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New hybrid model and switched PI observer for dry friction systems

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Abstract: In this paper, a polytopic approach is used to derive a new hybrid model of systems submitted to dry friction. The principal characteristics of the proposed approach are that it is easily comprehensible, has few parameters, allows the adjustment of the model complexity to the treated case, models the stick-slip phenomena, and has low simulation time. The proposed new dry friction model is applied to the modeling of a real experimental mechanical system. The model parameters are obtained using an adequate position control which is based on a controller with very low bandwidth. To estimate the states of such system, a polytopic PI observer is proposed using a H_∞ formulation. Its performance and robustness against model uncertainty are shown in simulation.

Keywords: Hybrid system, dry friction, switched system, polytopic observer, robust synthesis.

1. INTRODUCTION

Mechanical friction is present in most motion systems. It leads to lower precision and limit cycles may appear, particularly when dry friction is present. When models take into account Coulomb's force and Stribeck's effect, they catch the principal behavior of dry friction; see Olsson et al. [1998]. Such models are often implemented using the sign function of speed, then the simulation time can be very large when speed is near 0 because of the discontinuity of the sign function at null speed. More sophisticated models as Dahl's or LuGre models have been developed to account for other frictions phenomena as presliding displacement or stick-slip motion. Unfortunately, such models need a lot of parameters that are not easily obtained experimentally; see Canudas de Wit et al. [1995]. In most applications the friction parameters change with operating point and/or with wear, then the initial extreme precision of the model is not of major importance. Moreover, in many industrial applications the used position sensor doesn't permit to detect the presliding displacement. Then, it is of a real interest to have a simple basic model easily identifiable whose precision can be improved according to the application.

As said before, the dry friction parameters may change with time, then it is necessary to synthesize control laws that are robust with respect to uncertainties of the model. Many works have been done on control laws for such systems; see Ha et al. [2000], Vivas et al. [2002]. The main conclusion is the need of a good estimation of the system state, i.e the system motion or motionless ; see Friedland and Mentzelopoulou [1992].

In this paper, a polytopic approach is used to derive a new model for systems with dry friction, view as hybrid systems. The obtained model is simpler than the LuGre model with also low simulation time, but needs less parameters and could be easily improved as desired. To illustrate the proposed modeling, a real mechanical system submitted to dry friction is modeled. A simple approach is given to find its Coulomb's friction level which, with the used experimental system, depends on the mechanical position. The Stribeck effect is also taking into account.

In order to estimate the states of the system, the hybrid nature of the proposed model is used to design, using H_∞ synthesis, a switched PI (Proportional Integral) observer robust to model uncertainties, see Koenig [2005], Koenig et al. [2007b]. Recall that an hybrid dynamic system consists of a family of linear time-invariant subsystems and a switching law between them. Sun and Ge [2005] present a survey on basic problems in switched system stability and design. Most of the contributions in this field deal with stability analysis and control synthesis, see Daafouz et al. [2002], Lieberzon and Morse [1999]. Also, many works propose robust observers Takahashi and Peres [1996], Jabbari and Schmitendorf [1991], Magdi et al. [1999] but references about switched polytopic observers are more rarely, see El Messoussi et al. [2006], Nouailletas et al. [2007], Koenig et al. [2007a].

The paper is organized as follows. Section 2 presents the new model. Section 3 introduces the studied experimental system. The observer design is developed in section 4 and simulation results are given in section 5. Finally, section 6 concludes the paper.

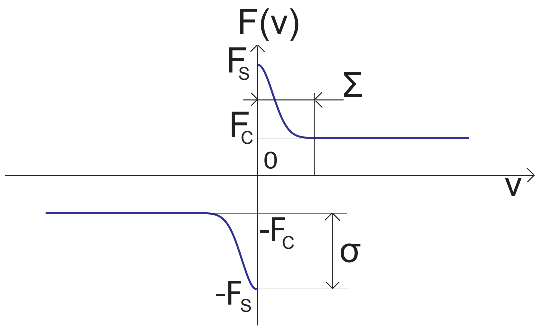

Fig. 1. Typical dry friction function accountig for Coulomb's force and Stribeck effect

Notation 1. $(\cdot)^T$ stands for transpose matrix and $(\cdot) > 0$ denotes a symmetric positive definite matrix. I represents the identity matrix with appropriate dimensions.

