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Resource-Constrained Sensor Routing for Parameter Estimation of Distributed Systems

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Abstract: We consider a setting where mobile nodes with sensing capacity form a network whose mission is to collect measurements for parameter estimation of a distributed parameter system (DPS). Two techniques to optimize node motions are presented which constitute a trade-off between the achievable accuracy of parameter estimates and limited motion resources of sensor network nodes. The framework is based on the use of the D-optimality criterion defined on the Fisher information matrix associated with the estimated parameters as a measure of the information content in the measurements. Restrictions on maximal distances traveled by sensor nodes are imposed so as to guarantee realizable solutions. The approach is to convert the problem to a canonical optimal control one in Mayer form, in which both the control forces of the sensors and the initial sensor positions are optimized. Numerical solutions are then obtained using the MATLAB PDE toolbox and the RIOTS_95 optimal control toolbox which handles various constraints imposed on the node motions. Illustrative numerical experiments with the proposed techniques are presented.

1. INTRODUCTION

The importance of measurement system design for estimation of unknown coefficients in distributed parameter systems (DPSs), i.e., systems described by partial differential equations (PDEs), has been recognized for a long time, but relatively few attempts have been made at solving this problem, cf. surveys in [Kubrusly and Malebranche, 1985, Uciński, 2005, 1999, Patan, 2004, van de Wal and de Jager, 2001]. It has found special interest in many areas, e.g., air quality monitoring systems, groundwater-resources management, recovery of valuable minerals and hydrocarbon, model calibration in meteorology and oceanography, chemical engineering, hazardous environments and smart materials, cf. [Nehorai et al., 1995, Porat and Nehorai, 1996, Jeremić and Nehorai, 1998, 2000, Navon, 1997, Daescu and Navon, 2004, Christofides, 2001, Banks et al., 1996, Sun, 1994, Uciński, 2005]. The operation and control of such systems usually requires precise information on the parameters which dictate the accuracy of the underlying mathematical model, but that information is only available through a limited number of possibly expensive sensors. Although the details of the approaches differ, in essence the underlying idea is to select those locations that lead to the best estimates of the process parameters. The optimality of the locations is judged by an appropriate measure of the estimate-error covariance matrix.

The sensor location problem was attacked from various angles, but the results communicated by most authors are limited to the selection of stationary sensor positions. A generalization which imposes itself is to apply sensors which are capable of tracking points providing at a given time moment the best information about the parameters. A possibility of using moving observations does arise in a variety of applications, e.g., air pollutants in the environment are often measured using data gathered by monitoring cars moving in an urban area and atmospheric variables are measured using instruments carried in a satellite. Other examples include scanning measurement of a surface temperature by optical pyrometers and measurement of vibrations and strains in materials using optical registration. However, communications in this field are rather limited. Rafajłowicz [1986] considers the determinant of the Fisher Information Matrix (FIM) associated with the parameters to be estimated as a measure of the identification accuracy and looks for an optimal time-dependent measure, rather than for the trajectories themselves. On the other hand, Uciński [2005, 1999, 2000], apart from generalizing Rafajłowicz's results, developed some computational algorithms based on the FIM framework. He reduces the problem to a state-constrained optimal-control one for which solutions are obtained via the methods of successive linearization which is capable of handling various constraints imposed on sensor motions.

It is worth pointing out that the optimal design of moving sensor trajectories is increasingly attracting attention in the context of sensor networks which play a role of importance in the research community, cf. [Zhao and Guibas, 2004, Cassandras and Li, 2005, Sinopoli et al., 2003, Chong and Kumar, 2003, Culler et al., 2004, Jain and Agrawal, 2005]. Technological advances in communication systems and the growing ease in making small, low power and inexpensive mobile systems now make it feasible to deploy a group of networked vehicles in a number of environments, cf. [Ögren et al., 2004]. A cooperated and scalable network of vehicles, each of them equipped with a single sensor, has the potential to substantially improve the performance of the observation systems. Applications in various fields of research are being developed and interesting ongoing projects include extensive experimentation based on test-beds. Our work on one of such experimental platforms, namely the MAS-net (mobile actuator and sensor networks) testbed being a distributed system equipped with two-wheeled differentially driven mobile robots capable of sensing the states of DPSs described by diffusion equation, cf. [Moore et al., 2004, Chen et al., 2004], revealed numerous deficiencies of the existing techniques of sensor location and commanded attention to aspects which, on one hand, are of paramount practical importance and, on the other hand, have been neglected in the literature so far. These, in turn, lead to non-trivial theoretical problems which still call for solutions. As a result, Uciński and Chen [2005] attempted to properly formulate and solve the time-optimal problem for moving sensors which observe the state of a DPS so as to estimate some of its parameters. In the same vein, Uciński and Chen [2006] used Turing's measure of conditioning to make the Hessian of the parameter estimation cost well conditioned.

The purpose of the investigations undertaken here was to establish a practical approach to properly formulate and solve one of such problems, namely the problem of taking account of limited motion capabilities of mobile nodes while guiding them so as to observe the state of a DPS and then estimate its unknown parameters. Motivations come from technical limitations imposed on the vehicles conveying the measurement equipment. These are inherent to mobile platforms carrying sensors, which are supplied with power from batteries. Unfortunately, the researchers' attention is predominantly focused only on the achieved precision while neglecting the problem of realizability of the produced solutions. Our main objective has thus been to produce results which can be useful when the distances traveled by the nodes are a crucial factor. Two formulations are thus proposed: the first for the specified limit on the longest path, and the other for the case in which the longest path length constitutes a decision variable, which leads to a minimax formulation. It is also shown that these formulations can be transformed into equivalent optimal control problems in canonical Mayer form, which can be efficiently solved by the MATLAB toolbox RIOTS_95, a high-performance general-purpose toolbox for solving optimal control problems, cf. [Schwartz et al., 1997].

