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H_∞ Tracking with Preview for Linear Systems with Impulsive Effects -State Feedback and Full Information Cases-

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Abstract: In this paper we study H_∞ tracking problems with preview by state feedback and full information for linear systems with impulsive effects on the finite time interval. We mainly consider the case that our systems are affected by discrete disturbance at jump instants. We mainly consider the problem that the reference signals are previewed in a fixed time interval and present state feedback and full information control laws for the H_∞ tracking problems. Our theory can be applied into robust H_∞ tracking problems with preview considering upper bounded uncertainties.

1. INTRODUCTION

It is well known that, for the design of tracking control systems, the preview information of reference signals is very useful for improving the performance of the closed-loop systems, and recently much work has been done for preview control systems. Considering the effect of modelling uncertainties or disturbance is also very important on preview control theory. U. Shaked et al. have studied the H_∞ tracking theory with preview for continuous- and discrete-time systems by the game theoretic approach ([1][2][5]).

Control theory for linear systems with impulsive effects (or linear jump systems), which contain linear continuous and discrete time systems, can be widely applied, for example, to mechanical systems, ecosystems, chemical processes, finantial engineering and so on. It has been reseached in detail by A. Ichikawa and H. Katayama([3]). Their theory can be also applied into the sampled-data control system with the control input realized through a zero-order hold and the sampled-observation.

In this paper we study the H_∞ tracking problems with preview in the state feedback and full information settings for linear systems with impulsive effects (or linear jump systems). Our systems are described by the ordinary differential equations with impulsive effects. We consider two different tracking problems according to the preview information structure and give the control strategies for them respectively. Our theory can be applied into the control system with the control input realized through a zero-order hold. Our theory can be also easily reduced to the case that only the preview information of a discrete reference signal is available, or the case that only whether the continuous input or the impulsive input is available.

2. PROBLEM FORMULATION

Consider the following linear system with impulsive effects.

$$\begin{aligned}
\dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B_2(t)u_c(t) + B_3(t)r_c(t) \\
&\qquad t \neq k\tau,\ x(0) = x_0 \\
x(k\tau^+) &= A_d(k)x(k\tau) + B_{1d}(k)w_d(k) \\
&\qquad + B_{2d}(k)u_d(k) + B_{3d}(k)r_d(k) \\
z_c(t) &= C_1(t)x(t) + D_{12}(t)u_c(t) \\
&\qquad + D_{13}(t)r_c(t),\ t \neq k\tau \qquad (1) \\
z_d(k) &= C_{1d}(k)x(k\tau) + D_{12d}(k)u_d(k) + D_{13d}(k)r_d(k)
\end{aligned}$$

where $x \in \mathbf{R}^\mathbf{n}$ is the state, $w \in \mathbf{R}^\mathbf{p}$ and $w_d \in \mathbf{R}^{\mathbf{p_d}}$ are the exogenous disturbances, $u_c \in \mathbf{R}^{\mathbf{m_c}}$ and $u_d \in \mathbf{R}^{\mathbf{m_d}}$ are the continuous and impulsive control input, $z_c \in \mathbf{R}^{\mathbf{k_c}}$ and $z_d \in \mathbf{R}^{\mathbf{k_d}}$ are the controlled output, $r_c(t) \in \mathbf{R}^{\mathbf{r_c}}$ and $r_d(k) \in \mathbf{R}^{\mathbf{r_d}}$ are known or mesurable reference signals, x_0 is an unknown initial state. We assume that all matrices are of compatible dimensions. Throughout this paper the dependence of the system matrices on t or k will be omitted for the sake of notation simplification.

The H_∞ tracking problems we address in this paper for the system (1) are to design control laws $u_c(t) \in L_2[0,T]$ and $u_d(k) \in l_2[0,N]$ over the finite horizon $[0,T]$, $N\tau < T < (N+1)\tau$ using the information available on the known parts of the reference signals $r_c(t)$ and $r_d(k)$ and minimizing the sum of the energy of $z_c(t)$ and $z_d(k)$, for the worst case of the initial condition x_0, the disturbances $w(t) \in \tilde{L}_2([0,T];\mathbf{R}^\mathbf{p})$ and $w_d(k) \in \tilde{l}_2([0,N];\mathbf{R}^{\mathbf{p_d}})$. We denote by $\tilde{L}_2([0,T];\mathbf{R}^\mathbf{k})$ and $\tilde{l}_2([0,N];\mathbf{R}^{\mathbf{k_d}})$ the space of nonanticipative signals. Considering the average of the performance index over the statistics of the unknown parts of r_c and r_d, we define the following performance index.

$$\begin{aligned}
J_T(x_0, u_c, u_d, w, w_d, r_c, r_d) &= -\gamma^2 x_0' R^{-1} x_0 \\
&+ \left\{ \sum_{k=0}^{N-1} \int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T} \right\} \mathbf{E}_{\bar{R}_s}\{\|z_c(s)\|^2\} ds \\
&+ \sum_{k=0}^{N} \mathbf{E}_{\bar{R}_k}\{\|z_d(k)\|^2\} - \gamma^2[\|w_d\|_2^2 + \|w\|_2^2] \qquad (2)
\end{aligned}$$

where $N\tau < T < (N+1)\tau$, $R = R' > O$ is a given weighting matrix for the initial state, $\mathbf{E}_{\bar{R}_s}$ and $\mathbf{E}_{\bar{R}_k}$ mean expectations over $\bar{R}_{s+h\tau}$ and $\bar{R}_{k+h}$, h is the preview length of $r_c(t)$ and $r_d(k)$, and $\bar{R}_s$ and $\bar{R}_j$ denote the future information on r_c and r_d at time s and $j\tau$ respectively, i.e.,$\bar{R}_s := \{r_c(l); s < l \le T\}$ and $\bar{R}_j := \{r_i; j < i \le N\}$.

