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Robust LPV Control of UAV with Parameter Dependent Performance

Jianchi Chen   Dawei Gu   Ian Postlethwaite   Kannan Natesan

* *Control and Instrumentation Research Group, Department of Engineering, University of Leicester LE2 7RH, UK*

Abstract: Existing control theory for linear parameter varying system uses a uniform upper bound on the induced-$\mathcal{L}_2$ norm for the varying parameters. In this paper, this constant is generalized as a function of parameters, the design provides varying induced-$\mathcal{L}_2$ performance in the presence of real-time variations and consequently gives rise to the superior robust performance of the local operating point. The controller synthesis incorporates known bounds on the rate of variation and as in the existing theory, the synthesis problem reduces to convex optimization involving linear matrix inequalities. An example, robust LPV flight control design of UAV demonstrator was used to illustrate the design criterion.

1. INTRODUCTION

Gain scheduling control problem of linear parameter varying (LPV) system has received plenty of attentions. An LPV system is a finite dimensional linear system that depends on one or more time-varying measurable parameters (e.g. families of linearized model for nonlinear aircraft system). LPV control design uses these measurements to provide good performance in the presence of real-time parameters. In Apkarian [1995], Apkarian [1995a], Becker [1994], the control problem is solved by single quadratic lypunov function synthesis (SQLF), one such method use a fixed quadratic Lyapunov function and H_∞ optimization technique to guarantee exponential stability. The approache in Apkarian [1998], Wu [1995],Feron [1996],Wu [1996] incorporates a parameter-dependent Lyapunov function (PDLF). This enables knowledge of the bounds on the rate of variation of the parameter trajectory to be used in the analysis test, unlike SQLF, which designed with arbitrarily fast variations in the parameters. The resulted controllers are characterized by solutions to linear matrix inequalities, and optimization reduced to convex programming. The controllers are themselves parameter dependent and require real time measurements of the varying parameters.

The performance criterion used by existing theory is a uniform upper bound γ on $\mathcal{L}_2$ norm. In the controller applications, if a uniform upper bound has been chosen for all the operating points, the resulted controller performance might be limited by a few troublesome operating points (e.g. aircraft operates on the boundary of flight envelope). One way to overcome the mediocre performance is to use parameter dependent performance. This approach gives rise to a more flexible norm bound that allows us to characterized "local" performance in the parameters, and subsequently evaluate the controllers with superior robust performance.

The rest of the paper is organized as follows. In §2 we present the robust performance problem for induced-$\mathcal{L}_2$ norm of LPV system. We generalized the existing synthesis method in §3 and discuss the solution computation and improved robust conditions with parameter dependent performances in §4. Finally in §5 we present a design example of robust longitudinal control of Eclipse UAV, and use the approaches we discuss in the early sections.

The following notations are used in additional to standard notations: R_+ and $R^{n\times n}$ denotes the sets of positive real numbers and positive-defined, real $n \times n$ matrices. $\mathcal{C}^1(V, W)$ denotes continuous functions from U to V which have the first order derivative. The vertices of ploytope $\mathcal{V}$ are denoted by vert($\mathcal{V}$).

2. PROBLEM FORMULATION

Considered the LPV plant $\Sigma_s(\rho)$ given by

$$\begin{bmatrix} \dot{x}(t) \\ e(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B_d(\rho(t)) & B_u(\rho(t)) \\ C_e(\rho(t)) & D_{ed}(\rho(t)) & D_{eu}(\rho(t)) \\ C_y(\rho(t)) & D_{yd}(\rho(t)) & D_{yu}(\rho(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \\ u(t) \end{bmatrix} \quad (1)$$

where $x(t) \in \mathbf{R}^n, d(t) \in \mathbf{R}_{n_d}, e(t) \in R_{n_e}$, and state space data (A, B, C, D) depend continuously on time varying parameters that are represented by the vector-valued signal $\rho \in \mathcal{C}^1(R_+, R^s)$. Varying parameters $\rho(t)$ and its rate of variation $\dot{\rho}(t)$ are contained in prespecified compact sets $\mathcal{P}$ and $\mathcal{V}$, where $\mathcal{V}$ is a convex polytope of the rate of variation $\dot{\rho}(t)$.

