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A multimodal model for an urban traffic control policy

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Abstract: The main aim of this paper is to present a multimodal model of urban traffic used as predictive model for a traffic control policy. The traffic control policy is based on Model Predictive Control (MPC). The predictive model focus on the private vehicle mode and public transport mode. For the first mode, the model is based essentially on the store-and-forward model. For the second mode, we present an innovative model based on mean behavior of public transport vehicles. Simulation tests show that the model of public transport mode is consistent and worthy to be implemented for a traffic control policy.

Keywords: Traffic control; Predictive model; Multimodal urban network.

1. INTRODUCTION

In last few decades, major improvements were realized to deal with the complex problem of the traffic control in urban areas. The main way to regulate the urban traffic is via traffic lights. To quote only the best known, TUC, PRODYN, UTOPIA, SCOOTS, CRONOS are strategies that acts on traffic lights using optimization methods to regulate the traffic of private vehicle. However most of these strategies regulate the traffic of public transport vehicle using only rules-based techniques. In other terms, these strategies do not consider the public transport as a full-fledged mode as the private vehicle mode. In addition, some strategies like PRODYN have a local point of view that is applicable only for single junctions. Considering the public transport vehicle as an urban mode in the optimization process is necessary for a best traffic regulation in a whole urban network. This is the aim of our work. To optimize or to control a traffic process, it is obvious that one have to dispose of a mathematical model of this complex process. The fact that these two models would be used for an automatic traffic control impose to have simple models that do not require great time calculations. The private vehicle model is based essentially on the model developed for TUC strategy. It is a macroscopic model. In the opposite, the public transport vehicle mode model is semi-macroscopic.

The goal of the paper is to depict a multimodal model of urban traffic used as predictive model for a traffic control strategy. The paper is organised as follows. Firstly, a description of urban network is given. Secondly, we introduce the general architecture of the traffic control based on the model predictive control (MPC). The detailled description of the model is given in the next section. For the private vehicle mode, the model is based essentially on the store-and-forward model. For the public transport mode, we present an innovative model based on mean behavior of public transport vehicles. The next section concerns the simulation tests. The results show that the model of public transport mode is consistent and worthy to be implemented for a real time traffic control policy. In the last section, conclusions and future works are given.

2. SYSTEM DESCRIPTION : MULTIMODAL URBAN ROAD NETWORK

A multimodal urban road network is an urban road network crossed by at least two transportation modes. For this study, we restrict to the case of two modes : private vehicles and public transport vehicles (buses). From now, we will talk about buses or public transport vehicles to assign the public transport mode. An urban road network comprises junctions (intersections), approaches (links) that links between two successive junctions and stations in which buses have to stop. A junction is formed by approaches that lead to a common cross area. The urban road network is crossed by public transport vehicles like buses or shuttle. A route is formed by succession of junctions and stations, in which buses have to stop for a time called the dwell time(Fig.1). The dwell time for each line at any station is considered fixed and known.

The traffic at a junction is divided into streams. A stream is formed by all vehicles that cross the junction from the same approach. Two streams are compatible if they can cross safely the junction simultaneously, they are conflicting otherwise. A junction is called signal controlled when the streams are controlled by a traffic light. A traffic light is described by five main variables (Papageorgiou [1999]):

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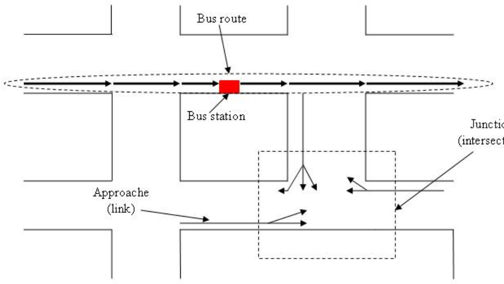

Fig. 1. Multimodal urban road network

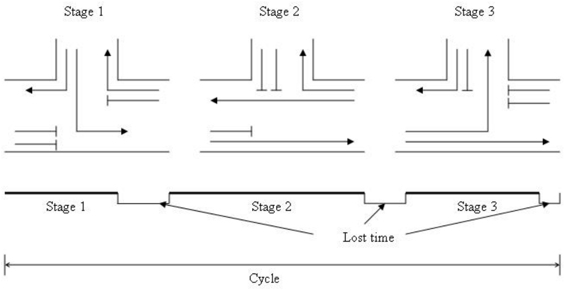

Fig. 2. Cycle time and example of signal cycle

- Stage (phase) : The duration into which a set of compatible streams is given the right of way.
- Lost time : The time inserted, between two consecutive stages, in order to avoid interferences between two conflicting streams.
- Cycle time : It is the duration of signal cycle. The signal cycle is formed by succession of stages and lost times (Fig.2).
- Split : It is the relative green duration of each stage, as a proportion of the cycle time.
- Offset : It is the time difference between cycles for successive junctions.

To improve the traffic conditions via traffic lights, there are four possible actions : cycle time specification(Allsop [1983]), Stage specification (Scemama [1994] and Heydecker and Dudgeon [1987]), offset specification (Stamatiadis and Gartner [1996]) and split specification according to the demand of the involved streams(Diakaki and al. [2002], Di Taranto and Mauro [1989],Barrière and al. [1986], Dotoli and al. [2005] and Bhouri and Lotito [2006]). This present work focus only on Split. Cycle time, stage specification and offset are considered known and fixed for the future developments.