We consider two different tracking problems according to the information structures (preview lengths) of r_c and r_d as follows.

Case a) $\mathbf{H}_\infty$ Fixed-Preview Tracking:
In this case, it is assumed that at the current time t ($k\tau^+ \le t \le (k+1)\tau$), $r_c(s)$ is known for $s \le \min(T, s+h\tau)$ and at the time $k\tau$, $r_d(i)$ is known for $i \le \min(N, k+h)$.

Case b) $\mathbf{H}_\infty$ Tracking of Noncausal $\{r_c(t)$ and $r_d(k)\}$:
In this case, the signals $\{r_c(t)\}$ and $\{r_d(k)\}$ are assumed to be known *a priori* for the whole time intervals $t \in [0^+, T]$ and $k \in [0, N]$.

In order to solve these problem, we formulate the following differential game problems for the system (1) and the performance index (2).

The $\mathbf{H}_\infty$ Tracking Problem by State Feedback:
Find $\{u_c^*\}$, $\{u_d^*\}$, $\{w^*\}$, $\{w_d^*\}$ and x_0^* satisfying the following (saddle point) condition:

$$\begin{aligned} &J_T(x_0, u_c^*, u_d^*, w, w_d, r_c, r_d) \\ &\quad \le J_T(x_0^*, u_c^*, u_d^*, w^*, w_d^*, r_c, r_d) \\ &\qquad \le J_T(x_0^*, u_c, u_d, w^*, w_d^*, r_c, r_d) \end{aligned}$$

where the control strategies $u_c^*(s)$, $0 \le s \le T$ and $u_d^*(k)$, $0 \le k \le N$, are based on the information $R_{s+h\tau} := \{r_c(l); 0 < l \le s+h\tau\}$ and $R_{k+h} := \{r_d(i); 0 < i \le k+h\}$ $(0 \le h \le N)$, and, in the state feedback case, based on the current state, in the full information case, based on the current state and the disturbances.

3. H_∞ TRACKING CONTROLLERS BY STATE FEEDBACK

In this section we present the theory of H_∞ tracking by state feedback.

For the system (1), we assume the following condition.

A1: $D_{12}'D_{12} > O$ and $D_{12d}'D_{12d} > O$

Now we consider the following Riccati equation with jump parts.

$$\dot{X} + A'X + XA + C_1'C_1 + \frac{1}{\gamma^2}XB_1B_1'X - \tilde{S}'\tilde{R}^{-1}\tilde{S} = O,\ t \ne k\tau \tag{3}$$

$$X(k\tau^-) = A_d'X(k\tau)A_d + C_{1d}'C_{1d} + R_1'T_1^{-1}R_1(k) - F_2'V_2F_2(k)\ \ k = 0, 1, \cdots \tag{4}$$

$$T_1(k) > a_X I \text{ for some } a_X > 0 \tag{5}$$

where

$$\begin{aligned} &\tilde{R} = D_{12}'D_{12},\ \tilde{S}(t) = B_2'X(t) + D_{12}'C_1, \\ &T_1(k) = \gamma^2 I - B_{1d}'X(k\tau)B_{1d}, \\ &T_2(k) = D_{12d}'D_{12d} + B_{2d}'X(k\tau)B_{2d}, \\ &S(k) = B_{2d}'X(k\tau)B_{1d}, \\ &R_1(k) = B_{1d}'X(k\tau)A_d, \\ &R_2(k) = D_{12d}'C_{1d} + B_{2d}'X(k\tau)A_d, \\ &V_2(k) = (T_2 + ST_1^{-1}S')(k), \\ &V_3(k) = (B_{2d}' + ST_1^{-1}B_{1d}')(k), \\ &F_2(k) = -V_2^{-1}(k)(R_2 + ST_1^{-1}R_1)(k). \end{aligned}$$

We obtain the following saddle point strategy for our game problem. (Also refer to the case of no any disturbances at jump parts in [4])

Proposition 3.1. Consider the system (1) and suppose **A1**. Then **the $\mathbf{H}_\infty$ Tracking Problem** is solvable **by State Feedback** if and only if there exists a matrix $X(t) > O$ satisfying the conditions $X(0^-) < \gamma^2 R^{-1}$ and $X(T) = O$ such that the Riccati equation (3)(4) with (5) holds over $[0, T]$. A saddle point strategy is given by

$$\begin{aligned} &x_0^* = [\gamma^2 R^{-1} - X(0^-)]^{-1}\theta(0) \\ &w^* = \gamma^{-2}B_1'Xx + \mathbf{C}_\theta\theta \\ &u_c^* = -\tilde{R}^{-1}\tilde{S}x - \mathbf{C}_u r_c - \mathbf{C}_{\theta u}\theta_c \\ &w_d^*(k) = T_1^{-1}(k)[R_1(k)x(k\tau) + S'(k)u_d(k)] \\ &\qquad\qquad + \mathbf{D}_w r_d(k) + \mathbf{D}_{\theta w}\theta(k\tau^+) \\ &u_d^*(k) = F_2(k)x(k\tau) + \mathbf{D}_u(k)r_d(k) + \mathbf{D}_{\theta u}\theta_c(k\tau^+) \end{aligned}$$