The induced-$\mathcal{L}_2$ norms associate with an LPV system can be found using a parameter dependent quadratic Lyapunov function and an integral quadratic constraint. The definition of parameter-dependent quadratic performance given below facilitates this characterization Wu [1995].

Lemma 1. Given a constant γ, the LPV system in (1) has parameter-dependent quadratic performance of level γ if some matrix function $W \in \mathcal{C}^1(R^s, R_+)$ satisfies

$$\begin{bmatrix} E(\rho,\dot{\rho}) & W(\rho)B(\rho) & C^T(\rho) \\ B^T(\rho)W(\rho) & -\gamma I_{n_d} & D^T(\rho) \\ C(\rho) & D(\rho) & -\gamma I_{n_e} \end{bmatrix} < 0 \quad (2)$$

at all $(\rho, \dot{\rho})$ for which $\rho \in \mathcal{P}$ and $\rho \in \mathcal{V}$ where

$$E(\rho,\dot{\rho}) = A^T(\rho)W(\rho) + W(\rho)A(\rho) + \sum_{i=1}^{s} \dot{\rho}_i \frac{\partial W(\rho)}{\partial \rho_i}$$

Supposed the LPV system Σ_s in (1) has parameter-dependent quadratic performance of level γ. Then, for any allowable parameter $\rho(t)$, the LPV system is exponential stable and

$$\int_0^\infty \frac{1}{\gamma(\rho(t))} e^T(t)e(t)dt < \int_0^\infty \gamma(\rho(t))d^T(t)d(t)dt$$

for all nonzero $d \in \mathcal{L}_2$, assuming zero initial conditions. This implies the norm bound $\|\Sigma_s(\rho)\|_{i2} < \sup_{t>0}\gamma(\rho(t))$, which for constant γ simply reduces to $\|\Sigma_s\|_{i2} < \sup_{t>0}\gamma$. In the above treatment of LPV performance problem, γ is constant, signifying performance for not only worst-case disturbances, but also worst-case parameter variations.

3. OUTPUT FEEDBACK SYNTHESIS

Consider the generalized LPV plant in (1), let the nth-order generalized simplified LPV system $\Sigma_{\mathcal{P}}^{\mathcal{V}}$ given as

$$\begin{bmatrix} \dot{x} \\ e_1 \\ e_2 \\ y \end{bmatrix} = \begin{bmatrix} A(\rho) & B_{d1}(\rho) & B_{d2}(\rho) & B_u(\rho) \\ C_{e1}(\rho) & 0 & 0 & 0 \\ C_{e2}(\rho) & 0 & 0 & I_{n_u} \\ C_y(\rho) & 0 & I_{n_y} & 0 \end{bmatrix} \begin{bmatrix} x \\ d_1 \\ d_2 \\ u \end{bmatrix} \quad (3)$$

where $\rho \in R^s, x, \in R^n, [d_1^T \quad d_2^T]^T \in^{n_d}, [e_1^T \quad e_2^T]^T \in R^{n_e}, u \in R^{n_u}$ and $y \in R_{n_u}$ respectively represent the parameters, states, disturbances, errors, controls and measurements. Note for simplicity of derivation, we assume $D_{ed} = 0, D_{yu} = 0$ and $D_{yd} = [0 \ \ I_{n_d}], D_{eu} = [0 \ \ I_{n_u}]^T$ has full row rank and column rank respectively for all $\rho \subset \mathcal{P}$. It is shown that we can statically, invertibly transform any general plant to satisfy the above assumptions using a loop transformation Safonov [1989].