3. TRAFFIC CONTROL

The automatic control theory gives many tools and methods to deal with this challenging problem (Papageorgiou [1983]). The Model Predictive Control (MPC) is one of the most effective methods for Optimal control policy. Its main goal is to optimize, over an open loop time sequence

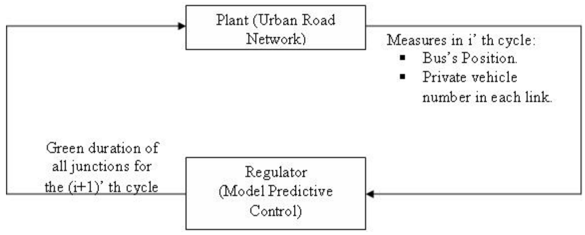

Fig. 3. Control architecture of the traffic process

of controls, the process response using a model to forecast the future process behavior over a prediction horizon(Rao [2000] and Garcia and al. [2000]). It is particularly well fitted for getting of an optimal traffic conditions because of the complexity of the traffic process on a multimodal urban road network.

At the beginning of each cycle time, specific traffic measures are gathered from the real Urban Road Network. According to this measures, the automatic control policy(MPC for our case) determines the suitable actions to take for a better service to the road user as well in private vehicle or in public transport vehicle.

Despite that the private vehicles and public transport vehicles share the same roads, these two modes have different features. The most important differences are :

- There are much more private vehicles than public transport vehicles.
- A public transport vehicle route is known in opposition to private vehicle.

These two last points lead up to consider :

- The number of vehicles in each approach as the state variable of private vehicle mode.
- The positions of vehicle in the network as the state variable of public transport mode.

From these measures, the Optimal Control policy determines the actions to take in order to minimize the number of private vehicles in each link and to minimize the difference between the current position of public transport vehicle and a pre-specified position. The decision-making variable or control variable is the green duration of each stage of all the junction of the network. The control architecture is based on model predictive control (Fig.3).

A General MPC Policy comprises two blocs :

- Modeling bloc : The model is used to predict the behavior of outputs of a dynamical system with respect to changes in the process inputs. The process is the multimodal traffic in an urban road network that is a discrete dynamical system. The inputs are

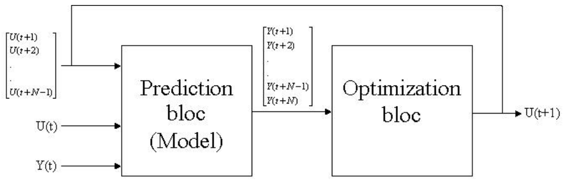

Fig. 4. Traffic regulator based on Model Predictive Control

the green duration of each stage of all junctions in the network. The outputs are the positions of public transport vehicles and the number of private vehicle on each link.

- Optimization bloc : This bloc is used to minimize the number of private vehicle on each link and the difference between the current position of public transport vehicle and a pre-specified position.

At time t, the current process state is measured and a cost minimizing control strategy is computed for a time horizon $T = N \cdot t$ in the future: $[t, t+T]$. Only the first step of the control strategy is applied to the real process, then the process state is measured again and the calculations are repeated starting from the now current state, yielding a new control and new predicted state path (Fig.4). The prediction horizon keeps being shifted forward(Garcia and al. [2000] and Mayne and Michalska [2000]).

The optimization bloc is iterative and based on meta-heuristic because of :

- The complex nature of traffic that leads to a nonlinear analytical model for the private vehicle.
- The non existence of an accurate analytical model for the circulation of public transport vehicles that leads to an hybrid model.
- The relative easiness of implementation of a meta-heuristic methods.

We will not give further explanations for the optimization bloc because it is not the goal of this paper, which is the model description.

4. MODEL DESCRIPTION

The process, to be modeled, is the circulation of both private vehicles and public transport vehicles in an urban road network. From the traffic state at current cycle time and the green duration of each stage of all junctions in the current cycle time, the model predicts the traffic state at the next cycle time. The traffic state is made of :

- The positions of all public transport vehicles in the network.
- The number of private vehicles in each link of the network.

Before evolving mathematical details of the model, we present notations and hypothesis that lead to the final model.

4.1 Notations and Hypothesis

Notations :

Indices

i :	link index.
l :	junction index.
j :	bus station index.
m :	bus line index.
n :	bus index for a given bus line.
k :	discrete time index.

Sets

I :	set of links.
L :	set of junctions.
J :	set of bus stations.
In_j :	set of incoming links for junction j.
M :	set of bus lines.
N_m :	set of bus for bus line m.
$G_k^{m,n}$:	set of green durations of traffic lights that may be crossed by the bus n of line m over the cycle k.
$St_k^{m,n}$:	set of dwell times in stations that may be crossed by the bus n of line m over the cycle k.
$Nb_k^{m,n}$:	set of the number of private vehicle ahead the bus n of line m in all the links that may be crossed over the cycle k.

General variables

C :	cycle time for all junctions.
a :	mean effective private vehicle length.
Vb :	maximum free bus speed.