2. A NEW MODEL FOR DRY FRICTION

A single-mass system with dynamics (1) is considered where x is the position and v the speed. It is submitted to an input force $u(t)$ and to friction represented by the force F which depends on speed, position,... and parameters as wear, Coulomb's friction level, Stribeck velocity... see Canudas de Wit et al. [1995].

$$\begin{array}{c} \dot{x}(t) = v(t) \\ m\dot{v}(t) = u(t) - F(t, v, x, ...) \end{array} \qquad (1)$$

The genesis of the proposed model for systems with dry friction starts from a simple report that is that a mechanical system has just two operating modes: it moves or it is motionless. So it can be seen as a state machine with two states: state 0 where the system is motionless with model (2), and state 1 where the system moves following (1). Equations (2) force the speed to converge quickly to zero. The pole p has to be chosen by the designer much more faster than the fastest dynamic of the system.

$$\begin{array}{c} \dot{x} = v \\ m\dot{v} = -pv,\ p \longrightarrow +\infty \\ F = u \end{array} \qquad (2)$$

It is also necessary to define how this hybrid system commutes between its two states. For that, the force F is first described. Later on, we will see that, with the used experimental system, the force dependence on the system position has to be taken into account. For now, the simpler case of figure (1) is considered where the dry friction force accounts for Coulomb's and Stribeck's effects: when the absolute value of the speed is less than Σ, the system is considered into state 0 (motionless) governed by (2). In this state, the unique way to switch to state 1 (motion) is that the absolute value of the input $|u(t)|$ becomes superior to F_s. After switching to state 1, the system stays in this one as long as the speed absolute value $|v(t)|$ is not less or equal to Σ or the input $|u(t)|$ is superior to F_c. Figure (2) summarizes this state machine. F is obtained experimentally as it will be shown in section 3. The model parameters Σ and σ are chosen to fit the observed experimental stick-slip phenomena.

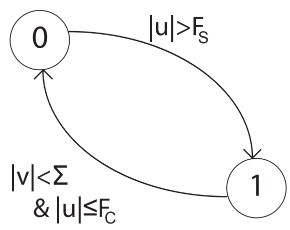

Fig. 2. State machine of dry friction

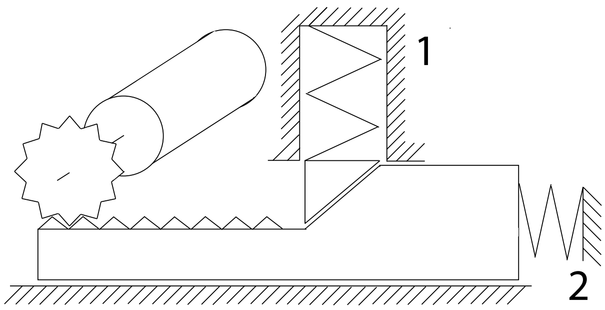

Fig. 3. Kinematic diagram of the studied system

The principal characteristics of the proposed approach to model dry friction are that the model is easily comprehensible, has few parameters (Σ, σ), allows the adjustment of the model complexity to the treated case (in next section the model takes into account the dry friction dependence to mechanical position), models the stick-slip phenomena, and has low simulation time since the *sign* function is not used.

3. MODEL OF THE STUDIED SYSTEM

Figure (3) gives the kinematic diagram of our mechanical system. It is composed of an electrical motor, a toothed rack and two springs. The spring 1 presses the toothed rack tangentially, spring 2 presses it via a piece of metal with a form of bevel. The bevel is designed to compensate the both springs: when spring 1 is compressed, spring 2 is uncompressed and conversely. Theoretically the resistant torque seen by the motor should be null, but due to dry friction, a residual torque, function of the position, is present. Instead of modeling all the parts of the system, a more practical approach has been used which consists in putting in F all the system dry friction depending on system position.