where

$$\begin{aligned} &\mathbf{C}_\theta = -\gamma^{-2}B_1',\ \mathbf{C}_{\theta u} = \tilde{R}^{-1}B_2',\ \mathbf{C}_u = \tilde{R}^{-1}D_{12}'D_{13} \\ &\mathbf{D}_w(k) = T_1^{-1}(k)B_{1d}'X(k\tau)B_{3d} \\ &\mathbf{D}_u(k) = -V_2^{-1}(k)(B_{2d}'X(k\tau)B_{3d} + D_{12d}'D_{13d} + S\mathbf{D}_w) \\ &\mathbf{D}_{\theta w}(k) = T_1^{-1}(k)B_{1d}' \\ &\mathbf{D}_{\theta u}(k) = -V_2^{-1}(k)(S(k)\mathbf{D}_{\theta w} + B_{2d}'). \end{aligned}$$

$\theta(t)$, $t \in [0, T]$, satisfies

$$\begin{cases} \dot{\theta}(t) = -\bar{A}_c'(t)\theta(t) + \bar{B}_c(t)r_c(t),\ t \ne k\tau \\ \theta(k\tau) = \bar{A}_d'(k)\theta(k\tau^+) + \bar{B}_d(k)r_d(k) \\ \theta(T) = 0 \end{cases} \tag{6}$$

where

$$\begin{aligned} &\bar{A}_c = A + \frac{1}{\gamma^2}B_1B_1'X - B_2\tilde{R}^{-1}\tilde{S} \\ &\bar{B}_c = -(XB_3 + C_1'D_{13}) + \tilde{S}'\mathbf{C}_u \\ &\bar{A}_d(k) = A_d + \mathbf{D}_{\theta w}'R_1(k) - \mathbf{D}_{\theta u}'V_2F_2(k) \\ &\bar{B}_d(k) = A_d'X(k\tau)B_{3d} + R_1'\mathbf{D}_w \\ &\qquad\qquad - F_2V_2\mathbf{D}_u(k) + C_{1d}'D_{13d} \end{aligned}$$

and $\theta_c(t)$ is the 'causal' part of $\theta(\cdot)$ at time t. This θ_c is the expected value of θ over $\bar{R}_s$ and $\bar{R}_k$ and given by

$$\begin{cases} \dot{\theta}_c(s) = -\bar{A}_c'(s)\theta_c(s) + \bar{B}_c(s)r_c(s),\ s \ne k\tau \\ \qquad t \le s \le t_f \\ \theta_c(j\tau) = \bar{A}_d'(j)\theta_c(j\tau^+) + \bar{B}_d(j)r_d(j), \\ \qquad k < j \le k_f\ (k_f\tau < t_f < (k_f+1)\tau) \\ \theta_c(t_f) = 0 \end{cases} \tag{7}$$

where, for $k\tau^+ \le t \le (k+1)\tau$,

$$\begin{cases} t_f = t + h\tau \text{ and } k_f = k + h \text{ if } (k+h)\tau < T \\ t_f = T \quad \text{and} \quad k_f = N \quad \text{if } (k+h)\tau \geq T. \end{cases}$$

Moreover, the value of the game is

$$J_T(x_0^*, u_c^*, u_d^*, w^*, w_d^*, r_c, r_d) = \left\{ \sum_{k=0}^{N-1} \int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T} \right\} \mathbf{E}_{\bar{R}_s}\{\|\tilde{R}^{1/2}\mathbf{C}_{\theta u}\theta_1(s)\|^2\}ds + \sum_{k=0}^{N} \mathbf{E}_{\bar{R}_k}\{\|\tilde{V}_2^{1/2}\mathbf{D}_{\theta u}(k)\theta_1(k\tau^+)\|^2\} + \bar{J}_c(r_c) + \bar{J}_d(r_d) \tag{8}$$

where $\theta_1(t) = \theta(t) - \theta_c(t),\ t \in [0, T]$,

$$\bar{J}_c(r_c) = \left\{ \sum_{k=0}^{N-1} \int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T} \right\} \mathbf{E}_{\bar{R}_s}\{\delta J_c(r_c) + 2\theta' B_3 r_c + \gamma^2\|\mathbf{C}_\theta \theta\|^2 - 2\theta'\mathbf{C}'_{\theta u}\tilde{R}'\mathbf{C}_u r_c - \|\tilde{R}^{1/2}\mathbf{C}_{\theta u}\theta\|^2\}ds,$$

$$\bar{J}_d(r_d) = \gamma^2\mathbf{E}_{\bar{R}_0}\{\|\theta(0^-)\|^2_{P_0}\} + \sum_{k=0}^{N} \mathbf{E}_{\bar{R}_k}\{\theta'(k\tau^+)\mathbf{D}'_{\theta w}T_1\mathbf{D}_{\theta w}(k)\theta(k\tau^+) + 2\theta'(k\tau^+)\mathbf{D}'_{\theta w}T_1\mathbf{D}_w(k)r_d(k) - \theta'(k\tau^+)\mathbf{D}'_{\theta u}\tilde{V}_2\mathbf{D}_{\theta u}(k)\theta(k\tau^+) - 2\theta'(k\tau^+)\mathbf{D}'_{\theta u}\tilde{V}_2\mathbf{D}_u(k)r_d(k) + 2\theta'(k\tau^+)B_{3d}r_d(k) + J_{d,k}(r_d)\}$$

and $P_0 = [R^{-1} - \gamma^{-2}X(0^-)]^{-1}$.

