbance attenuation

The "equivalent" transfer matrix $P_4(s)$ for the standard problem of Fig. 4 is given by:

$$\begin{bmatrix} \tilde{z} \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} [T(s)\ \ 0] & -I \\ [G_f(s)\ \ G_d(s)] & 0 \end{bmatrix}}_{P_4(s)} \begin{bmatrix} d_1 \\ \hat{z} \end{bmatrix} \tag{27}$$

If the state space realization of transfer matrix $T(s)$ is

$$T(s) = \left[\begin{array}{c|c} A_T & B_T \\ \hline C_T & D_T \end{array}\right] \tag{28}$$

the "equivalent" transfer matrix $P_4(s)$ for the standard problem of Fig. 4 is given by:

$$P_4(s) = \left[\begin{array}{cc|ccc} A_T & 0 & B_T & 0 & 0 \\ 0 & A & R_1 & E_1 & 0 \\ \hline C_T & 0 & D_T & 0 & -I \\ 0 & C & R_2 & E_2 & 0 \end{array}\right] \tag{29}$$

The sensitivity transfer matrix for this standard H_∞ formulation is given by:

$$\begin{aligned} G_{\tilde{z}f}(s) &= LFT(P_4(s), K(s)) \\ &= [T(s)\ \ 0] - K(s)[G_f(s)\quad G_d(s)] \end{aligned} \tag{30}$$

To estimate the filtered fault with the disturbance attenuation property within the standard H_∞ formulation, an optimal filter $K(s) \in RH_\infty$ should be found to satisfy the following condition:

$$\|G_{\tilde{z}f}(s)\|_\infty < \gamma \tag{31}$$

The signal $\hat{z}(t)$ in Figs.3 & 4 can be used as a residual signal as well as an estimate of the filtered fault.

5. ROBUSTNESS ISSUES

In Section 4, the estimation of the filtered fault with the disturbance attenuation property is studied. This filtered fault estimate can be used as a residual. Since the disturbance affect on this residual is minimized and the residual has been made close to the filtered fault, the robust FDI in terms of the disturbance effect minimization and fault effect maximization is achieved. However, the robustness is only achieved on the assumption that there are no modeling errors or the modeling errors have been approximately transformed into disturbances using techniques developed in Patton and Chen [1992, 1993]. In reality, the modeling errors always exist and cannot totally be "transformed" into disturbances. This problem can be tackled using techniques exist in H_∞ control since we have formulated the FDI problem as a special H_∞ problem. To start with the investigation, let us ignore the control input and the system model is:

$$y(s) = (I + \Delta_d(s))G_d(s)d(s) + (I + \Delta_f(s))G_f(s)f(s) \tag{32}$$

To solve the problem of estimating faults for systems with disturbances and modeling errors, the standard H_∞ problem in Fig. 4 should be reformulated to incorporate the uncertainty block as shown in Fig. 5.

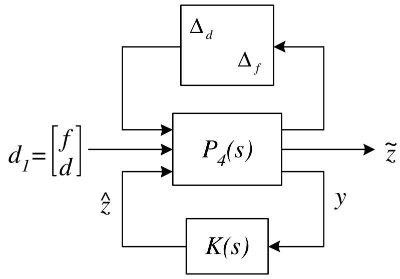

Fig. 5. Standard H_∞ formulation of robust fault estimation

The control input can only be ignored when there are no modeling errors in the transfer matrix between control input and the system output, i.e. $G_u(s)$. However, this is not always the case. A complete description of a system with all kinds of uncertainties is:

$$\begin{aligned} y(s) = &(I + \Delta_u(s))G_u(s)u(s) + (I + \Delta_f(s))G_f(s)f(s) \\ &+ (I + \Delta_d(s))G_d(s)d(s) \end{aligned} \tag{33}$$

It is not easy to incorporate the uncertainty $\Delta_u(s)$ in the standard problem formulation in Fig. 5. The only way is to transform the modeling error $\Delta_u(s)$ into an equivalent disturbance. This solution is feasible but it does not fully utilize the potential in H_∞ design. One way to solve this problem is the integrated design, i.e. to design controller and residual generator (fault estimator) simultaneously. The integrated design problem can be formulated according to Fig. 6 where y_e is the signal used to evaluate the control performance, $K_o(s)$ is the fault estimator and $K_c(s)$ is the controller.

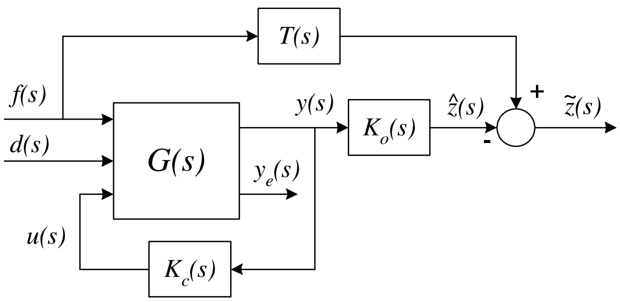

Fig. 6. Formulation of integrated design

The integrated design problem Fig. 6 can be transformed into a standard H_∞ problem as given in Fig. 7.