The dry friction function shown in figure (1) is difficult to identify experimentally. First, only the Coulomb's force F_c is identified. For that, a control law with very low bandwidth has been designed and a reference position as described in figure (4) has been applied to the system. With the chosen reference position, the speed of the system is very small and almost constant. So, the viscous force, $fv(t)$, and the acceleration can be neglected. Then, the control input u is supposed to be equal to dry friction F. Figure (5) gives the result of the experimentation and an approximation by a polynomial function in each case

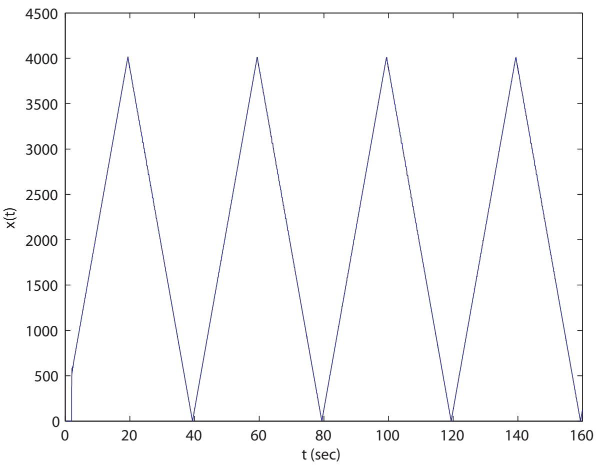

Fig. 4. Reference position to estimate dry friction

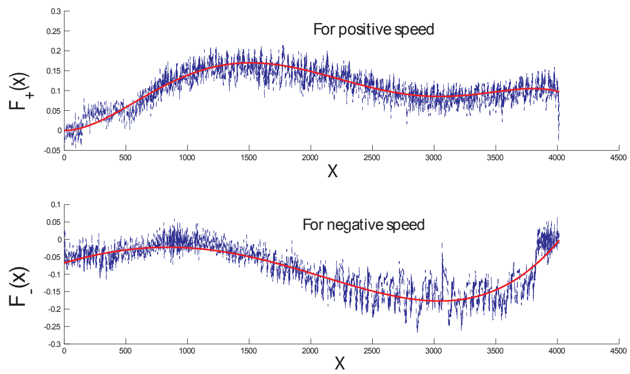

Fig. 5. The measured (dashed) and estimated (solid) Coulomb's resistant torques

(positive and negative speeds). Finally, the system model in moving state 1 is given by the following equations:

$$\begin{gathered}\dot{x} = v \\ m\dot{v} = u - fv - F(x, v) \\ F_+(x) = F(x, v > 0) = F_c(x, v > 0)\left(\sigma e^{-\left(\frac{v-\Sigma}{v_s}\right)^2} + 1\right) \\ F_-(x) = F(x, v < 0) = F_c(x, v < 0)\left(\sigma e^{-\left(\frac{v+\Sigma}{v_s}\right)^2} + 1\right)\end{gathered} \tag{3}$$

where v_s and σ are parameters to tune the Stribeck effect.

The system is now defined by two states with equations (2) and (3), $x \in [0, x_{\max}]$ and $v \in [v_{\min}, v_{\max}]$ are respectively the position and the speed of the rotor of the motor. For the moving state, the dry friction force depends on the speed and the position: if the speed is positive (negative) $F(x, v)$ is equal to $F_+(x)$ $(F_-(x))$. Finally, the system is an hybrid one with 3 sub-models. To select the activated model at time t, a scalar i is used: when $i = 0$ the motionless model is used, when $i = 1(-1)$, the positive (negative) motion model is active.