2. OPTIMAL SENSOR LOCATION PROBLEM

Let $\Omega \subset \mathbb{R}^n$ ($n = 1, 2$ or 3) be a bounded spatial domain with sufficiently smooth boundary Γ, and $T = (0, t_f]$ a bounded time interval. Consider a distributed parameter system (DPS) whose scalar state at a spatial point $\boldsymbol{x} \in \Omega \subset \mathbb{R}^n$ and time instant $t \in T$ is denoted by $y(\boldsymbol{x}, t)$. Mathematically, the system state is governed by the partial differential equation (PDE)

$$\frac{\partial y}{\partial t} = \mathcal{F}\Big(\boldsymbol{x}, t, y, \boldsymbol{\theta}\Big) \quad \text{in } \Omega \times T, \tag{1}$$

where $\mathcal{F}$ is a well-posed, possibly nonlinear, differential operator which involves first- and second-order spatial derivatives and may include terms accounting for forcing inputs specified *a priori*. The PDE (1) is accompanied by the appropriate boundary and initial conditions

$$\mathcal{B}(\boldsymbol{x}, t, y, \boldsymbol{\theta}) = 0 \qquad \text{on } \Gamma \times T, \tag{2}$$

$$y = y_0 \qquad \text{in } \Omega \times \{t = 0\}, \tag{3}$$

respectively, $\mathcal{B}$ being an operator acting on the boundary Γ and $y_0 = y_0(\boldsymbol{x})$ a given function. Conditions (2) and (3) complement (1) such that the existence of a sufficiently smooth and unique solution is guaranteed. We assume that the forms of $\mathcal{F}$ and $\mathcal{B}$ are given explicitly up to an m-dimensional vector of unknown constant parameters $\boldsymbol{\theta}$ which must be estimated using observations of the system. The implicit dependence of the state y on the unknown parameter vector $\boldsymbol{\theta}$ will be reflected by the notation $y(\boldsymbol{x}, t; \boldsymbol{\theta})$.

We assume that the vector $\boldsymbol{\theta} \in \mathbb{R}^m$ is to be estimated from measurements made by N moving sensors over the observation horizon T. We call $\boldsymbol{x}^j : T \to \Omega_{\text{ad}}$ the trajectory of the j-th sensor, where $\Omega_{\text{ad}} \subset \Omega \cup \Gamma$ is a compact set representing the area where measurements can be made. The observations are of the form

$$z^j(t) = y(\boldsymbol{x}^j(t), t) + \varepsilon(\boldsymbol{x}^j(t), t), \quad t \in T, \quad j = 1, \ldots, N, \tag{4}$$

where ε constitutes the measurement noise which is assumed to be is zero-mean, Gaussian, spatial uncorrelated and white [Quereshi et al., 1980, Omatu and Seinfeld, 1989, Amouroux and Babary, 1988], i.e.,

$$\mathrm{E}\Big\{\varepsilon(\boldsymbol{x}^j(t), t)\varepsilon(\boldsymbol{x}^i(\tau), \tau)\Big\} = \sigma^2 \delta_{ji} \delta(t - \tau), \tag{5}$$

where σ^2 defines the intensity of the noise; δ_{ij} and $\delta(\,\cdot\,)$ stand for the Kronecker and Dirac delta functions, respectively. Although white noise is a physically impossible process, it constitutes a reasonable approximation to any disturbance whose adjacent samples are uncorrelated at all time instants for which the time increment exceeds some value which is small compared with the time constants of the DPS. The white-noise assumption is consistent with most of the literature on the subject.

In the presented framework, the parameter identification problem is usually formulated as follows: Given the model (1)–(3) and the outcomes of the measurements z^j along the trajectories $\boldsymbol{x}^j$, $j = 1, \ldots, N$, determine an estimate $\widehat{\boldsymbol{\theta}} \in \Theta_{\text{ad}}$ (Θ_{ad} being the set of admissible parameters) which minimizes the output least-squares fit-to-data functional given by (for details, see [Banks and Kunisch, 1989, Omatu and Seinfeld, 1989])

$$\widehat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\vartheta} \in \Theta_{\text{ad}}} \sum_{j=1}^{N} \int_T \Big[z^j(t) - y(\boldsymbol{x}^j(t), t; \boldsymbol{\vartheta})\Big]^2 \,\mathrm{d}t \tag{6}$$

where y now solves (1)–(3) when $\boldsymbol{\theta}$ replaced by $\boldsymbol{\vartheta}$.