The gain-scheduling output-feedback control problem consists of finding a dynamic LPV controller, $K(\rho,\dot{\rho})$, with state-space equations

$$\begin{bmatrix} \dot{x}_k(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_K(\rho(t),\dot{\rho}(t)) & B_K(\rho(t),\dot{\rho}(t)) \\ C_K(\rho(t),\dot{\rho}(t)) & D_K(\rho(t),\dot{\rho}(t)) \end{bmatrix} \begin{bmatrix} x_k(t) \\ y(t) \end{bmatrix} \quad (4)$$

Define $x_c^T := [X_c^T \quad x_k^T]$. The closed loop system can be written as

$$\begin{bmatrix} \dot{x}_c(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} A_c(\rho(t),\dot{\rho}(t)) & B_c(\rho(t),\dot{\rho}(t)) \\ C_c(\rho(t),\dot{\rho}(t)) & D_c(\rho(t),\dot{\rho}(t)) \end{bmatrix} \begin{bmatrix} x_c(t) \\ d(t) \end{bmatrix} \quad (5)$$

where

$$A_c(\rho,\dot{\rho}) = \begin{bmatrix} A + B_u D_K C_y & B_u C_K \\ B_K C_y & A_K \end{bmatrix}$$

$$B_c(\rho,\dot{\rho}) = \begin{bmatrix} B_{d2} & B_{d2} + B_u D_K \\ 0 & B_K \end{bmatrix}$$

$$C_c(\rho,\dot{\rho}) = \begin{bmatrix} C_{e1} & 0 \\ C_{e2} + D_K C_y & C_K \end{bmatrix}$$

$$D_c(\rho,\dot{\rho}) = \begin{bmatrix} 0 & 0 \\ 0 & D_K \end{bmatrix}$$

The parameter dependent terms in above matrices are omitted. With the inequalities defined in (2), we have the following main theorem for solving output feedback synthesis problem in Wu [1995].

Theorem 2. Given the LPV plant in (3), and a constant scalar γ, the parameter dependent quadratic performance problem is solvable if and only if there exist matrix function $X, Y \in \mathcal{C}^1(R^s, R_+^{n\times n})$ that satisfy

$$\begin{bmatrix} E_Y(\rho,\dot{\rho}) & Y(\rho)C_{e1}^T(\rho) & B_d(\rho) \\ C_{e1}(\rho)Y(\rho) & -\gamma(\rho)I_{n_{e1}} & 0 \\ B_d^T(\rho) & 0 & -\gamma(\rho)I_{n_d} \end{bmatrix} < 0 \quad (6)$$

$$\begin{bmatrix} E_X(\rho,\dot{\rho}) & X(\rho)B_{d1}^T(\rho) & C_e^T(\rho) \\ B_{d1}^T(\rho)X(\rho) & -\gamma(\rho)I_{n_{d1}} & 0 \\ C_e^T(\rho) & 0 & -\gamma(\rho)I_{n_e} \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} Y(\rho) & I_n \\ I_n & X(\rho) \end{bmatrix} < 0 \quad (8)$$

at all $(\rho,\dot{\rho})$ for which $\rho \subset \mathcal{P}$ and $\dot{\rho} \subset \mathrm{vert}(\mathcal{V})$, where

$$\begin{aligned} E_Y(\rho,\dot{\rho}) &= \hat{A}(\rho)Y(\rho) + Y(\rho)\hat{A}^T(\rho) - \gamma(\rho)B_u(\rho)B_u^T(\rho) \\ &\quad - \textstyle\sum_{i=1}^{s} \dot{\rho}_i \frac{\partial Y(\rho)}{\partial \rho_i} \\ E_X(\rho,\dot{\rho}) &= \tilde{A}^T(\rho)Y(\rho) + X(\rho)\tilde{A}(\rho) - \gamma(\rho)C_y^T(\rho)C_y(\rho) \\ &\quad + \textstyle\sum_{i=1}^{s} \dot{\rho}_i \frac{\partial X(\rho)}{\partial \rho_i} \end{aligned} \quad (9)$$

and

$$\tilde{A}(\rho) = A(\rho) - B_{d2}(\rho)C_y(\rho) \qquad \hat{A}(\rho) = A(\rho) - B_u(\rho)C_{e2}(\rho);$$

If we drop the derivation terms of X and Y in the equation (9), the above theorem is equivalent to the single quadratic Lyapunov function synthesis problem.