Next, utilizing the above saddle point strategy for our game problem, we present a solution to each of the three H_∞ tracking problems by state feedback.

Theorem 3.1. Consider the system (1) and suppose **A1**. Then each of **the $\mathbf{H}_\infty$ tracking problems** is solvable **by state feedback** if and only if there exists a matrix $X(t) > O$ satisfying the conditions $X(0^-) < \gamma^2 R^{-1}$ and $X(T) = O$ such that the Riccati equation (3)(4) and the condition (5) hold over $[0, T]$. Moreover, the following results hold using

$$K_{x,c} = -\tilde{R}^{-1}\tilde{S},\ K_{r_c} = -\tilde{R}^{-1}D'_{12}D_{13},$$
$$K_{\theta,c} = -\tilde{R}^{-1}B'_2\ K_{x,d} = F_2(k),$$
$$K_{r_d} = -V_2^{-1}(k)(D'_{12d}D_{13d} + V_3(k)X(k\tau)B_{3d}),$$
$$K_{\theta,d} = -V_2^{-1}V_3(k).$$

Case a) The control law for **the $\mathbf{H}_\infty$ fixed-preview tracking** is

$$u_{s,a,c}(t) = K_{x,c}x(t) + K_{r_c}r_c(t) + K_{\theta,c}\theta_c(t),\ t \neq k\tau$$
$$u_{s,a,d}(k) = K_{x,d}x(k\tau) + K_{r_d}r_d(k) + K_{\theta,d}\theta_c(k\tau^+)$$

with $\theta_c(\cdot)$ given by (7). Moreover, $J_T(x_0^*, u_{s,a,c}, u_{s,a,d}, w^*, w_d^*, r_c, r_d)$ coincides with (8) .

Case b) The control law for **the $\mathbf{H}_\infty$ tracking of noncausal $\mathbf{r_c}(\cdot)$ and $\mathbf{r_d}(\cdot)$** is

$$u_{s,b,c}(t) = K_{x,c}x(t) + K_{r_c}r_c(t) + K_{\theta,c}\theta(t),\ t \neq k\tau$$
$$u_{s,b,d}(k) = K_{x,d}x(k\tau) + K_{r_d}r_d(k) + K_{\theta,d}\theta(k\tau^+)$$

with $\theta(\cdot)$ given by (6) and

$$J_T(x_0^*, u_{s,b,c}, u_{s,b,d}, w^*, w_d^*, r_c, r_d) = \bar{J}_c(r_c) + \bar{J}_d(r_d)$$

since $\theta(t) = \theta_c(t),\ \forall\ t \in [0, T]$.

4. PROOF OF PROPOSITION 3.1

In this section, we describe the proof of Proposition 3.1.

(**Proof of Proposition 3.1**)

Sufficiency: Let $X(t)$ be a solution to (3) over $[0, T]$, $t \neq k\tau$, such that $X(0^-) < \gamma^2 R^{-1}$. By considering (1) and (3), in the genaral case that $\{r_c(\cdot)\}$ is arbitrary but not including any preview information, it can be shown that the following equality holds.

$$\mathbf{E}_{\bar{R}_s}\int_{k\tau^+}^{(k+1)\tau} \frac{d}{ds}\{x'(s)X(s)x(s)\}ds = \mathbf{E}_{\bar{R}_s}\int_{k\tau^+}^{(k+1)\tau} \{\gamma^2[\|w\|^2 - \|w - \gamma^{-2}B'_1Xx(s)\|^2] - \|C_1x + D_{12}u_c + D_{13}r_c\|^2 + \|u_c(s) + \tilde{R}^{-1}\tilde{S}x(s) + \mathbf{C}_u r_c\|^2_{\tilde{R}} - 2x'\bar{B}_c(s)r_c(s) + \delta J_c(r_c)\}ds \tag{9}$$

where $\delta J_c(r_c) = \|D_{13}r_c\|^2 - \|\tilde{R}^{1/2}\mathbf{C}_u r_c\|^2$.

Now

$$2\int_{k\tau^+}^{(k+1)\tau} \mathbf{E}_{\bar{R}_s}\{\frac{d}{ds}(\theta'(s)x(s))\}ds = 2\int_{k\tau^+}^{(k+1)\tau} \mathbf{E}_{\bar{R}_s}\{\dot{\theta}'x + \theta'(\bar{A}_c x + B_1\hat{w} + B_2\hat{u}_c + B_3 r_c)\}ds \tag{10}$$

where $\hat{w} = w - \gamma^{-2}B'_1Xx$ and $\hat{u}_c = u_c + \tilde{R}^{-1}\tilde{S}x$. Adding (10) to (9),

$$\mathbf{E}_{\bar{R}_s}\int_{k\tau^+}^{(k+1)\tau} \{\frac{d}{ds}(x'(s)X(s)x(s)) + 2\frac{d}{ds}(\theta'(s)x(s))\}ds = \mathbf{E}_{\bar{R}_s}\int_{k\tau^+}^{(k+1)\tau} \{\gamma^2[\|w\|^2 - \|\hat{w} - \mathbf{C}_\theta\theta(s)\|^2] - \|C_1x + D_{12}u_c + D_{13}r_c\|^2 + \|\hat{u}_c(s) + \mathbf{C}_u r_c + \mathbf{C}_{\theta u}\theta(s)\|^2_{\tilde{R}} + \delta\bar{J}_c(r_c)\}ds \tag{11}$$

where

$$\delta\bar{J}_c(r_c) = \delta J_c(r_c) + 2\theta' B_3 r_c + \gamma^2\|\mathbf{C}_\theta\theta\|^2 - 2\theta'\mathbf{C}'_{\theta u}\tilde{R}'\mathbf{C}_u r_c - \|\tilde{R}^{1/2}\mathbf{C}_{\theta u}\theta\|^2$$

$$= \delta J_c(r_c) + 2\theta' B_3 r_c + \gamma^{-2}\|B_1'\theta\|^2$$
$$-2\theta' B_2 \tilde{R}^{-1} D_{12}' D_{13} r_c - \|\tilde{R}^{-1/2} B_2'\theta\|^2$$

and we have used the continuous part

$$\dot{\theta} + \bar{A}_c'\theta - \bar{B}_c r_c = 0$$

of the dynamics (6) to get rid of the terms that mix r_c, θ and x.