We feel, intuitively, that the parameter estimate $\widehat{\boldsymbol{\theta}}$ depends on the trajectories $\boldsymbol{x}^j$ since the right-hand side of eqn. (6) does so. This fact suggests that we may attempt to select these design variables so as to produce best estimates of the system parameters after performing the actual experiment. To form a basis for the comparison of different trajectories, a quantitative measure of the 'goodness' of particular trajectories is required. A logical approach is to choose a measure related to the expected accuracy of the parameter estimates to be obtained from the data collected (note that the design is to be performed off-line, before taking any measurements). Such a measure is usually based on the concept of the *Fisher Information Matrix* (FIM), cf. [Sun, 1994, Rafajłowicz, 1986], which is widely used in optimum experiment design (OED) theory for lumped systems, see [Walter and Pronzato, 1997, Fedorov and Hackl, 1997, Atkinson and Donev, 1992]. When the time horizon is large, the nonlinearity of the model with respect to its parameters is mild and the measurement errors are independently distributed and have small magnitudes, the inverse of the FIM constitutes a good approximation of the covariance matrix for the estimate of $\boldsymbol{\theta}$, cf. [Walter and Pronzato, 1997, Fedorov and Hackl, 1997, Atkinson and Donev, 1992].

For simplicity of notations, let us write

$$\boldsymbol{q}(t) = \begin{bmatrix} \boldsymbol{x}^1(t) \\ \vdots \\ \boldsymbol{x}^N(t) \end{bmatrix}, \quad \forall\, t \in T. \tag{7}$$

The FIM has the following representation, cf. [Uciński, 2005, Quereshi et al., 1980]:

$$\boldsymbol{M}(\boldsymbol{q}) = \sum_{j=1}^{N} \int_T \boldsymbol{p}(\boldsymbol{x}^j(t), t) \boldsymbol{p}^{\mathsf{T}}(\boldsymbol{x}^j(t), t) \,\mathrm{d}t, \tag{8}$$

where

$$\boldsymbol{p}(\boldsymbol{x}, t) = \nabla_{\boldsymbol{\vartheta}} y(\boldsymbol{x}, t; \boldsymbol{\vartheta})\Big|_{\boldsymbol{\vartheta} = \boldsymbol{\theta}^0} \tag{9}$$

denotes the vector of the so-called *sensitivity coefficients*, $\boldsymbol{\theta}^0$ being a prior estimate to the unknown parameter vector θ [Uciński, 2000, 2005].

The sought optimal design settings can be found by maximizing some scalar function Ψ of the information matrix. The introduction of the design criterion permits to cast the sensor location problem as an optimization problem, and the criterion itself can be treated as a measure of the information content of the observations. Several choices exist for such a function, cf. [Walter and Pronzato, 1997, Fedorov and Hackl, 1997, Atkinson and Donev, 1992], and the most popular one is the D-optimality criterion

$$\Psi[\boldsymbol{M}] = \log \det(\boldsymbol{M}). \tag{10}$$

Its use yields the minimal volume of the confidence ellipsoid for the estimates. In what follows, we shall restrict our attention to this D-optimality criterion.

3. MOBILE NODE DYNAMICS AND CONSTRAINTS

3.1 Node Dynamics

With no loss of generality, we assume that all sensors are carried by identical vehicles whose motions are described by

$$\dot{\boldsymbol{x}}^j(t) = \boldsymbol{f}(\boldsymbol{x}^j(t), \boldsymbol{u}^j(t)) \quad \text{a.e. on } T, \quad \boldsymbol{x}^j(0) = \boldsymbol{x}_0^j \tag{11}$$

where the given function $\boldsymbol{f} : \mathbb{R}^n \times \mathbb{R}^\ell \to \mathbb{R}^n$ is required to be continuously differentiable; $\boldsymbol{x}_0^j \in \mathbb{R}^n$ defines an initial sensor configuration, and $\boldsymbol{u}^j : T \to \mathbb{R}^\ell$ is the control signal to be decided which must satisfy

$$\boldsymbol{u}_l \le \boldsymbol{u}^j(t) \le \boldsymbol{u}_u \quad \text{a.e. on } T \tag{12}$$

for given constant vectors $\boldsymbol{u}_l$ and $\boldsymbol{u}_u$, $j = 1, \ldots, N$.

For each $j = 1, \ldots, N$, given any initial position $\boldsymbol{x}_0^j$ and any control signal vector $\boldsymbol{u}^j$, there is a unique absolutely continuous function $\boldsymbol{x}^j : T \to \mathbb{R}^n$ which satisfies (11) a.e. on T. In what follows, we will call it the sensor trajectory corresponding to $\boldsymbol{x}_0^j$ and $\boldsymbol{u}^j$.

For notational simplicity, in lieu of (11) we shall subsequently use one vector system of ODEs

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{d}(\boldsymbol{q}(t), \boldsymbol{u}(t)) \quad \text{a.e. on } T, \quad \boldsymbol{q}(0) = \boldsymbol{q}_0, \tag{13}$$

where

$$\boldsymbol{u}(t) = \begin{bmatrix} \boldsymbol{u}^1(t) \\ \vdots \\ \boldsymbol{u}^N(t) \end{bmatrix}, \quad \boldsymbol{q}_0 = \begin{bmatrix} \boldsymbol{x}_0^1 \\ \vdots \\ \boldsymbol{x}_0^N \end{bmatrix}, \tag{14}$$

$$\boldsymbol{d}(\boldsymbol{q}(t), \boldsymbol{u}(t)) = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}^1(t), \boldsymbol{u}^1(t)) \\ \vdots \\ \boldsymbol{f}(\boldsymbol{x}^N(t), \boldsymbol{u}^N(t)) \end{bmatrix}. \tag{15}$$