The necessary and sufficient conditions in Theorem 2 form a system of LMIs with functional variables X, Y and γ. Although these constraints are infinite dimensional as written, they can be solved approximately using griding method which will be discussed later. The resulting LMI solution can then be used to construct explicit controller formula:

Theorem 3. Suppose $X, Y \in \mathcal{C}^1(R^s, R_+^{n\times n})$ and $\gamma \in \mathcal{C}(R^s, R_+)$ satisfying (6)-(8) at all $(\rho,\dot{\rho})$ for which $\rho \in \mathcal{P}$ and $\dot{\rho} \in \mathrm{vert}(\mathcal{V})$. Let $N, M \in \mathcal{C}^1(R^s, R_+^{n\times n})$ be matrix functions that satisfy the identity

$$X(\rho)Y(\rho) + N(\rho)M^T(\rho) = I_n \quad (10)$$

Then the parameter-varying state-feedback and output injection gains

$$F = -(\gamma B_u^T Y^{-1} + D_{eu}^T C_e)$$
$$L = -(\gamma X^{-1} C_y^T + B_d D_{yd}^T)$$

and parameter varying state space matrices

$$\begin{aligned} C_K &= FYM^{-T} \\ B_K &= N^{-1}XL \\ A_K &= -N^{-1}\{A^T + X(A + B_u F + LC_y)Y \\ &\quad + C_e^T(C_e + D_{eu}F)Y/\gamma + X(B_d + LD_{yd})B_d^T/\gamma \\ &\quad + \dot{X}Y + \dot{N}M^T\}M^{-T} \end{aligned} \quad (11)$$

define a strictly proper ($D_k = 0$) LPV controller that solve the γ-performance problem for LPV plant (3).

From (11) we can see the formula of A_K in general explicitly depends on $\dot{\rho}$ via $(\dot{X}, \dot{N})$. This makes the associated controller impractical to implemented. Nevertheless, the product rule in (10) implies the identity

$$\dot{X}Y + \dot{N}M^T = -X\dot{Y} - N\dot{M}^T,$$

so that the derivation term in (11) can be eliminated by adding either one of the following constraints:

- X is constant, $(N, M) = (I_n, I_n - YX)$
- Y is constant, $(N, M) = (I_n - YX, I_n)$

4. COMPUTATION OF SOLUTION

The constraints given by the LMIs (6)–(8) are clearly infinite-dimensional; in order to compute solutions and optimize γ using convex programming tools, some approximation must be made. First of all, we can relieve the infinite-dimensionality of the constraints by approximating the parameter set $\mathcal{P}$ by a finite grid $\mathcal{P}_{\text{grid}} = \{p^{(l)}\}_{l=1}^{L} \subset \mathcal{P}$ and resorting the continuity arguments to extend the feasibility solution to entire parameter set $\mathcal{P}$. Secondly, we approximate the parameter-dependent matrix variables by linear combination of user defined set of continuously differentiable functions. For any matrices $\{X_i\}_{i=1}^{N}, X_i \in \mathcal{S}^{n\times n}$ and $\{Y_i\}_{i=1}^{N}, Y_i \in \mathcal{S}^{n\times n}$ the function

$$X(\rho) := \sum_{i=1}^{N} f_i(\rho)X_i, \quad Y(\rho) := \sum_{i=1}^{N} g_i(\rho)Y_i \tag{12}$$

are continuous differentiable on $R^s \to \mathcal{S}^{n\times n}$. So once basis function f_i and g_i are chosen, we can attempt to solve the LMIs synthesis problem over the matrices $X_i, Y_i \in \mathcal{S}^{n\times n}$. There is no systematic rule of choosing the basis functions but a practical technique has been proposed in Wu [1996], the key is to mimic the dependency of the parameter of the original plant.