On the other hand, with respect to jump parts, define

$$\phi_k := x'(k\tau^+)X(k\tau)x(k\tau^+) - x'(k\tau)X(k\tau^-)x(k\tau)$$

We first consider the case that $\{r_d(\cdot)\}$ is arbitrary but not including any preview information. It can be shown that the following equality holds, using the jump parts (4) of the Riccati equation.(cf. [3])

$$\begin{aligned}\mathbf{E}_{\bar{R}_k}\{\phi_k\} = \{&\gamma^2\|w_d(k)\|^2 - \|z_d(k)\|^2 \\ &-\|T_1^{1/2}(k)[w_d(k) - T_1^{-1}(k)\{R_1(k)x(k\tau) \\ &+S'(k)u_d(k)\} - \mathbf{D}_w r_d(k)]\|^2 \\ &+\|V_2^{1/2}(k)[u_d(k) - F_2(k)x(k\tau) - \mathbf{D}_u r_d(k)]\|^2 \\ &+2x'(k\tau)\bar{B}_d(k)r_d(k) + J_{d,k}(r_d)\}\end{aligned}$$

Notice that, in the right hand side of this equality, $J_{d,k}(r_d)$, which means the jump parts of the tracking error not considering the effect of the preview information, is added. In fact,

$$\begin{aligned}J_{d,k}(r_d) = r_d'(k)[&\mathbf{D}_w' T_1 \mathbf{D}_w(k) - \mathbf{D}_u' V_2 \mathbf{D}_u(k) \\ &+B_{3d}' X(k\tau) B_{3d} + D_{13d}' D_{13d}] r_d(k).\end{aligned}$$

Now introducing the vector θ, which can include some preview information of the tracking signals,

$$\begin{aligned}&\mathbf{E}_{\bar{R}_k}\{\phi_k + 2\{\theta'(k\tau^+)x(k\tau^+) - \theta'(k\tau)x(k\tau)\}\} \\ &= \mathbf{E}_{\bar{R}_k}\{\gamma^2\|w_d(k)\|^2 - \|z_d(k)\|^2 \\ &\quad -\|T_1^{1/2}(k)[w_d(k) - T_1^{-1}(k)\{R_1(k)x(k\tau) \\ &\quad +S'(k)u_d(k)\} - \mathbf{D}_w r_d(k)]\|^2 \\ &\quad +\|V_2^{1/2}(k)[u_d(k) - F_2(k)x(k\tau) - \mathbf{D}_u r_d(k)]\|^2 \\ &\quad +2x'(k\tau)\bar{B}_d(k)r_d(k) + J_{d,k}(r_d) \\ &\quad +2\theta'(k\tau^+)(A_d x(k\tau) + B_{1d}w_d(k) + B_{2d}u_d(k) \\ &\quad +B_{3d}r_d(k)) - 2\theta'(k\tau)x(k\tau)\} \\ &= \mathbf{E}_{\bar{R}_k}\{\gamma^2\|w_d(k)\|^2 - \|z_d(k)\|^2 \\ &\quad -\|T_1^{1/2}(k)[w_d(k) - T_1^{-1}(k)\{R_1(k)x(k\tau) \\ &\quad +S'(k)u_d(k)\} - \mathbf{D}_w r_d(k) - \mathbf{D}_{\theta w}\theta(k\tau^+)]\|^2 \\ &\quad +\|V_2^{1/2}(k)[u_d(k) - F_2(k)x(k\tau) - \mathbf{D}_u r_d(k) \\ &\quad -\mathbf{D}_{\theta u}\theta(k\tau^+)]\|^2 \\ &\quad +\bar{J}_{d,k}(r_d)\}\end{aligned} \tag{12}$$

where we have used the jump part

$$\theta(k\tau) = \bar{A}_d'(k)\theta(k\tau^+) + \bar{B}_d(k)r_d(k).$$

of the dynamics (6) to get rid of the terms that mix r_c, θ and x, and $\bar{J}_{d,k}(r_d)$, which means the tracking error including the preview information vector θ, can be expressed by

$$\begin{aligned}\bar{J}_{d,k}(r_d) = &\theta'(k\tau^+)\mathbf{D}_{\theta w}' T_1 \mathbf{D}_{\theta w}(k)\theta(k\tau^+) \\ &+2\theta'(k\tau^+)\mathbf{D}_{\theta w}' T_1 \mathbf{D}_w(k) r_d(k) \\ &-\theta'(k\tau^+)\mathbf{D}_{\theta u}' \tilde{V}_2 \mathbf{D}_{\theta u}(k)\theta(k\tau^+) \\ &-2\theta'(k\tau^+)\mathbf{D}_{\theta u}' \tilde{V}_2 \mathbf{D}_u(k) r_d(k) \\ &+2\theta'(k\tau^+)B_{3d} r_d(k) + J_{d,k}(r_d).\end{aligned}$$