To validate the identification method, the reference position shown in figure (4) and the same low bandwidth control law are applied to the model. If parameter Σ is chosen very low and no Stribeck effect is considered (i.e. $\sigma = 0$) then estimated curves of figure (5) are obtained. After adjusting parameter Σ and including the Stribeck effect in the model, i.e $\sigma > 0$ and $v_s \simeq \Sigma$, the excellent results of figure (6) are obtained.

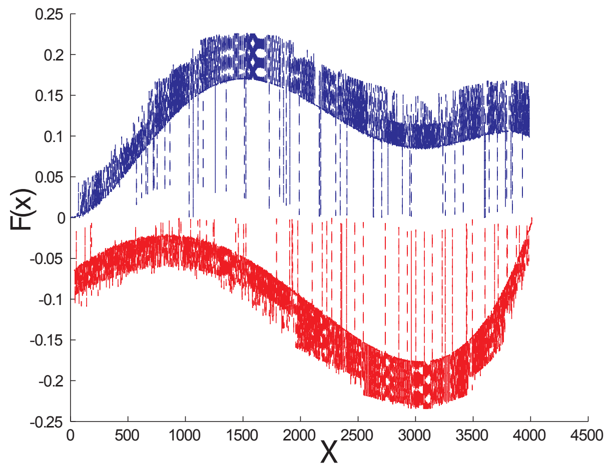

Fig. 6. Residual resistant torques obtained in simulation

Now, a discrete-time model (4) of (2) and (3) is derived using the Euler approximation with sampling time $T_e = 1ms$. This model will be used to design a switched discrete-time observer of the system in order to estimate in real time, only from the position measurement, the system motion or motionless, and its speed.

$$\begin{gathered}X(k+1) = A_i X(k) + B_i u(k) + Q_i(X(k)) \\ y(k) = CX(k)\end{gathered} \tag{4}$$

where

$$X(k) = \begin{pmatrix} x(k) \\ v(k) \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$A_1 = A_{-1} = \begin{pmatrix} 1 & T_e \\ 0 & 1 - \frac{T_e}{m} f \end{pmatrix}, A_0 = \begin{pmatrix} 1 & T_e \\ 0 & 1 - T_e p \end{pmatrix}$$

$$B_1 = B_{-1} = \begin{pmatrix} 0 \\ \frac{T_e}{m} \end{pmatrix}, B_0 = \begin{pmatrix} 0 \\ \frac{T_e}{m} \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 0 \\ \frac{TeF_+(x(k))}{m} \end{pmatrix}, Q_{-1} = \begin{pmatrix} 0 \\ \frac{TeF_-(x(k))}{m} \end{pmatrix},$$

$$Q_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

4. PROPORTIONAL INTEGRAL OBSERVER DESIGN

4.1 Problem formulation

Consider a switched discrete-time system with model uncertainty described by:

$$\begin{gathered}X(k+1) = A_{\beta(k)} X(k) + B_{\beta(k)} u(k) + Q_{\beta(k)}(X(k)) \\ y(k) = C_{\beta(k)} X(k)\end{gathered} \tag{5}$$

where $X(k) \in \mathbb{R}^l$ is the state vector, $y(k) \in \mathbb{R}^m$ is the output vector. The signal $u(k) \in \mathbb{R}^p$ is the input vector and $A_{\beta(k)}, B_{\beta(k)}, C_{\beta(k)}$ are known matrices with appropriate dimensions. The non-linear function $Q_{\beta(k)}$ is rewritten as:

$$Q_{\beta(k)}(X(k)) = \bar{Q}_{\beta(k)}(X(k)) + \tilde{Q}_{\beta(k)} \Delta(X(k))$$