3.2 Induced State Constraints

In reality, some restrictions on the motions are inevitably induced. First of all, all sensors should stay within the admissible region Ω_{ad} where measurements are allowed. We assume that it is a compact set defined as follows:

$$\Omega_{\mathrm{ad}} = \{\boldsymbol{x} \in \Omega \cup \Gamma \mid b_i(\boldsymbol{x}) \leq 0,\ i = 1, \dots, I\} \tag{16}$$

where b_i's are given continuously differentiable functions. Accordingly, the conditions

$$b_i(\boldsymbol{x}^j(t)) \leq 0, \quad \forall\, t \in T \tag{17}$$

must be fulfilled, where $1 \leq i \leq I$ and $1 \leq j \leq N$. To further simplify the notation, after relabeling, we rewrite constraints (17) in the form

$$\gamma_l(\boldsymbol{q}(t)) \leq 0, \quad \forall\, t \in T, \tag{18}$$

where γ_l, $l = 1, \dots, \nu$ tally with (17), $\nu = IN$.

4. PROPOSED FORMULATIONS TAKING ACCOUNT OF LIMITED PATH LENGTHS

As mentioned in Sec. 1, motion capabilities of the nodes may be severely limited. In the sequel, we shall be primarily concerned with restrictions imposed on their path lengths. The distance traveled by the j-th sensor over the interval $[0, t]$ is

$$s_j(t) = \int_0^t \|\dot{\boldsymbol{x}}^j(\tau)\| \,\mathrm{d}\tau = \int_0^t \|\boldsymbol{f}(\boldsymbol{x}^j(\tau), \boldsymbol{u}^j(\tau))\| \,\mathrm{d}\tau, \tag{19}$$

where $\|\cdot\|$ signifies the Euclidean norm.

4.1 Trajectory design with hard constraints on path lengths

The distances traveled by sensor nodes are a critical factor especially in the context of a cooperative mobile sensor network formed from a number of wheeled mobile robots (e.g., differential drives, synchronous drives, etc.), if a major problem in the design is the power consumption by the robots. Then we demand that the lengths of the trajectories do not exceed a given limit $s_{\max}$, i.e., we require that

$$s_j(t_f) \leq s_{\max}, \quad j = 1, \dots, N. \tag{20}$$

The goal in the optimal measurement problem is to determine the forces (controls) applied to each vehicle conveying a sensor, which minimize a design criterion $\Psi[\boldsymbol{M}(\boldsymbol{q})]$ defined on the set of all real-valued information matrices of the form (8) under the constraints (12) on the magnitude of the controls and induced state constraints (18). In order to increase the degree of optimality, in our approach we will regard $\boldsymbol{q}_0$ as the design parameter vector to be chosen in addition to the control signal vector $\boldsymbol{u}$.

Clearly, in order to guarantee the correctness of such a formulation and further derivations, it is necessary to put some restrictions on the sensitivity coefficients $\boldsymbol{p}$. In the remainder of this paper we require $\boldsymbol{p}$ to be continuously differentiable.

Since sensor trajectories $\boldsymbol{q}$ are unequivocally determined as solutions to the state equation (13), the above control problem can be interpreted as an optimization problem over the set of feasible triples

$$\begin{aligned}\mathcal{P} = \Big\{(\boldsymbol{q}_0, \boldsymbol{u}) \mid \boldsymbol{x}_0^j \in \Omega_{\mathrm{ad}}, \boldsymbol{u}^j : T \to \mathbb{R}^\ell,& \\ \boldsymbol{u}_l \leq \boldsymbol{u}^j(t) \leq \boldsymbol{u}_u \text{ a.e. on } T,\ j = 1, \dots, N\Big\}.&\end{aligned} \tag{21}$$

This leads to the following formulation:

Problem 1. Find the pair $(\boldsymbol{q}_0, \boldsymbol{u}) \in \mathcal{P}$ *which maximizes*

$$J_1(\boldsymbol{q}_0, \boldsymbol{u}) = \Psi[\boldsymbol{M}(\boldsymbol{q})] \tag{22}$$

subject to the constraints (13), (18) *and* (20).

4.2 Minimum-length trajectories with guaranteed efficiency

Sometimes, a limited energy budget raises the question of how to minimize the distances run by sensor nodes while guaranteeing an acceptable level of the information content of the collected observations. The compromise proposed here relies on the notion of the D-efficiency which quantifies the suboptimality of given trajectories. In much the same way as in the classical optimum experimental design, cf. [Atkinson and Donev, 1992, Walter and Pronzato, 1997], we define it here as follows:

$$E_D(\boldsymbol{q}) = \left\{ \frac{\det(\boldsymbol{M}(\boldsymbol{q}))}{\det(\boldsymbol{M}(\widehat{\boldsymbol{q}}))} \right\}^{1/m}, \tag{23}$$

where $\widehat{\boldsymbol{q}}$ stands for the D-optimal trajectories obtained for the observations with no constraints on the traveled distances. The value of $\det(\boldsymbol{M}(\widehat{\boldsymbol{q}}))$ can be determined beforehand, and setting a reasonable positive threshold $\eta < 1$, we can introduce the constraint relation

$$E_D(\boldsymbol{q}) \geq \eta, \tag{24}$$

which guarantees a suboptimal yet reasonable solution. It is easily seen that (24) is equivalent to the constraint

$$\Psi[\boldsymbol{M}(\boldsymbol{q})] \geq C, \tag{25}$$

where $C = \Psi[\boldsymbol{M}(\widehat{\boldsymbol{q}})] + m \log(\eta)$.