By summing up these quantities ((11)),(12)) from $t = 0$ to $t = T$ piecewise,

$$\begin{aligned}&-\gamma^2 x'(0)R^{-1}x(0) \\ &\quad +\sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\|z_d(k)\|^2 - \gamma^2\|w_d(k)\|^2\} \\ &+\left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\{\|z_c\|^2 - \gamma^2\|w\|^2\}ds \\ &\quad +2\left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\left\{\frac{d}{ds}(\theta' x)\right\}ds \\ &+\sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\phi_k + 2(\theta'(k\tau^+)x(k\tau^+) - \theta'(k\tau)x(k\tau))\} \\ &\quad +\left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\left\{\frac{d}{ds}(x'(s)X(s)x(s))\right\}ds \\ &= -\gamma^2 x'(0)R^{-1}x(0) \\ &\quad +\left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\{-\gamma^2\|\hat{w} - \mathbf{C}_\theta\theta(s)\|^2 \\ &\qquad +\|\hat{u}_c(s) + \mathbf{C}_{\theta u}\theta(s)\|_{\tilde{R}}^2\}ds \\ &\quad +\sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\|\hat{u}_d(k) - \mathbf{D}_{\theta u}(k)\theta(k\tau^+)\|_{\tilde{V}_2(k)}^2 \\ &\qquad -\|\hat{w}_d(k) - \mathbf{D}_{\theta w}\theta(k\tau^+)\|_{T_1(k)}^2\} \\ &\qquad +\bar{J}_c(r_c) + \bar{J}_d(r_d)\end{aligned}$$

where

$$\begin{aligned}\hat{w}_d(k) &= w_d(k) - T_1^{-1}(k)\{R_1(k)x(k\tau) + S'(k)u_d(k)\} \\ &\quad -\mathbf{D}_w r_d(k), \\ \hat{u}_c(t) &= u_c(t) + \tilde{R}^{-1}\tilde{S}x(t) + \mathbf{C}_u r_c(t), \\ \hat{u}_d(k) &= u_d(k) - F_2(k)x(k\tau) - \mathbf{D}_u r_d(k).\end{aligned}$$

Since the left hand side reduces to

$$\begin{aligned}&\left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\{\|z_c(s)\|^2\}ds \\ &\quad +\sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\|z_d\|^2\} \\ &\qquad -\gamma^2[\|w\|_2^2 + \|w_d\|_2^2 + x_0' R^{-1} x_0] \\ &\qquad -2\theta'(0)x(0) - x_0' X(0^-) x_0\end{aligned}$$

considering $X(T) = O$ and $\theta(T) = 0$, we obtain

$$\begin{aligned}
&J_T(x_0, u_c, u_d, w, w_d, r_c, r_d) \\
&= -\gamma^2 \mathbf{E}_{\bar{R}_0}\{\|x_0 - \gamma^{-2}P_0\theta(0)\|^2_{P_0^{-1}}\} \\
&+ \left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\{-\gamma^2\|\hat{w} - \mathbf{C}_\theta\theta(s)\|^2 \\
&\qquad + \|\hat{u}_c(s) + \mathbf{C}_{\theta u}\theta(s)\|^2_{\tilde{R}}\}ds \\
&+ \sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\|\hat{u}_d(k) - \mathbf{D}_{\theta u}(k)\theta(k\tau^+)\|^2_{\tilde{V}_2(k)} \\
&\qquad - \|\hat{w}_d(k) - \mathbf{D}_{\theta w}\theta(k\tau^+)\|^2_{T_1(k)}\} \\
&\qquad + \bar{J}_c(r_c) + \bar{J}_d(r_d).
\end{aligned}$$

Note that $\bar{J}_c(r_c)$ and $\bar{J}_d(r_d)$ are independent of u_c, u_d, w, w_d and x_0. Since the average of θ_1 over $\bar{R}_k$ is zero, including the 'causal' part $\theta_c(t)$ of $\theta(\cdot)$ at time t, we adopt

$$\hat{u}_c^*(t) = -\mathbf{C}_{\theta u}\theta_c(t)$$
$$\hat{u}_d^*(k) = \mathbf{D}_{\theta w}(k)\theta_c(k\tau^+)$$

as the minimizing control strategy. Then

$$\begin{aligned}
&J_T(x_0^*, u_c, u_d, w^*, w_d^*, r_c, r_d) \\
&= \left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\{\|\hat{u}_c(s) + \mathbf{C}_{\theta u}\theta(s)\|^2_{\tilde{R}}\}ds \\
&+ \sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\|\hat{u}_d(k) + \mathbf{D}_{\theta u}(k)\theta(k\tau^+)\|^2_{\tilde{V}_2(k)}\} \\
&\qquad + \bar{J}_c(r_c) + \bar{J}_d(r_d) \\
&\geq \left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\{\|\mathbf{C}_{\theta u}\theta_1(s)\|^2_{\tilde{R}}\}ds \\
&\qquad + \sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\|\mathbf{D}_{\theta u}(k)\theta_1(k\tau^+)\|^2_{\tilde{V}_2(k)}\} \\
&\qquad = J_T(x_0^*, u_c^*, u_d^*, w^*, w_d^*, r_c, r_d).
\end{aligned}$$