where $\bar{Q}_{\beta(k)}$ is a known non-linear function of $X(k)$, $\tilde{Q}_{\beta(k)}$ is a constant matrix with appropriate dimensions, and $\Delta(X(k))$ is an unknown state-varying uncertainty. As $X(k)$, $\Delta(X(k))$ varies with time so it will be noted $\Delta(k)$. $\beta(k)$ is a piecewise constant switching signal taking values from a finite index set $\varepsilon = \{1, 2, ..., n\}$. We assume that the switching time sequence is real-time accessible, depending on the input or on the measured output, or using a finite automation or any strategy. $\{(A_i, B_i, C_i, Q_i) : i \in \varepsilon\}$ are a family of matrices parameterized by an index set $\varepsilon = \{1, 2, ..., n\}$. Moreover at time k, $\beta(k) = i$ means that the matrices (A_i, B_i, C_i, Q_i) are activated. For the rest of the paper we note $\beta(k) = i$ and $\beta(k+1) = j$, so for an easier reading, (5) becomes:

$$\begin{aligned} X(k+1) = A_i X(k) + B_i u(k) + \bar{Q}_{\beta(k)}(X(k)) \\ +\tilde{Q}_{\beta(k)}\Delta(X(k)) \\ y(k) = C_i X(k) \end{aligned} \tag{6}$$

This type of linear parameter variation (LPV) discrete-time system with state-space realization is developed in many references as Millerioux et al. [2004]. In Nouailletas et al. [2007], the system is submit to an uncertain linear function on $X(k)$. Here, we assume that Q_i is a continuous non-linear function on $X(k)$, which can be discontinuous when the system switches. This means that $Q_i(X(k))$ can be different from $Q_j(X(k+1))$ when T_e goes to 0.

4.2 Switched observer with integral effect

Our objective is to design a robust observer which estimates a linear combination of the state vector $X(k)$. More precisely, it should minimize: $z(k) = \bar{C}_i\left(X(k) - \hat{X}(k)\right)$, where $\hat{X}(k)$ is the estimated state and $\bar{C}_i$ is a matrix chosen by the designer. The full order proportional integral switched observer is constructed in the following form:

$$\begin{aligned} \hat{X}(k+1) = A_i\hat{X}(k) + B_i u(k) + \bar{Q}_i(X(k)) + B_i w(k) \\ +L_i\left(y(k) - C_i\hat{X}(k)\right) \\ w(k+1) = w(k) + G_i\left(y(k) - C_i\hat{X}(k)\right) \end{aligned} \tag{7}$$

where L_i and G_i are respectively the proportional and integral observer gains of the current model i. We assume in a first time that the mode i of system (6) is known. Applying the PI observer (7) to the system (6) and with the estimation error $e(k) = X(k) - \hat{X}(k)$, the dynamic of state estimation error becomes:

$$\begin{aligned} e(k+1) = (A_i - L_iC_i)e(k) + B_i w(k) + \tilde{Q}_i\Delta(k) \\ w(k+1) = w(k) + G_iC_ie(k) \end{aligned} \tag{8}$$

To simplify the notations, (8) could be written:

$$e_a(k+1) = (A_{ai} - L_{ai}C_{ai})e_a(k) + \tilde{Q}_{ai}\Delta(k) \tag{9}$$

where

$$e_a(k) = \begin{pmatrix} e(k) \\ w(k) \end{pmatrix}, A_{ai} = \begin{pmatrix} A_i & B_i \\ 0 & 1 \end{pmatrix}, \tag{10}$$

$$C_{ai} = \begin{pmatrix} C_i & 0 \end{pmatrix}, \tilde{Q}_{ai} = \begin{pmatrix} \tilde{Q}_i \\ 0 \end{pmatrix}, L_{ai} = \begin{pmatrix} L_i \\ G_i \end{pmatrix} \tag{11}$$

For H_∞ formulation, we need to define the output $z_a(k)$:

$$z_a(k) = \bar{C_{ai}}e_a(k), \bar{C_{ai}} = \begin{pmatrix} \bar{C}_i & 0 \end{pmatrix} \tag{12}$$

We can now formulate the observer synthesis problem as follows:

Problem 1. The H_∞-observer problem is to find the gain L_{ai} of the observer (7) such that the following specifications are obtained:

- S_1: the state estimation error $e_a(k)$ is globally asymptotically stable when $\Delta(k) = 0$
- S_2: the error $z(k) = \bar{C}_i\left(X(k) - \hat{X}(k)\right)$ guarantees, under zero-initial condition, $\|z(k)\|_2 \le \gamma\|\Delta(k)\|_2$ for all nonzero $\Delta(k) \in l_2[0, \infty)$ and a given positive constant γ.