The approximation procedure can then be solved with all grid points $(\rho, \dot{\rho})$ satisfying $\rho \in \mathcal{P}_{\text{grid}}$ and $\dot{\rho} \in \text{vert}(\mathcal{V})$. Since $\rho \subset R^s$, it will require approximately L^s points to grid $\mathcal{P}$ with approximately L points in each dimension. So the optimization problem to determine appropriate X_i and Y_i is approximately $L^s(2^{s+1}+1)$ affine matrix inequalities in the matrix variables $(X_1, Y_1, \ldots, X_N, Y_N)$. Here we assume 2^s vertices for $\mathcal{V}(\rho)$. In the above approximation procedure, the LMIs are satisfied only at the grid point of $\mathcal{P}$, the synthesis conditions might not be satisfied at some parameter values that lie between the grid points. One generally needs to check the computed solution against a sufficiently fine grid, if the check fail, repeat the synthesis procedure using more grid points.

If γ is restricted to be constant scalar, the computation is straightforward, just minimize γ subject to (6)–(8) with decision variable X_i, Y_i, γ . As in our early discussion, this is guarantee the worst case $\mathcal{L}_2$ performance for all the grid points. Nevertheless, The LPV controller design using a constant γ tends to be conservative at relatively "benign" operating points, because the overall controller performance has to be sacrificed by a few "troublesome" operating points which correspond to the worst case performance. If we allow γ to be parameter-varying, we do not have to limited the overall controller performance. A varying γ allows the designer to tolerate the unavoidable mediocre performance at a few operating points without sacrifice superior performance for parameter trajectory that avoids the worst case conditions. For the parameter varying γ we can define a weight $w(\rho)$, such that we can minimized some convex combination $\delta(\gamma) := w(p)\gamma(p)$ subject to (6)-(8), while $\gamma(\rho)$ corresponds to L scalar variables $\{\gamma_l\}_{l=1}^{L}$ evaluated at each grid points. In absence of prior information about γ, we can initially choosing the weight $w(\rho)$ to be uniform and update the weights with an iterative design procedure to search for "optimal" γ . For example, we can find out the "difficult" operating point by a few trials with constant weights for every grid points, then penalized those grid points more heavily with larger weights. Alternatively, we can approximate the γ simply as a linear combination $\gamma(\rho) = \sum_{k=1}^{N} h_k(\rho)\gamma_k$ where $h_k(\rho) \in \mathcal{C}(R^s, R)$ is continuous basis function.

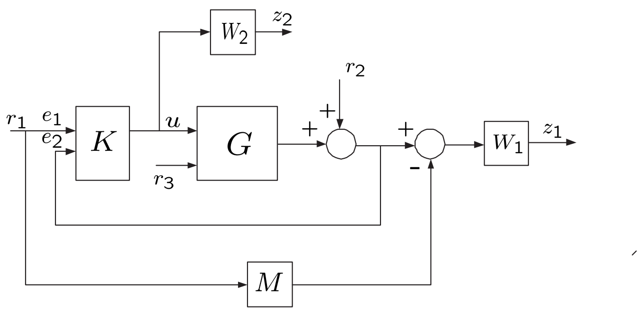

Fig. 1. Two Degree of Freedom control structure

5. DESIGN EXAMPLE

In this section we will investigate an gain-scheduling controller design for the longitudinal axis of the Eclipse UAV demonstrator in Kannan [2006a]. The problem is revisited using the methods discussed in the previous sections. Single quadratic lypunov function synthesis (SQLF) and parameter dependent lypunov function synthesis (PDLF) are applied respectively to the longitudinal LPV controller design and comparisons are made. The use of parameter varying γ are discussed at the end of section.