From Fig.7, it can be seen that there are two sub-blocks in the "controller" block. A similar scheme was investigated by Stoustrup et al. [1997] and they named it as the two-parameters integrated control structure. Stoustrup et al. [1997] also pointed out the four-parameters integrated control structure studied in

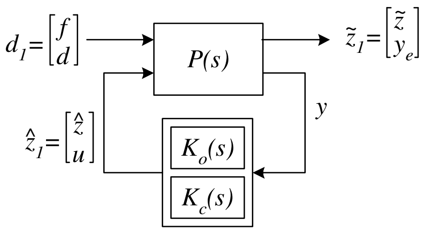

Fig. 7. Standard H_∞ formulation of integrated design

Jacobson and Nett [1991], is just a special case of the two-parameters structure discussed here. The solutions for this integrated control structure for both nominal and robust cases are developed by Stoustrup *et al* Stoustrup et al. [1997], Stoustrup and Grimble [1997]. With this standard H_∞ setting, all modeling error blocks can be considered and the idea is depicted in Fig. 8.

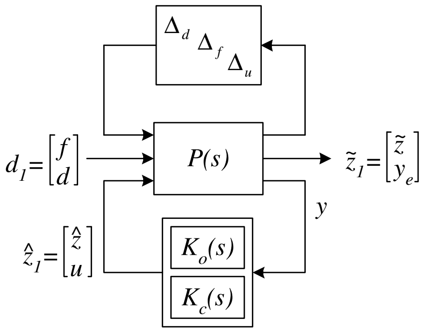

Fig. 8. Standard H_∞ formulation of integrated design with robustness consideration

6. A DESIGN EXAMPLE

Consider the following system with parameter uncertainties

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} d(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} f(t) + \begin{bmatrix} 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} d(t)$$

A sensor fault is modeled by $f(t)$ in the measurement $y(t)$. The parameters α_1, α_2 and α_3 vary in the range $[0.2, 1.8]$, $[0.4, 3.6]$ and $[0.6, 5.4]$. The estimator is designed for a nominal model $\alpha_1 = 1$, $\alpha_2 = 2$ and $\alpha_3 = 3$. The sensor fault effect on residuals designed via an H_∞ robust fault detection observer can be considered by testing $\|G_{rf}\|_\infty / \|G_{rd}\|_\infty$ and $\|G_{rf}\|_- / \|G_{rd}\|_\infty$. To compare the robust fault detection observer designed with a non-robust design, an observer is designed using pole-placement. For the Riccati equation solution, $M = I$, $\gamma_{opt} = 2$ and the following gain matrix give the best performance testing ratios. Fig.9 shows a comparison of the performance testing ratios $k_1 = \|G_{rf}\|_\infty / \|G_{rd}\|_\infty$ and $k_2 = \|G_{rf}\|_- / \|G_{rd}\|_\infty$.

$$K_o = \begin{bmatrix} 1.8799 & 0.3331 & -0.7908 \\ 0.3331 & 1.3148 & -0.0403 \end{bmatrix}$$

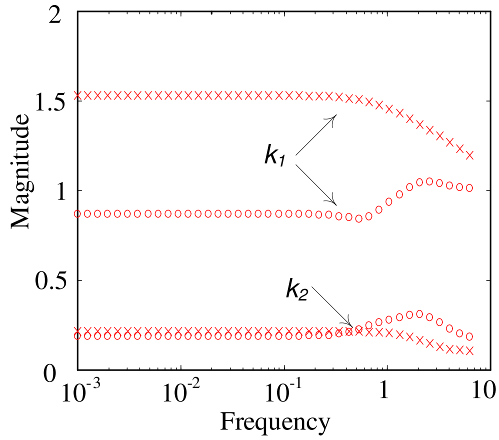

Fig. 9. Comparison of H_∞ robust estimator with pole-placement design ("x" denotes the H_∞ design and "o" denotes pole-placement design)

7. SUMMARY

This paper has shown that a robust fault detection problem can be formulated into a standard H_∞ filtering with the aid of the linear fraction transformation, then the ARE-based solution can be found. Both robust fault detection and fault estimation problems have been formulated in this paper. One advantage of this approach is the simplicity because its close association with H_∞ filter problems. The most important advantage is that it provides a framework to deal with modeling errors. The modeling uncertainties can easily be incorporated into the standard H_∞ formulation and then robust solutions can be found using the techniques in robust control such as μ synthesis. This paper has formulated robust fault detection and estimation problems on H_∞ setting. It is expected the standard H_∞ control techniques can be used to solve the problem formulated in this paper. However, there is still more research work to be done before the detailed design procedure is delivered. The great potential of this approach, especially in the integrated design, is waiting to be further exploited.

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