Theorem 1. For $i, j \in \{1, 2, ..., n\}$ (where the couple (i, j) represents all the possible commutations of the system), if all the pairs (A_{ai}, C_{ai}), $i \in \varepsilon$ are detectable, and if there exists a scalar $\gamma > 0$ such that the following LMI in $P_i \in \mathbb{R}^{l\times l} > 0$ and $V_i \in \mathbb{R}^{l\times m} > 0$ is feasible for all couples (i, j)

$$\begin{pmatrix} -P_i + \bar{C_{ia}}^T\bar{C_{ia}} & 0 & A_{ai}^TP_i - C_{ai}^TV_i \\ * & -\gamma^2 I & \tilde{Q}_{ai}^TP_i \\ * & * & P_j - 2P_i \end{pmatrix} < 0 \tag{13}$$

then observer (7) for system (6) guaranteeing S_1 and S_2 exists and the observer gains are given by $L_{ai} = V_i^TP_i^{-1}$.

The two next lemmas are recalled for the proof of Theorem 1.

Lemma 1. Boyd et al. [1994] Let $Q > 0, S > 0$ and R be given matrices. Then the following statements are equivalents:

$$a: \begin{pmatrix} Q & R \\ R^T & S \end{pmatrix} > 0 \tag{14}$$

$$b: Q - RS^{-1}R^T > 0$$

$$c: S - R^TQ^{-1}R > 0$$

Lemma 2. Let $R > 0$ and $S > 0$ be given matrices and the following quadratic inequality: $(R - S)S^{-1}(R - S) \ge 0$. A simple development yields:

$$-RS^{-1}R \le S - 2R$$

Proof 1. In order to establish sufficient conditions for existence of (7) according to specifications S_1 and S_2, we should verify the following inequality:

$$\begin{aligned} e_a^T(k+1)P_je_a(k+1) - e_a^T(k)P_ie_a(k) \\ +z_a^T(k)z_a(k) - \gamma^2\Delta^T(k)\Delta(k) < 0 \end{aligned} \tag{15}$$

where P_i and P_j are two positive definite matrices. From (15) and with expression (9), it comes:

$$\begin{pmatrix} (A_{ai} - L_{ai}C_{ai})^T P_j (A_{ai} - L_{ai}C_{ai}) - P_i + \bar{C_{ia}}^T\bar{C_{ia}} & (A_{ai} - L_{ai}C_{ai})^T P_j\tilde{Q}_{ai} \\ * & \tilde{Q}_{ai}^TP_j\tilde{Q}_{ai} - \gamma^2 I \end{pmatrix} < 0 \tag{16}$$

Applying the Schur's complement ($b \to a$), inequality (16) becomes:

$$\begin{pmatrix} -P_i + \bar{C_{ia}}^T\bar{C_{ia}} & 0 & (A_{ai} - L_{ai}C_{ai})^T \\ * & -\gamma^2 I & \tilde{Q}_{ai}^T \\ * & * & -P_j^{-1} \end{pmatrix} < 0 \tag{17}$$

Now, pre- and post-multiply the above inequality (17) by $diag\{I, I, P_i\}$, we obtain:

$$\begin{pmatrix} -P_i + \bar{C_{ia}}^T \bar{C_{ia}} & 0 & (A_{ai} - L_{ai}C_{ai})^T P_i \\ * & -\gamma^2 I & \tilde{Q}_{ai}^T P_i \\ * & * & -P_i P_j^{-1} P_i \end{pmatrix} < 0 \quad (18)$$

According to Lemma 2, (18) could be majorized by :

$$\begin{pmatrix} -P_i + \bar{C_{ia}}^T \bar{C_{ia}} & 0 & A_{ai}^T P_i - C_{ai}^T L_{ai}^T P_i \\ * & -\gamma^2 I & \tilde{Q}_{ai}^T P_i \\ * & * & P_j - 2P_i \end{pmatrix} < 0 \quad (19)$$

Let $V_i = L_{ai}^T P_i$ to obtain expression (13).