We can thus formulate the following version of Problem 1:

Problem 2. Find the pair $(\boldsymbol{q}_0, \boldsymbol{u}) \in \mathcal{P}$ *which minimizes*

$$J_2(\boldsymbol{q}_0, \boldsymbol{u}) = \max_{j=1,\dots,N} s_j(t_f) \tag{26}$$

subject to the constraints (13), (18) *and* (25).

The above minimax problem is slightly more difficult than Problem 1. Therefore, in what follows, we shall fix our attention on Problem 2 only, as Problem 1 can be addressed in much the same way.

5. EQUIVALENT CANONICAL OPTIMAL CONTROL PROBLEM

It is clear that Problems 1 and 2 are highly nonlinear and we are not capable of finding closed-form formulae for their solutions. Accordingly, we must resort to numerical techniques. This section discusses the conversion of Problem 2 into a canonical optimal control problem with inequality-constrained trajectories and an endpoint cost, cf. [Polak, 1997]. Such a transcription makes it possible to employ existing software packages for numerically solving dynamic optimization and optimal control problems.

To simplify our notation further, consider the function svec : $\mathbb{S}^m \to \mathbb{R}^{m(m+1)/2}$, where $\mathbb{S}^m$ denotes the subspace of all symmetric matrices in $\mathbb{R}^{m\times m}$, that takes the lower triangular part (the elements only on the main diagonal and below) of a symmetric matrix $\boldsymbol{A}$ and stacks them into a vector $\boldsymbol{a}$:

$$\begin{aligned}\boldsymbol{a} &= \operatorname{svec}(\boldsymbol{A}) \\ &= \operatorname{col}[A_{11}, A_{21}, \dots, A_{m1}, A_{22}, A_{32}, \dots, A_{m2}, \dots, A_{mm}].\end{aligned} \tag{27}$$

Similarly, let $\boldsymbol{A} = \operatorname{Smat}(\boldsymbol{a})$ be the symmetric matrix such that $\operatorname{svec}(\operatorname{Smat}(\boldsymbol{a})) = \boldsymbol{a}$ for any $\boldsymbol{a} \in \mathbb{R}^{m(m+1)/2}$.

To set forth our basic idea, define first the matrix-valued function

$$\boldsymbol{\Pi}(\boldsymbol{q}(t), t) = \sum_{j=1}^{N} \boldsymbol{p}(\boldsymbol{x}^j(t), t)\boldsymbol{p}^{\mathsf{T}}(\boldsymbol{x}^j(t), t). \tag{28}$$

Setting $\boldsymbol{r} : T \to \mathbb{R}^{m(m+1)/2}$ as the solution of the differential equations

$$\dot{\boldsymbol{r}}(t) = \operatorname{svec}(\boldsymbol{\Pi}(\boldsymbol{q}(t), t)), \quad \boldsymbol{r}(0) = \boldsymbol{0}, \tag{29}$$

we have

$$\boldsymbol{M}(\boldsymbol{q}) = \operatorname{Smat}(\boldsymbol{r}(t_f)), \tag{30}$$

i.e., maximization of $\Psi[\boldsymbol{M}(\boldsymbol{r})]$ thus reduces to maximization of a function of the terminal value of the solution to (29).

The distances s_j traveled by sensor nodes, cf. (19), are easily incorporated into the usual optimal control formulation by augmenting the system dynamics with additional states

$$\boldsymbol{s}(t) = \operatorname{col}[s_1(t), \dots, s_N(t)] \tag{31}$$

being the solutions of the Cauchy problems

$$\dot{\boldsymbol{s}}(t) = \boldsymbol{h}(\boldsymbol{q}(t), \boldsymbol{u}(t)), \quad \boldsymbol{s}(0) = 0, \tag{32}$$

where

$$\begin{aligned}&\boldsymbol{h}(\boldsymbol{q}(t), \boldsymbol{u}(t)) \\ &\quad = \operatorname{col}[\|\boldsymbol{f}(\boldsymbol{x}^1(t), \boldsymbol{u}^1(t))\|, \dots, \|\boldsymbol{f}(\boldsymbol{x}^N(t), \boldsymbol{u}^N(t))\|].\end{aligned} \tag{33}$$

It is clear that the non-differentiability of the norm under the integrand (19) when the function f is equal to zero may lead to some difficulties during the minimization, as common non-linear programming packages usually have strong requirements regarding the smoothness of the cost functions. Obviously, it is possible to use some specialized procedures, but an alternative solution, which is very suitable here, consist in replacing the original non-smooth problem by minimization of a smooth function being an approximation to the minimal distance. Being more precise, it is possible to exchange the mentioned norm by a smooth symmetric exponential penalty function [Polak, 1997, p.248].