Link specified variables

$X_i(k)$:	number of private vehicles in link i at cycle k.
$G_i(k)$:	total green duration of approach i at cycle k .
S_i :	saturation flow of link i.
$q_i(k)$:	inflow of the link i at cycle k.
$u_i(k)$:	outflow of the link i at cycle k.
$\tau_{w,i}$:	the turning rate from link w toward link i.
$Lf_{m,i}$:	position of light line of link i referenced to the start of bus line m.

public transport variables

T_j :	dwell time of a bus in station j.
$P_{m,n}(k)$:	position of bus n of line m in the beginning of cycle k.
$i_k^{m,n}$:	number of links (traffic lights) that may be crossed by the bus n of line m during the cycle k.
$j_k^{m,n}$:	number of bus stations that may be crossed by the bus n of line m during the cycle k.
tr_n, tr_p :	new and past remaining time before the end of present cycle.
$Ls_{m,j}$:	position of bus station j referenced to the start of bus line m.

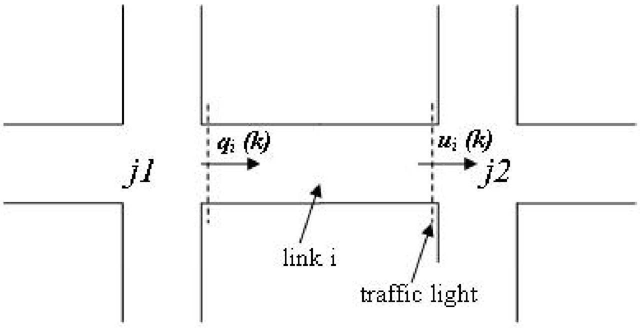

Fig. 5. An urban road link

$Nb^i_{m,n}(k)$: Number of private vehicles ahead of bus n of line m in link i at cycle k.

Hypothesis :

Hypothesis 1. The free bus speed Vb_i in link i is smaller than free private vehicle speed.

Hypothesis 2. For a cycle time C, if the green duration of an approach i is $G_i(k)$ then for a given time tr, the green duration allocated to the approach i, during the time tr, is $\frac{tr}{C} \times G_i(k)$.

Hypothesis 3. The inflow of private vehicles, during a cycle time, into a given link are divided up uniformly along the link. For example : for a link of length L and into it Nv private vehicles drive in, the number of private vehicles existing in a section of length l is $\frac{l}{L} \times Nv$.

Hypothesis 4. If a bus reach a traffic light line, which its green duration is $G_i(k)$, at tr seconds before the end of the current cycle, the bus will be stopped for a time of $\frac{1}{2} \times tr \times (1 - \frac{G_i(k)}{C})$.

4.2 private vehicle Mode Modeling

For the mathematical model of both private and public transport vehicles, the urban road network is represented as a direct graph with links $i \in I$ and junctions $j \in J$. Each link i is controlled by a traffic light with green duration $G_i(k)$. The private vehicle Mode Model is based essentially on the model developed by Diakaki and al. [2002] for The strategy TUC. However an important improvement was introduced to take into account the case of undersaturated traffic conditions as introduced by Dotoli and al. [2005]. For a link i connecting two junctions $j1$ and $j2$ (Fig.5), the dynamics of this link is expressed by the conservation equation :

$$X_i(k+1) = X_i(k) + C \cdot (q_i(k) - u_i(k)) \qquad (1)$$

Where $X_i(k)$ is the number of private vehicles within link i at cycle time k, $q_i(k)$ and $u_i(k)$ are respectively the inflow and outflow of link i over the period $[k \cdot C, (k+1) \cdot C]$. The inflow of link i may be expressed by :

$$q_i(k) = \sum_{w \in In_{j1}} \tau_{w,i} \cdot u_w(k)$$

where $\tau_{w,i}$ is the turning rate from link w toward link i and In_{j1} is the set of incoming links for junction $j1$. In the TUC strategy, the calculation of the outflow of link i is given by using the store-and-forward model(Gazis [1964] and Gazis and Potts [1963])leading to this equation of outflow of link i :

$$u_i(k) = S_i \cdot \frac{G_i(k)}{C}$$

where S_i is the saturation flow of link i. The store-and-forward model is fitted to the case of oversaturated traffic conditions. In order to take into account undersaturated traffic conditions, the final expression of outflow of link i is given by :

$$u_i(k) = min(S_i \cdot \frac{G_i(k)}{C}, \frac{X_i(k)}{C})$$

Introducing all the above in (1), the final dynamics of link i is expressed by :

$$\begin{aligned} X_i(k+1) &= X_i(k) \\ &+C \cdot [\sum_{w \in I_{j1}} \tau_{w,i} \cdot min(S_w \cdot \frac{G_w(k)}{C}, \frac{X_w(k)}{C})] \\ &-min(S_i \cdot \frac{G_i(k)}{C}, \frac{X_i(k)}{C})) \end{aligned} \qquad (2)$$

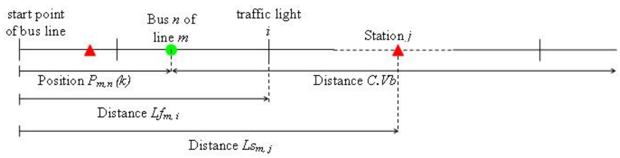

Fig. 6. Bus route

4.3 public transport vehicle Mode Modeling

The Urban road network is crossed by many bus lines and it is possible to have many buses of the same line crossing simultaneously the network. The dynamics of bus n of the bus line m is represented by the position $P_{m,n}(k)$ in the network, referenced to the start point of the bus line. Over the cycle time C, or control interval, the bus may stride, at maximum, the distance of $C \cdot Vb$ where Vb is the free speed of the bus in urban area. Along this distance, the bus may cross over $i^{m,n}_k$ traffic lights which their green duration are spliced into the set $G^{m,n}_k$ and over $j^{m,n}_k$ stations which their dwell time are spliced into the set $St^{m,n}_k$ (Fig.6).