Finally we obtain

$$\begin{aligned}
&J_T(x_0, u_c^*, u_d^*, w, w_d, r_c, r_d) \\
&= -\gamma^2 \mathbf{E}_{\bar{R}_0}\{\|x_0 - \gamma^{-2}P_0\theta(0)\|^2_{P_0^{-1}}\} \\
&+ \left\{\sum_{k=0}^{N-1}\int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T}\right\}\mathbf{E}_{\bar{R}_s}\{-\gamma^2\|\hat{w} - \mathbf{C}_\theta\theta(s)\|^2 \\
&\qquad + \|\mathbf{C}_{\theta u}\theta_1(s)\|^2_{\tilde{R}}\}ds \\
&+ \sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{\|\mathbf{D}_{\theta u}(k)\theta_1(k\tau^+)\|^2_{\tilde{V}_2(k)} \\
&\qquad - \|\hat{w}_d(k) - \mathbf{D}_{\theta w}\theta(k\tau^+)\|^2_{T_1(k)}\} \\
&\qquad + \bar{J}_c(r_c) + \bar{J}_d(r_d) \\
&\qquad \leq J_T(x_0^*, u_c^*, u_d^*, w^*, w_d^*, r_c, r_d)
\end{aligned}$$

which concludes the proof of sufficiency.

Necessity: Because of arbitrality of the reference signals $r_c(\cdot)$ and $r_d(\cdot)$, by considering the case of $r_c(\cdot) \equiv 0$ and $r_d(\cdot) \equiv 0$, one can easily deduce the necessity for the solvability of our game problem. One can also get the saddle point strategy by applying the standard dynamic programming method. In detail, refer to ([3]) and so on. (QED.)

5. FULL INFORMATION CASE

In this section, we present the solution of the H_∞ tracking problem in the case of full information such that, at each jump instant, all states and disturbances are observable. The H_∞ tracking theory in the full information setting, presented in this section, can be applied the H_∞ tracking problem by output feedback for linear jump systems which are affected by discrete disturbance at jump instants and the robust H_∞ tracking problem by output feedback for linear jump systems.

In this section, we do not consider the effect of any continuous inputs. Namely we assume $B_2 \equiv O$ and $D_{12} \equiv O$ on the system (1).

We consider the following Riccati equation with jump parts for the system (1) with $B_2 \equiv O$ and $D_{12} \equiv O$.

$$\dot{X} + A'X + XA + C_1'C_1 + \frac{1}{\gamma^2}XB_1B_1'X = O, \ t \neq k\tau \tag{13}$$

$$X(k\tau^-) = A_d'X(k\tau)A_d + C_{1d}'C_{1d} - R_2'T_2^{-1}R_2(k) + F_1'V_1F_1(k) \ k = 0, 1, \cdots \tag{14}$$

$$V_1(k) > a_X I \text{ for some } a_X > 0 \tag{15}$$

where

$$V_1(k) = (T_1 + S'T_2^{-1}S)(k),$$
$$F_1(k) = V_1^{-1}(k)(R_1 - S'T_2^{-1}R_2)(k).$$

Then we obtain the following thorem, which gives the necessary and sufficient condition for the solvability of the H_∞ tracking problem in the full information setting and the saddle point strategy for this game problem.

Theorem 5.1. Consider the system (1) with $B_2 \equiv O$ and $D_{12} \equiv O$ and suppose **A1**. Then **the H_∞ tracking problems** is solvable **in the full information setting** if and only if there exists a matrix $X(t) > O$ satisfying the conditions $X(0^-) < \gamma^2 R^{-1}$ and $X(T) = O$ such that the equation (13)(14) and the condition (15) hold over $[0, T]$. Then

$$x_0^* = [\gamma^2 R^{-1} - X(0^-)]^{-1}\theta(0)$$
$$w^* = \frac{1}{\gamma^2}B_1'(Xx + \theta)$$
$$w_d^*(k) = F_1(k)x(k\tau) + \mathbf{R}_w(k)r_d(k) + \mathbf{R}_{\theta w}(k)\theta(k\tau^+)$$
$$u_d^*(k) = -[T_2^{-1}R_2(k)x(k\tau) + T_2^{-1}S(k)w_d(k) + \mathbf{R}_u(k)r_d(k) + \mathbf{R}_{\theta u}(k)\theta_c(k\tau^+)]$$

where

$$\mathbf{R}_w(k) = V_1^{-1}(k)[B_{1d}'X(k\tau)B_{3d}$$

$$-S'T_2^{-1}(k)(B'_{2d}X(k\tau)B_{3d} + D'_{12d}D_{13d})]$$
$$\mathbf{R}_u(k) = T_2^{-1}(k)(B'_{2d}X(k\tau)B_{3d} + D'_{12d}D_{13d})$$
$$\mathbf{R}_{\theta w}(k) = V_1^{-1}(k)(B'_{1d} - S'T_2^{-1}(k)B'_{2d})$$
$$\mathbf{R}_{\theta u}(k) = T_2^{-1}(k)B'_{2d}.$$