Defining the augmented state vector

$$\boldsymbol{w}(t) = \begin{bmatrix} \boldsymbol{q}(t) \\ \boldsymbol{r}(t) \\ \boldsymbol{s}(t) \end{bmatrix}, \tag{34}$$

we can have

$$\boldsymbol{w}_0 = \boldsymbol{w}(0) = \begin{bmatrix} \boldsymbol{q}_0 \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}. \tag{35}$$

Then, the equivalent canonical optimal control problem consists of finding a triple $(\boldsymbol{q}_0, \boldsymbol{u}, \varrho) \in \mathcal{P} \times \mathbb{R}_+$ which minimizes the performance index

$$J_2(\boldsymbol{q}_0, \boldsymbol{u}, \varrho) = \varrho \tag{36}$$

subject to

$$\begin{cases} \dot{\boldsymbol{w}}(t) = \boldsymbol{g}(\boldsymbol{w}(t), \boldsymbol{u}(t)), \\ \boldsymbol{w}(0) = \boldsymbol{w}_0, \\ \phi(\boldsymbol{w}(t_f)) \le C, \\ \beta_j(\boldsymbol{w}(t_f)) \le \varrho, \quad j = 1, \dots, N, \\ \bar{\gamma}_\ell(\boldsymbol{w}(t)) \le 0, \quad \forall\, t \in T, \quad \ell = 1, \dots, \nu, \end{cases} \tag{37}$$

where

$$\boldsymbol{g}(\boldsymbol{w}(t), \boldsymbol{u}(t)) = \begin{bmatrix} \boldsymbol{d}(\boldsymbol{q}(t), \boldsymbol{u}(t) \\ \operatorname{svec}(\boldsymbol{\Pi}(\boldsymbol{q}(t), t)) \\ \boldsymbol{h}(\boldsymbol{q}(t), \boldsymbol{u}(t)) \end{bmatrix}, \tag{38}$$

$$\phi(\boldsymbol{w}(t)) = \Psi[\operatorname{Smat}(\boldsymbol{q}(t))], \tag{39}$$

$$\beta_j(\boldsymbol{w}(t)) = s_j(t), \tag{40}$$

$$\bar{\gamma}_\ell(\boldsymbol{w}(t)) = \gamma_\ell(\boldsymbol{q}(t)). \tag{41}$$

The problem formulated in this way can be solved using existing packages for numerically solving dynamic optimization problems, such as RIOTS_95, cf. [Schwartz et al., 1997], DIRCOL, cf. [von Stryk, 1999], or MISER, cf. [Jennings et al., 2002]. We employed RIOTS_95in this work, which is designed as a MATLAB toolbox written mostly in C and runs under Windows 98/2000/XP and Linux. It provides an interactive environment for solving a very broad class of optimal control problems. The users's problems can be prepared purely as M-files and no compiler is required to solve them. To speed up the solution process, the functions defining the problem can be coded in C and then compiled and linked with some pre-built linking libraries. The implemented numerical methods are supported by the theory outlined in [Polak, 1997], which uses the approach of consistent approximations. Systems dynamics can be integrated with fixed step-size Runge-Kutta integration, a discrete-time solver or a variable step-size method. The software automatically computes gradients for all functions with respect to the controls and any free initial conditions. The controls are represented as splines, which allows for a high-degree of function approximation accuracy without requiring a large number of control parameters. There are three main optimization routines, each suited for different levels of generality, and the most general one is based on sequential quadratic programming methods (it was also used in our computations reported in the next section).

6. SIMULATION EXAMPLE

As a suitable example presenting the delineated approach, let us consider an atmospheric pollutant transport-chemistry process over an urban area being normalized to a unit square. At the point $x_0 = (0.3, 0.6)$ an active source of pollution is located, which leads to changes in the pollutant concentration $y(x,t)$. The entire process over the observation interval $T = [0, 1]$ and velocity field $v(x,t)$ varying in space and time according to the following model (cf. Fig. 1):

$$v(x,t) = \big(2(y + 1/4), 2(t - x)\big), \tag{42}$$

can be described by the following advection-diffusion-reaction equation:

$$\begin{aligned} \frac{\partial y(x,t)}{\partial t} &+ \nabla \cdot \big(\boldsymbol{v}(x,t) y(x,t)\big) \\ &= \nabla \cdot \big(\kappa(x) \nabla y(x,t)\big) + f(x),\ x \in \Omega \end{aligned} \tag{43}$$

subject to the following boundary and initial conditions:

$$\frac{\partial y(x,t)}{\partial n} = 0 \text{ on } \Gamma \times T, \quad y(x,0) = 0 \text{ in } \Omega, \tag{44}$$

where the term $f(x) = e^{-100\|x - x_0\|^2}$ represents an intensity of active source of pollutant, and $\partial y / \partial n$ stands for the partial derivative of y with respect to the outward normal to the boundary Γ. In our illustrative simulations, the following form of the turbulent diffussivity coefficient was applied

$$\kappa(x) = \theta_1 + \theta_2 x_1^2 + \theta_3 x_2^2, \tag{45}$$

so parameters $\theta = (\theta_1, \theta_2, \theta_3)$ need to be estimated based on measurement data. The values $\theta_1 = 0.02$, $\theta_2 = \theta_3 = 0.005$ were taken as the initial estimate.

Our goal is to determine the optimal trajectories for two movable sensors subject to the assumed estimation accuracy level. In order to verify the proposed approach, the MATLAB program was written using a PC equipped with Pentium M740 processor (1.73GHz, 1 GB RAM) running Windows XP and MATLAB 7 (R14). First, the system of PDEs was solved using some routines of the MATLAB PDE toolbox for a spatial mesh composed of 1800 triangles and 961 nodes and uniformly divided time interval (30 subintervals). Since the PDE toolbox is not particularly designed to solve the considered class of problems it was supplemented with additional procedures which take into account the advection term (being not self-adjoint operator) based on the modified Discontinuous-Galerkin method [Ern and Guermond, 2006].