Strictly speaking, the dynamics of buses is given by the general equation :

$$P_{m,n}(k+1) = F(P_{m,n}(k), G^{m,n}_k, Nb^{m,n}_k, St^{m,n}_k)$$

where F is the function to determine and $Nb^{m,n}_k$ is the vector formed by the number of private vehicles ahead of bus n of line m in all the links that may be crossed by the bus over the period $[k \cdot C, (k+1) \cdot C]$. The vector $Nb^{m,n}_k$ is calculated using the private vehicle model and hypothesis (3). This model is called semi-macroscopic in the sense that the interaction between the public transport vehicles and private vehicles is treated by a macroscopic way. The bus, from its position $P_{m,n}(k)$ at the beginning of cycle k, will cross over, one by one, stations or traffic lights until the end of the current cycle k. After passing over station or traffic light, the new position P_n and the remaining time tr_n, before the end of the cycle, are re-evaluated. Depending on whether it's a station or traffic

```
trp = C ;

While (trp > 0) do

        If (next_stop ==station) then
                [Pn, trn] = g_st (Pp, trp, Tst) ;

        Else //next_stop = traffic light
                [Pn, trn] = g_light (Pp, trp, G, Nb) ;
        End if;

        trp = trn;
        Pp = Pn;

End while;

Pm, n (k) = Pn;
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Fig. 7. Algorithm of function F

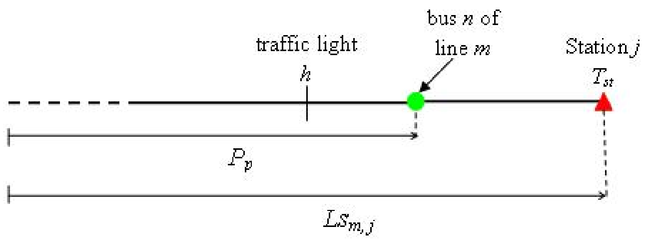

Fig. 8. Situation of a bus crossing a station

light, the re-evaluation of the position and the remaining time is carried out by the function g_st or the function g_light. The algorithm of function F is given in the figure 7.

The function g_st takes as input variables, the past position P_p, the past time remaining tr_p and the dwell time of the bus in the station j. It computes the new position P_n of the bus and the new time remaining tr_n.Obviously, the g_st function requires, for the calculation, the free bus speed Vb and the position of the station $Ls_{m,j}$. The figure 8 may help to understand the simple algorithm of this function given in figure 9.

The function g_light takes as input variables, the past position P_p, the past time remaining tr_p, the green duration of traffic light i and the number of private vehicle Nb^i ahead the bus in the link. It computes the new position P_n of the bus and the new time remaining tr_n.Obviously, the g_light function requires, for the calculation, the free bus speed Vb and the position of the traffic light position $Lf_{m,i}$.

The function g_light is the most important function for the development of the model and is worthy of further explanations. The figure 10 may be important to understand the underneath discussion.

(1) 1st case :

$$Lf_{m,i} - P_p - a \cdot Nb^i \geq tr_p \cdot V_b$$

The bus has not the possibility to reach the private vehicle queue before the end of current cycle. In this

```
[Pn, trn] = g_st (Pp, trp, Tst)

 If  (trp.Vb < Lsm, j - Pp)

        Pn = Pp + trp.Vb;
        trn = 0 ;
 Else

        Pn = Lsm, j;
         trn  = trp - Tst - (Lsm, j - Pp)/Vb;