$\theta(t)$, $t \in [0,T]$Csatisfies

$$\begin{cases} \dot{\theta}(t) = -\bar{A}'_{c,f}(t)\theta(t) + \bar{B}_{c,f}(t)r_c(t), \ t \neq k\tau \\ \theta(k\tau) = \bar{A}'_{d,f}(k)\theta(k\tau^+) + \bar{B}_{d,f}(k)r_d(k) \\ \theta(T) = 0 \end{cases} \quad (16)$$

where

$$\bar{A}_{c,f} = A + \frac{1}{\gamma^2}B_1B_1'X$$
$$\bar{B}_{c,f} = -(XB_3 + C_1'D_{13})$$
$$\bar{A}_{d,f}(k) = A_d + \mathbf{R}'_{\theta w}V_1F_1(k) - \mathbf{R}'_{\theta u}R_2(k)$$
$$\bar{B}_{d,f}(k) = A'_dX(k\tau)B_{3d} + (R_1' - R_2'T_2^{-1}S)\mathbf{R}_w(k) - R_2'\mathbf{R}_u(k) + C'_{1d}D_{13d}$$

and $\theta_c(t)$ is the 'causal' part of $\theta(\cdot)$ at time t. This θ_c is the expected value of θ over $\bar{R}_s$ and $\bar{R}_k$ and given by

$$\begin{cases} \dot{\theta}_c(s) = -\bar{A}'_{c,f}(s)\theta_c(s) + \bar{B}_{c,f}(s)r_c(s), \ s \neq k\tau, \\ \qquad t \leq s \leq t_f \\ \theta_c(j\tau) = \bar{A}'_{d,f}(j)\theta_c(j\tau^+) + \bar{B}_{d,f}(j)r_d(j) \\ \qquad k < j \leq k_f \ (k_f\tau < t_f < (k_f+1)\tau) \\ \theta_c(t_f) = 0 \end{cases} \quad (17)$$

where, for $k\tau^+ \leq t \leq (k+1)\tau$,

$$\begin{cases} t_f = t + h\tau \text{ and } k_f = k + h \text{ if } (k+h)\tau < T \\ t_f = T \quad \text{and} \quad k_f = N \quad \text{if } (k+h)\tau \geq T. \end{cases}$$

Moreover, the value of the game is

$$J_T(x_0^*, u_d^*, w^*, w_d^*, r_c, r_d) = \sum_{k=0}^{N} \mathbf{E}_{\bar{R}_k}\{\|T_2^{1/2}\mathbf{R}_{\theta u}(k)\theta_1(k\tau^+)\|^2\} + \bar{J}_c(r_c) + \bar{J}_d(r_d) \quad (18)$$

where $\theta_1(t) = \theta(t) - \theta_c(t)$, $t \in [0,T]$,

$$\bar{J}_c(r_c) = \left\{ \sum_{k=0}^{N-1} \int_{k\tau^+}^{(k+1)\tau} + \int_{N\tau^+}^{T} \right\} \mathbf{R}_{\bar{R}_s}\{\|D_3r_c\|^2 + \gamma^{-2}\|B_1'\theta\|^2 + 2\theta'B_3r_c\}ds, \quad (19)$$

$$\bar{J}_d(r_d) = \gamma^2\mathbf{E}_{\bar{R}_0}\{\|\theta(0^-)\|^2_{P_0}\} + \sum_{k=0}^{N}\mathbf{E}_{\bar{R}_k}\{-\|\mathbf{R}_u(k)r_d(k) + \mathbf{R}_{\theta u}(k)\theta(k\tau^+)\|^2_{T_2(k)} + \|\mathbf{R}_w(k)r_d(k) + \mathbf{R}_{\theta w}(k)\theta(k\tau^+)\|^2_{V_1(k)} + r_d'(k)[B'_{3d}X(k\tau)B_{3d} + D'_{13d}D_{13d}]r_d(k) + 2\theta'(k\tau^+)B_{3d}r_d(k)\} \quad (20)$$

and $P_0 = [R^{-1} - \gamma^{-2}X(0^-)]^{-1}$.

Corollary 5.1. If **Full Information $\mathbf{H_\infty}$ Tracking Problem** for the system (1) with $B_2 \equiv O$ and $D_{12} \equiv O$ and the performance index (2) is solvable, the following results hold, using the gains

$$K_x = -T_2^{-1}R_2(k), \ K_{w_d} = -T_2^{-1}S(k),$$
$$K_{r_d} = -\mathbf{R}_u(k), \ K_\theta = -\mathbf{R}_{\theta u}(k),$$

Case a) The FI control law for **the $\mathbf{H_\infty}$ fixed-preview tracking** is

$$u_{f,a,d}(k) = K_xx(k\tau) + K_{w_d}w_d(k) + K_{r_d}r_d(k) + K_\theta\theta_c(k\tau^+)$$

with $\theta_c(\cdot)$ given by (17). Moreover, $J_T(x_0^*, u_{f,a,d}, w^*, r_c, r_d)$ coincides with (18).

Case b) The FI control law for **the $\mathbf{H_\infty}$ tracking of noncausal $\mathbf{r_c}(\cdot)$ and $\mathbf{r_d}(\cdot)$** is

$$u_{f,b,d}(k) = K_xx(k\tau) + K_{w_d}w_d(k) + K_{r_d}r_d(k) + K_\theta\theta(k\tau^+)$$

with $\theta(\cdot)$ given by (16) and

$$J_T(x_0^*, u_{f,b,d}, w^*, w_d^*, r_c, r_d) = \bar{J}_c(r_c) + \bar{J}_d(r_d)$$

since $\theta(t) = \theta_c(t)$, $\forall\ t \in [0,T]$.

6. CONCLUSION

In this paper, we have presented the H_∞ tracking theory with preview for linear systems with impulsive effects (linear jump systems) in the state feedback and full information cases. Our state feedback theory can be applied to the robust H_∞ tracking theory by state feedback, and our full information theory can be also applied to the robust H_∞ tracking theory by output feedback for uncertain linear systems with impulsive effects, where the auxiliary disturbance is introduced at impulsive parts. Their theory will be reported elsewhere.

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