The sensitivity coefficients were then linearly interpolated and stored. Finally, to determine the optimal mobile sensor trajectories, the package RIOTS_95was applied. We adopt a simple sensors dynamics

$$\dot{q}(t) = u(t), \quad q(0) = q_0,$$

and impose the following bounds for u

$$|u_i(t)| \le 0.7, \quad \forall t \in T. \tag{46}$$

In order to avoid the convergence to the local minima, the simulations were restarted several times from different initial starting points. Each simulation took about 10–20 minutes of computation time.

The optimal trajectories are presented in Fig. 2. For comparison, the left subplot of Fig. 2 shows the D-optimum trajectories obtained for the setting without any constraints imposed on the lengths of sensor paths. We can observe how sensors try to follow the complex pollutant concentration changes leading to quite sophisticated sensor motions. As for Problem 1, after shortening the maximal trajectory length to $s_{\max} = 0.4$, the optimal trajectories are regularized and still explore the same subregion of the spatial domain. However, the complexity of the considered process induce the strong nonlinearity in the dependence between trajectory lengths and efficiency of the solutions. Therefore, a significant reduction of maximum length leads in this case to dramatic decrease in the efficiency of the experiment (which is decreased to $E_D = 41.24\%$). It becomes clear that the Problem 2 is more flexible to control the quality of the observation process and the lengths of trajectories can be significantly shortened (sparing the valuable resources) with guaranteed level of efficiency (cf. the right subplot of Fig. 2). The control input signals are shown in Fig. 3 for both Problems 1 and 2. Note the actuator saturation in Problem-2, prompting a more aggressive actuation for shorter distance traveled.

7. CONCLUSION

This work can be considered as an attempt to establish an interconnection between the quality of the parameter estimation in DPSs and the limited motion resources of sensor network nodes. As a result, two formulations of the related problem have been proposed: the first for a specified constraints on the distances traveled by sensors, and the other for the case in which the maximal sensor trajectory length is treated as a decision variable with additional terminal inequality constraint representing the accuracy of the estimation. The proper reformulation of those problems into equivalent optimal control problems in Mayer form is also briefly delineated. Then, they can be solved using the dedicated efficient existing tools, such as MATLAB package RIOTS_95. The future research will focus on the further extension of the proposed approach to the different control objectives motivated in real-world engineering applications.

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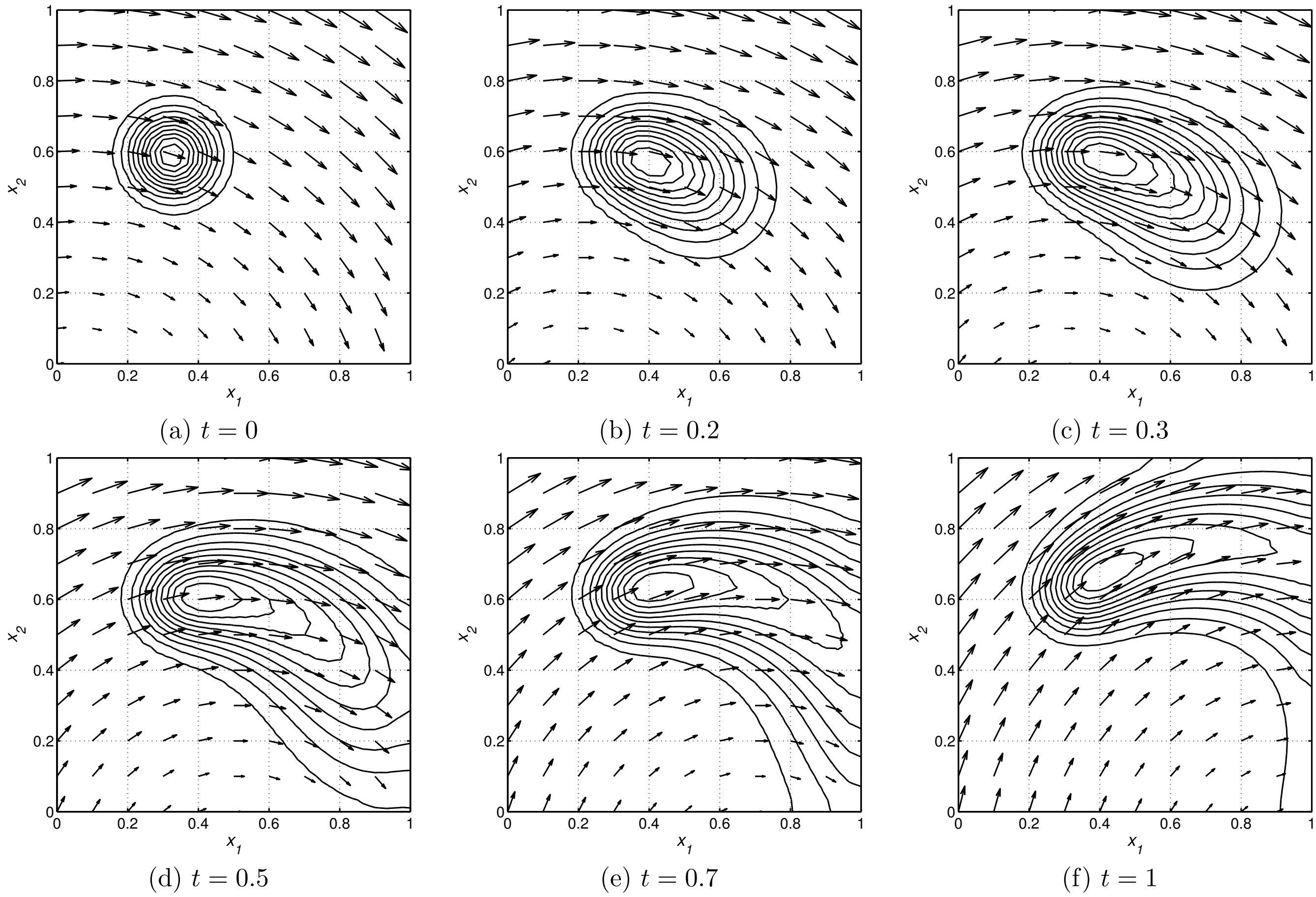

Fig. 1. Temporal changes in the wind velocity field and pollutant concentration.