 End if
```

Fig. 9. Algorithm of function g_st

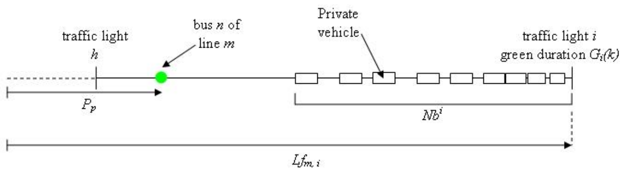

Fig. 10. Situation of a bus crossing a traffic light

case, P_n and tr_n are given by :

$$\begin{aligned} P_n &= P_p + tr_p \cdot V_b \\ tr_n &= 0 \end{aligned} \quad (3)$$

(2) 2nd case :

$$Lf_{m,i} - P_p - a \cdot Nb^i \leq tr_p \cdot V_b$$

The bus has the possibility to reach the private vehicle queue before the end of current cycle. In this case, the bus will reach the queue at time $tr_p + Y$. Where

$$Y = C \cdot \frac{Lf_{m,i} - P_p - a \cdot Nb^i}{C \cdot V_b - a \cdot S_i \cdot G_i(k)}$$

The details of calculation of Y and the proof that $Y \geq 0$ are given in Appendix A. In this case, we have to distinguish between the case in which the bus reaches the queue before the traffic light line and the case in which the bus have the opportunity to reach the queue after the traffic light line.

(a) $Y \cdot V_b \geq Lf_{m,i} - P_p$: The bus have the opportunity to reach the queue after the traffic light line. The bus rides with his free bus speed Vb until the traffic light line and reach this line at time $tr_p + \frac{Lf_{m,i} - P_p}{V_b}$. Here, there are also two possibilities :

- $tr_p \cdot V_b \leq Lf_{m,i} - P_p$ and thus P_n and tr_n are given by :

$$\begin{aligned} lP_n &= P_p + tr_p \cdot V_b \\ tr_n &= 0 \end{aligned} \quad (4)$$

- $tr_p \cdot V_b \geq Lf_{m,i} - P_p$. The bus reaches the traffic light line at time $C - tr_p + \frac{Lf_{m,i}-P_p}{V_b}$. Before the end of the current cycle, the remaining time is $tr_p - \frac{Lf_{m,i}-P_p}{V_b}$. According to the hypothesis (4), the bus have to be stopped for a time of $\frac{1}{2} \times (tr_p - \frac{Lf_{m,i}-P_p}{V_b}) \times (1 - \frac{G_i(k)}{C})$ and when leaving the link, the bus have the time of $(tr_p - \frac{Lf_{m,i}-P_p}{V_b}) \times \frac{G_i(k)}{C}$, before the end of current cycle, to ride in the downstream network. Finally, we have :

$$\begin{aligned} P_n &= Lf_{m,i} \\ tr_n &= \frac{1}{2} \cdot (tr_p - \frac{Lf_{m,i} - P_p}{V_b}) \\ &\cdot (1 + \frac{G_i(k)}{C}) \end{aligned} \quad (5)$$

(b) $Y \cdot V_b \leq Lf_{m,i} - P_p$: The bus reaches the queue before the traffic light line. According to the hypothesis (2), the number of private vehicles ahead the bus is $Nb^i - \frac{S_i \cdot Y \cdot G_i(k)}{C}$. The remaining time, before the end of current cycle time, is $tr_p - Y$ in which $(tr_p - Y) \times \frac{G_i(k)}{C}$ is green light (Hypothesis (2)). Again we have to distinguish two cases :

(i)

$$Nb^i \geq \frac{S_i \cdot G_i(k) \cdot tr_p}{C}$$

The queue can not disappear in this cycle. There are two possibilities depending on whether $tr_p \times V_b$ is higher or lower than $Lf_{m,i} - P_p - a \times (Nb^i - \frac{S_i \cdot G_i(k) \cdot tr_p}{C})$. For the first case, we have :

$$\begin{aligned} P_n &= P_p + tr_p \cdot V_b \\ tr_n &= 0 \end{aligned} \quad (6)$$

And the second case, we have :

$$\begin{aligned} P_n &= Lf_{m,i} \\ &-a \cdot (Nb^i - \frac{S_i \cdot G_i(k) \cdot tr_p}{C}) \\ tr_n &= 0 \end{aligned} \quad (7)$$

(ii)

$$Nb^i \leq \frac{S_i \cdot G_i(k) \cdot tr_p}{C}$$

The queue disappears in this cycle and the bus reaches the traffic light line. P_n and tr_n are given by :

$$\begin{aligned} P_n &= Lf_{m,i} \\ tr_n &= \frac{1}{2} \cdot (tr_p - Y) \cdot (1 + \frac{G_i(k)}{C}) \\ &- \frac{Nb^i}{S_i} + \frac{Y \cdot G_i(k)}{C} \end{aligned} \quad (8)$$

In order to sum up the public transport modeling, we consider :

(1) Conditions :

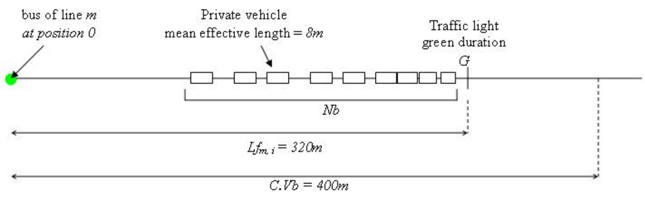

Fig. 11. Simulation of function g_light

A : $Lf_{m,i} - P_p - a \cdot Nb^i \geq tr_p \cdot V_b$
B : $Nb^i \geq \frac{S_i \cdot G_i(k) \cdot tr_p}{C}$
C : $tr_p \cdot V_b \leq Lf_{m,i} - P_p - a \cdot (Nb^i - \frac{S_i \cdot G_i(k) \cdot tr_p}{C})$
D : $G_i(k) \geq \frac{C \cdot Nb^i \cdot V_b}{S_i \cdot (Lf_{m,i} - P_p)}$
E : $tr_p \cdot V_b \leq Lf_{m,i} - P_p$

(2) Equations :
EQ 1 : $P_n = P_p + tr_p \cdot V_b$ and $tr_n = 0$