5. SIMULATION RESULTS

The switched observer (7) has been designed from model (4). For its tuning, it is supposed that for both negative and positive speeds, the uncertainty on F_- and F_+ are bounded by $\tilde{Q}_i$ which is a tuning parameter. So the observer has just two sub-models. Moreover, the observer is tuned without Stribeck effect. Finally, the H_∞ problem is solved to obtain observer gains L_{ai}.

In order to validate the proposed observer and its robustness, simulations have been carried out. The system is simulated with its switched continuous time model with functions F_- and F_+ including the Stribeck effect. To test the robustness of the observer, it does not take into account the Stribeck effect and the functions F_- and F_+ used by the observer are different from those of the system as it is shown in figure (7). Moreover, the parameter Σ is inferior to the one of the system.

The observer inputs are u, the system control input, and x, the measured position. So it has to estimate at each T_e the system state (motion or motionless). In the case where the commutation rate of the system is faster than $1/T_e$, the observer will not detect it.

The tests are made in closed loop (with the control law of section 3). A reference position filtered by a butterworth filter with a band-width of $20rad/s$ is applied, see figure 8. The sequence is divided in two parts: firstly a sequence of crawls and after 4s, a cycle of $2s$ period steps. The observer is initialized with a position at 1000, a null speed and null integral state, while the system position and speed are zero.

Figures (8), (9), (10) and (11) give simulation results. Despite some commutation errors, the observer works well and converges quickly to the right values. This robustness is due to the robust synthesis method and the integral effect which compensates the average error of the resistant torque estimation. The commutation errors are due to the differences between the parameters of the system and those of the observer.

6. CONCLUSION

This paper has presented a new model for dry friction. It is considered that systems with dry friction are hybrid systems. The main advantages of the proposed modeling are: easy implementation, low computation time, model versatility. Moreover a switched PI-observer robust to

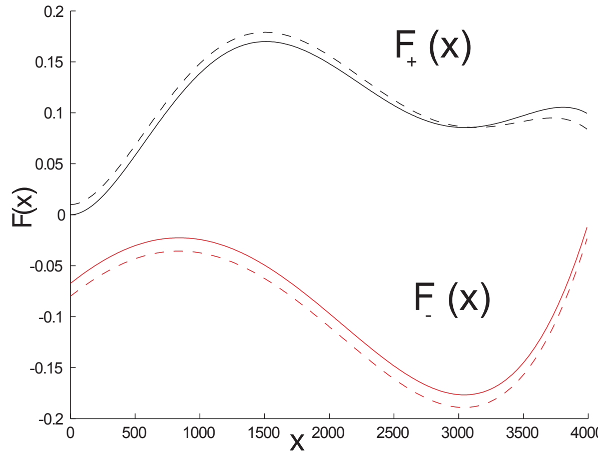

Fig. 7. The function F used by the system (solid) and the one used by the observer (dashed)