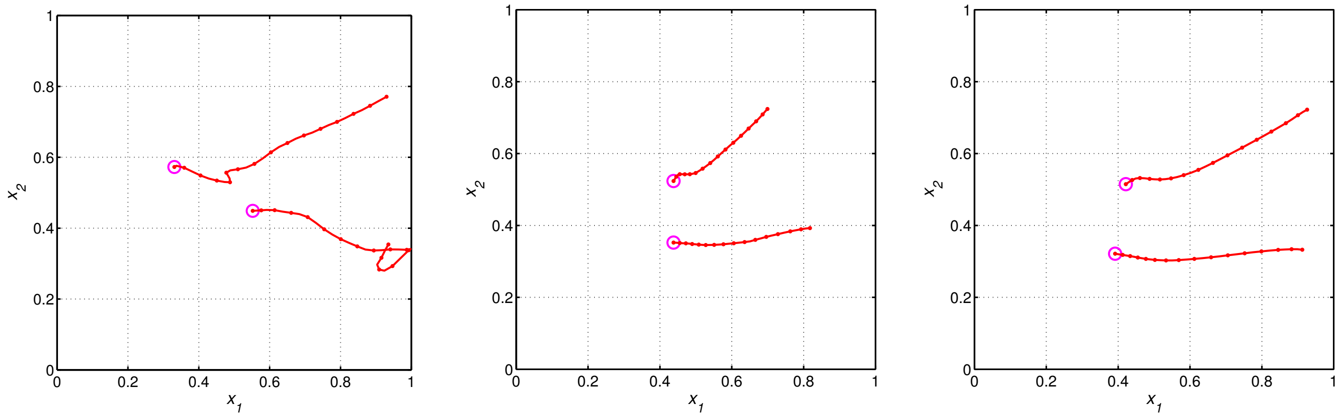

Fig. 2. **Left plot:** Optimal sensor trajectories: D-optimal solution (the maximal trajectory length is 0.71); **Middle plot:** Problem 1 for maximal trajectory lengths set to $s_{\max} \leq 0.4$ (efficiency level is $E_D = 41.24\%$) and **Right plot:** Problem 2 for the guaranteed D-efficiency value set to $E_D \geq 0.75$ (the maximal length of trajectory is $J_2 = 0.55$). In all plots, the initial sensor positions are marked with open circles, and the sensors positions at the consecutive points of the time grid are denoted by discs.

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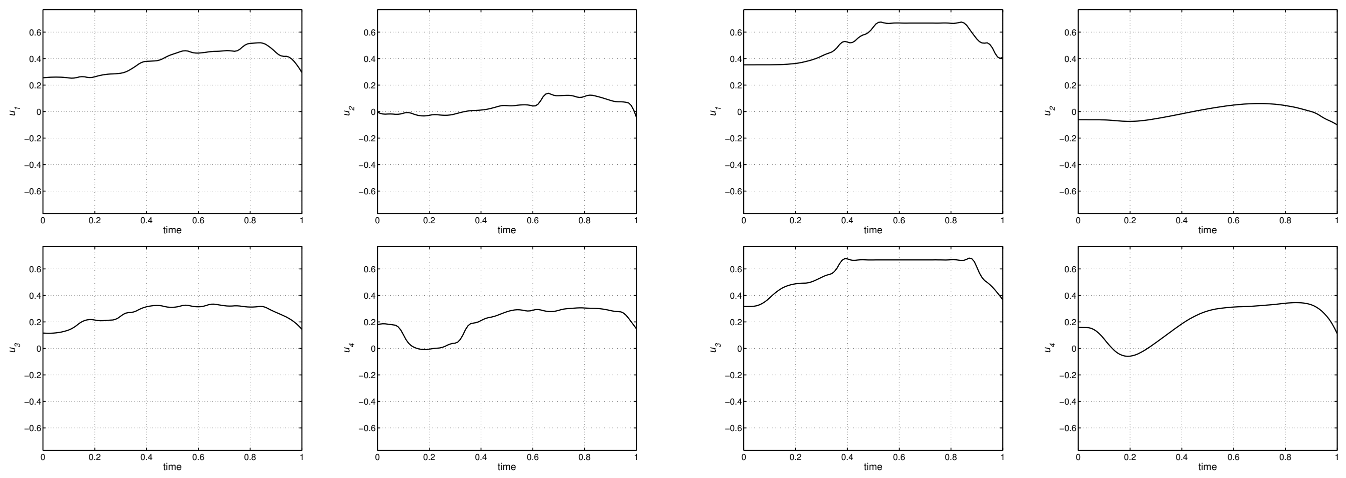

Fig. 3. Optimal controls: Problem 1 (left 4 plots) and Problem 2 (right 4 plots).

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