EQ 2 : $P_n = Lf_{m,i} - a \cdot (Nb^i - \frac{S_i \cdot G_i(k) \cdot tr_p}{C})$ and $tr_n = 0$
EQ 3 : $P_n = Lf_{m,i}$ et $tr_n = \frac{1}{2} \cdot (tr_p - Y) \cdot (1 + \frac{G_i(k)}{C}) - \frac{Nb^i}{S_i} + \frac{Y \cdot G_i(k)}{C}$
EQ 4 : $P_n = Lf_{m,i}$ et $tr_n = \frac{1}{2} \cdot (tr_p - \frac{Lf_{m,i} - P_p}{V_b}) \cdot (1 + \frac{G_i(k)}{C})$

With these last considerations, the algorithm is given by :

if $(A \mid (\overline{A} \times D \times E) \mid (\overline{A} \times B \times C \times \overline{D}))$	then EQ1;
if $(\overline{A} \times B \times \overline{C} \times \overline{D})$	then EQ2;
if $(\overline{A} \times \overline{B} \times \overline{D})$	then EQ3;
if $(\overline{A} \times D \times \overline{E})$	then EQ4;

Where | refers to "or", $\times$ to "and" and $\overline{A}$ to the negation of A.

5. SIMULATION RESULTS

5.1 Function $g\_$ light

The main contribution of this work is the model developed for the public transport vehicle. This model is based especially on the way in which the public transport vehicle rides over a traffic light. This way was described in the $g\_$ *light* function. In order to verify the consistency of this important function, a simulation test was carried out. This test is illustrated by the figure 11.

The test was carried out with varying the number of private vehicle ahead the bus Nb between 0 and 40 and the green duration G between 0 and 80. The new position $P_n$of the bus is given in the figure 12.

Firstly, one can see clearly that fewer private vehicles are ahead the bus higher the new position of the bus is and vice-versa. This remark is simply logical because : higher the traffic is congested more difficult is the progress of vehicles especially buses.

Secondly, one can see that : higher green duration is allocated to the link crossed by the bus, higher is the new position of the bus and vice-versa.

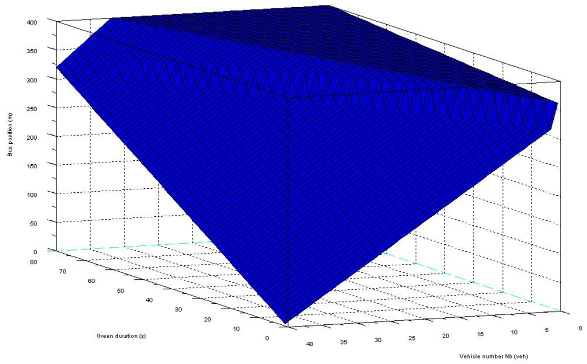

Fig. 12. Bus position

Finally, the curve shows two important points :

(1) For $Nb = 40veh$, the downstream of the bus is completely congested, and for $G = 0s$, no green light, the new position $P_n = 0m$ synonym that the bus did not move.

(2) For $G = 80s = C$, no red light, and for $Nb \leq 32veh$, the bus reaches its maximum distance of $400m$.

These all remarks show that the function g_light is globally coherent with the real behavior of a bus crossing over a traffic light in urban area. It remains to verify the consistency of the whole model in an urban network.

5.2 Network simulation

The consistency of the private vehicle model was widely proved in the simulation test and real-life implementation of TUC strategy (Diakaki and al. [2002], Bhouri and Lotito [2006] and Kachroudi and Bhouri [2007]). In the opposite, the public transport vehicle model is recently developed. For these reasons, only the public transport vehicle model will have a particular attention.

The simulation of the whole model is carried out using the network illustrated in the figure 13.

The network comprises 16 junctions, 11 Entrances and 49 links. It is crossed by two bus lines. The first pulls in by the entrance 3, cross the junctions 4, 3, 7, 11, 10, 9, 13 before leaving the network. Its transit frequency is $1bus/3cycles$. The second line pulls in by the entrance 10, cross the junctions 5, 6, 7, 11, 12, 16 before leaving. Its transit frequency is $1bus/7cycle$.

The simulations are performed in order to show the behavior of the public transport mode model according to the traffic demand and the state of traffic light of junctions crossed by the bus lines. The simulations are divided into two groups depending on whether we act on the green durations of junctions crossed by bus line 1 or bus line 2. Group 1 corresponds to the bus line 1 and Group 2 corresponds to the bus line 2. The horizon of simulations

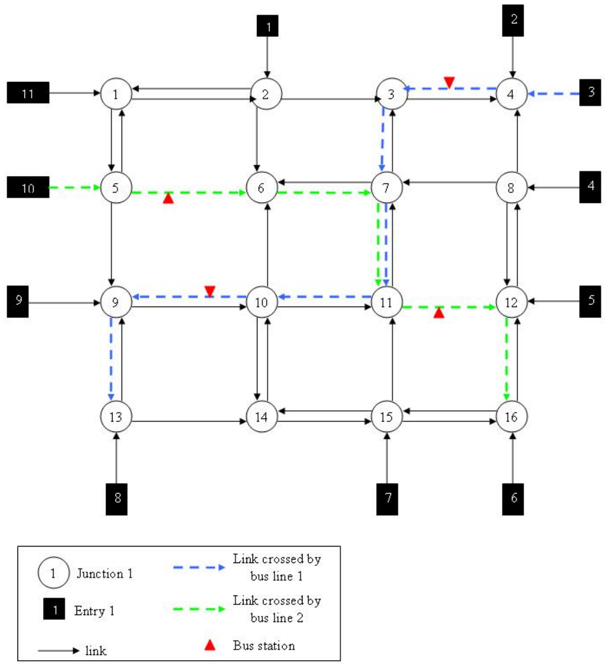

Fig. 13. The example network

is 100 cycles. As the traffic demand fluctuates around a mean value, the simulation results presented here are a mean value of all public transport vehicle that crossed the network over the simulation horizon.