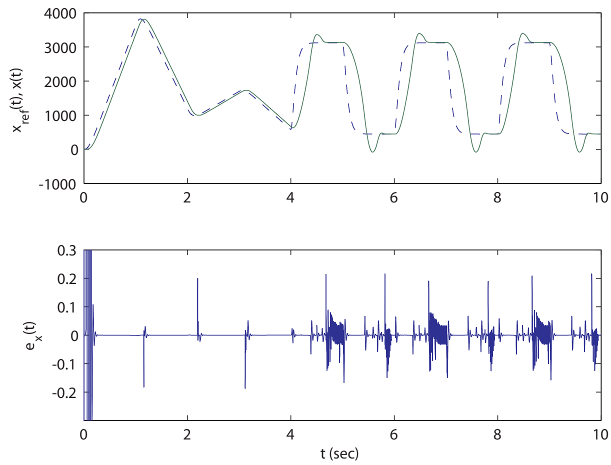

Fig. 8. Simulation result: $x(t)$ (solid), x_{ref} (dashed) and the position estimation error

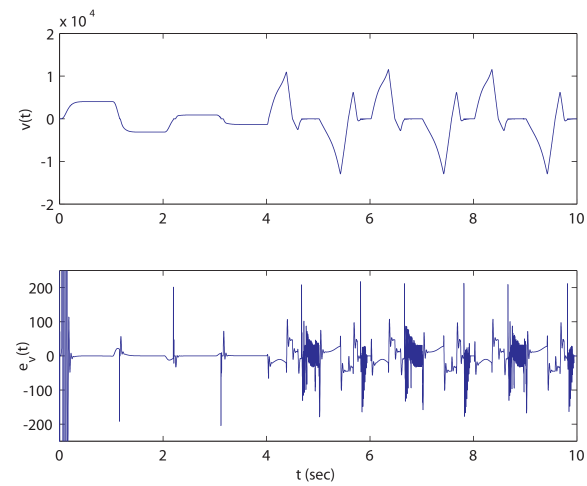

Fig. 9. Simulation result: $v(t)$ and the speed estimation error

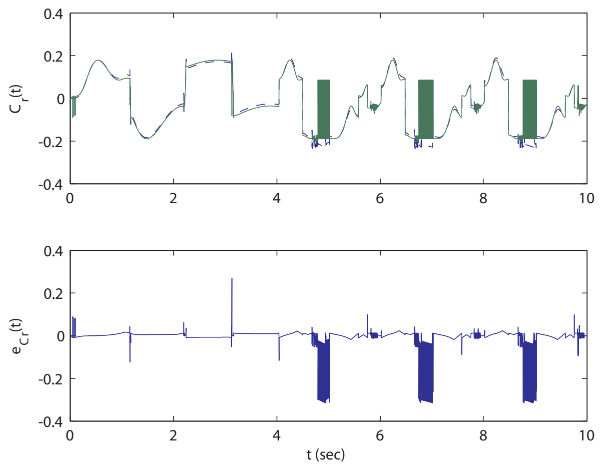

Fig. 10. Simulation result: $F(t)$ (dashed), $F(t)$ estimated (dashed) and the resistant torque error

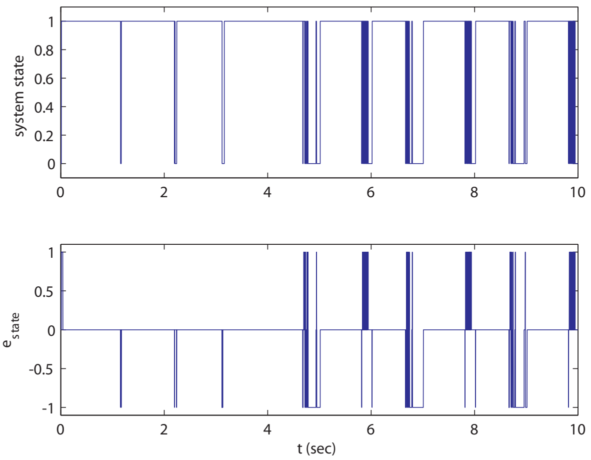

Fig. 11. Simulation result: State of the system and error of commutation between the system and the observer

model uncertainties has been developed. Existing conditions of such observer have been given and proved with a H_∞ formulation. Based on a real experimental system, the efficiency of the proposed modeling method for dry friction has been shown. Work is in progress to validate experimentally the designed observer.

7. ACKNOWLEDGEMENT

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