Each group contains four scenarios.

(1) ordinary demand in all and ordinary green duration allocated to links crossed by the bus.

(2) ordinary demand in all and low green duration allocated to links crossed by the bus.

(3) high demand in all and ordinary green duration allocated to links crossed by the bus.

(4) high demand in all and low green duration allocated to links crossed by the bus.

Group 1 The goal is to see the progress of the bus line 1 in the network for different scenarios of demand and green durations of traffic light crossed by the bus. The results are presented in the figure 14 which underlines the position of the bus of line 1 in the network.

The examination of the curve shows these points :

(1) For a same traffic demand (scenarios $(1, 2)$ and $(3, 4)$), the allocated green duration have a great impact on the progress of bus line 1. For scenario 1 and at the end of the 4^{th} cycle, the bus is at the position of about $1025m$ while for the scenario 2, the position is about $470m$.

(2) For a same green duration (scenarios $(1, 3)$ and $(2, 4)$), a higher traffic demand penalizes more the progess of the bus in the network. For the scenario 1, the bus reaches the position of $1025m$ while for scenario 3,

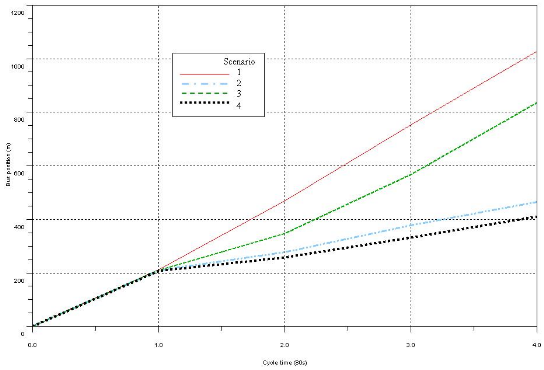

Fig. 14. Comparison between the scenarios for bus line 1 and for group simulation 1

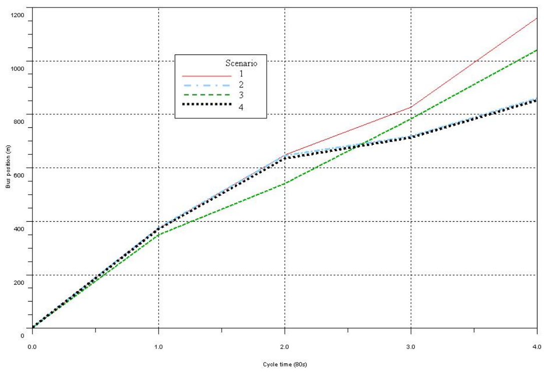

Fig. 15. Comparison between the scenarios for bus line 2 and for group simulation 1

the bus can not go beyond $830m$.

For the line 2, the bus position is given, for the 4 scenarios, in the figure 15.

Contrary to the bus line 1, the traffic demand has small impact on the progress of bus line 2 (scenarios $(1, 3)$ and $(2, 4)$). This can be explained by the fact that even with a high demand, the traffic remains smooth along the route of bus line 2. The comparison between scenarios $(1, 2)$ and even between scenarios $(3, 4)$ shows that penalizing the bus line 1 by allocating lower green durations penalize also the progress of bus line 2. This is due to the fact that bus line 2 and bus line 1 have in common a portion of their routes.

Group 2 The simulations are exactly the same as for the group 1 but now with the bus line 2. The results are in the figure 16 which underlines the position of the bus of line 2 in the network.

For the line 1, the bus position is given, for the 4 scenarios, in the figure 17.

The analysis of the curve of figure 16 confirms that the traffic demand have a small impact on the progress of bus line 2 (scenarios $(1, 3)$ and scenarios $(2, 4)$). It confirms also the great impact of the green durations on the the progress of the bus.

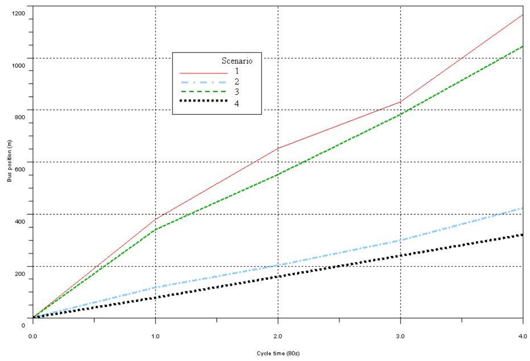

Fig. 16. Comparison between the scenarios for bus line 2 and for group simulation 2

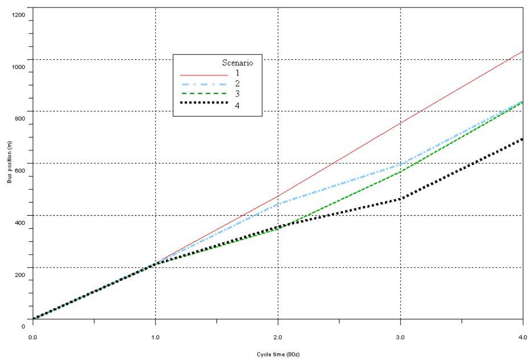

Fig. 17. Comparison between the scenarios for bus line 1 and for group simulation 2

The simulation tests carried out in an urban network composed of 16 junctions show that the public transport vehicle model is consistent with the real behavior of public transport vehicles. It confirm that the whole model (private and public transport vehicles) can be used as a predictor for a traffic control strategy because of the low calculation time.

6. CONCLUSION

The paper presents a predictive multimodal model of urban traffic involving two modes : private vehicle mode and public transport vehicle mode. The private vehicle model is based on the model used in TUC strategy and its efficiency is proved by many authors. The public transport vehicle model is recent and innovative. The simulation results focus only on the public transport mode and show that the model is consistent and efficient. This model reconcile accuracy and simplicity.

The multimodal urban traffic model will be used, in future, as a predictor for a Model Predictive Control policy. This control policy have the challenging aim of regulation of the urban traffic for both modes : private vehicles and public transport vehicles.

REFERENCES

Allsop, R.E. Optimization of timings of traffic signals. In AIRO conference, pages 103-120, 1983.

Barrière, J.F., J.L. Farges and J.J. Henry. Decentralization vs Hierarchy in Optimal Traffic control. In 5^{th} IFAC, IFIP, IFORS Coference on Control in Transportation Systems, pages 321-326, 1986.

Bertsekas, D.P. Dynamic programming and stochastic control. Academic press, 1976.

Bhouri, N. and P. Lotito. Régulation du trafic urbain multimodal avec priorité pour les transports en commun. MOSIM'06, 2006.

Diakaki, C., M. Papageorgiou and K. Aboudolas. A multivariable regulator approach to traffic-responsive network-wide signal control. In Control Engineering Practice, Pergamon, pages 183-195, 2002.

Di Taranto and V. Mauro. UTOPIA. In IFAC Control, Computers, Communications in Transportation, pages 575-597, 1989.

Dotoli, M., M.P. Fanti and C. Meloni. A signal timing plan formulation for urban traffic control. In Control Engineering Practice 14, Elsevier, pages 1297-1311, 2005.

Garcia, Prett and Morari. Model predictive control: theory and practice. In Automatica, pages 335-348, 1989.

Gazis, D.C. Optimum control of a system of oversaturated intersections. In Operation Research, 12, pages 815-831, 1964.

Gazis, D.C. and R.B. Potts . The oversaturated intersection. In Proceedings of the second international symposium on traffic theory, pages 221-237, 1963.

Heydecker, B.G. and I.W. Dudgeon. Calculation of signal settings to minimize delay at a junction. In Transportation and Traffic theory, Elsevier, pages 159-178, 1987.

Kachroudi, S. and N. Bhouri. Communication interprocessus entre Dynasim et Scilab pour l'évaluation d'une stratégie de régulation du trafic urbain multimodal. Congress ATEC, 2007.

Mayne and Michalska. Receding horizon control of nonlinear systems. In IEEE Transactions on Automatic Control, 35, pages 814-824, 1990.

Papageorgiou, M. Traffic control. In Handbook of transportation engineering, R.H Hall, Editor, Kluwer Academic Publishers, in press, 1999.

Papageorgiou, M. Application of automatic control concepts to traffic flow modeling and control. Springer-Verlag, 1983.

Rao, C.V. Moving Horizon Strategies for the Constrained Monitoring and Control of Nonlinear Discrete-Time Systems. Dissertation, University of Wisconsin Madison, 2000.

Scemama, G. La conception de plan de feux : une modélisation par la programmation sous contraintes. In Recherche-Transports-Sécurité(45), pages 3-14, 1994.

Stamatiadis, C. and N.H. Gartner. Multiband-96 : A program for Variable-Bandwidth Progression Optimization of Multiarterial Traffic Networks. In Transportation Research Record 1554, pages 9-17, 1996.

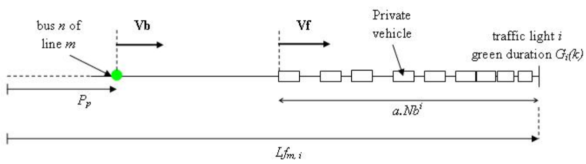

Fig. A.1. A bus line reaching the end of queue vehicle

Appendix A. CALCULATION OF Y AND PROOF OF $Y \geq 0$

Firstly, let's illustrate by a figure the situation that leads to the calculation of Y.

The bus speed, known and fixed, is Vb and the speed of disappearing of the queue is Vf. According to the store-and-forward model, the number of private vehicle exiting the link over the cycle k is $S \cdot G_i(k)$. Therefore, the distance that disappears from the queue, over the duration cycle C, is $a \cdot S \cdot G_i(k)$. Thus one can easily concludes that $Vf = \frac{a \cdot S \cdot G_i(k)}{C}$.

This Appendix aims to calculate the time Y corresponding to the time when the bus reaches the queue vehicle. Y have to satisfy the simple equation $P_p + Vb \cdot Y = Lf_{m,i} - a \cdot Nb^i + Vf \cdot Y$. This equation gives the final expression of Y :

$$Y = C \cdot \frac{Lf_{m,i} - P_p - a \cdot Nb^i}{C \cdot V_b - a \cdot S_i \cdot G_i(k)}$$

$Y \geq 0$ if $Vb \geq \frac{a \cdot S \cdot G_i(k)}{C}$. But knowing that $G_i(k) \leq C$, $a \cdot S \cong 4$ and $Vb \succ 5$, it is easy to verify that $Vb \geq \frac{a \cdot S \cdot G_i(k)}{C}$ and so $Y \geq 0$.