5.1 Problem Setup

The longitudinal dynamics of the UAV is given by Kannan [2006a]. The perturbation states in Equation are the velocity u along the x-axis of the body axes coordinate system, velocity w along the z-axis of the body axes, and q is pitch rate. There are two inputs: elevator input and throttle input which denoted by δ_t and δ_e. The system has three outputs measurements available: pitch rate q, height h and total velocity V_t. The velocity is considered ranging between [22 72]m/s. The linear model in equation (13) is obtained from a trim sub-routine, by linearizing the 6DoF nonlinear system at different operating point, i.e. different airspeed. The resulted 4th order equation of motion is:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} X_u(\rho) & X_w(\rho) & -g\cos(\theta(\rho)) & X_q(\rho) \\ Z_u(\rho) & Z_w(\rho) & Z_\rho(\rho) & Z_q(\rho) \\ 0 & 0 & 0 & 1 \\ M_u(\rho) & M_w(\rho) & M_\rho(\rho) & M_q(\rho) \end{bmatrix} \begin{bmatrix} u \\ w \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} X_{\delta e}(\rho) & X_{\delta t}(\rho) \\ Z_{\delta e}(\rho) & Z_{\delta t}(\rho) \\ 0 & 0 \\ M_{\delta e}(\rho) & M_{\delta t}(\rho) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix} \tag{13}$$

and the output channels are

$$\begin{bmatrix} q \\ h \\ V_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \cos(\alpha_t) & \sin(\alpha_t) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ \theta \\ q \\ h \end{bmatrix} \tag{14}$$

In the design of kannan [2006], Kannan [2006a], in order to obtain the affine parameter dependent LPV system, the static and dynamic derivatives in the equation (13) are approximated by lease square curve fit as continuous functions of the velocity. Since the height of the UAV does not vary much, density is assumed to be constant and does not change the LPV model and the coefficients X_q, Z_ρ, Z_q, M_u and $Z_{\delta e}$ are found to have the most significant effects on the plant dynamics. In this design no approximation need to be made, the LMIs can be solved at each grid point independently.

The system design structure is shown in Fig.1. In this system r_1 are the reference inputs, r_2 and r_3 are perturbations input, z_1 are the performance inputs, z_2 are the weighted control outputs, e_1 are the controller reference inputs, e_2 are the feedback measurements and u are the actuator control signals. The second and third output (h, V_t) are choose to match the ideal model W_m, as only 2 outputs are being matched, we augment the matching model to become

$$\begin{bmatrix} q_m \\ h_m \\ V_m \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{s^2+2s+1} & 0 \\ 0 & 0 & \frac{1}{s^2+2s+1} \end{bmatrix} \begin{bmatrix} q_d \\ h_d \\ V_d \end{bmatrix} \tag{15}$$

the performance weight W_1 are selected as

$$W_1 = \begin{bmatrix} \frac{100s+1}{s^2+20s+1} & 0 & 0 \\ 0 & \frac{2.5}{s+\epsilon} & 0 \\ 0 & 0 & \frac{1}{s+\epsilon} \end{bmatrix} \tag{16}$$

where $\epsilon = 10^{-6}$. Integral terms in performance weight are chosen for the design parameters V_t and h. Increased gain gives the closed loop system better matching to the ideal model and increase bandwidth of the disturbance rejection.

The control weight W_2 needs to limit the high frequency activity and to allow low frequency tracking. Hence W_2 are selected as high-pass filter to bound this requirement. A typical choice of the weighting function is $\frac{ks+w}{s+kw}$, parameter w is chosen so that high frequency control is minimised above a certain threshold frequency and k is chose to allow low frequency control effort. A sensible choice is $k = 10, w = 200$rad/s, thus

$$W_2 = \begin{bmatrix} \frac{10s+200}{s+2000} & 0 \\ 0 & \frac{10s+200}{s+2000} \end{bmatrix} \tag{17}$$

5.2 Controller synthesis and results

The augmented closed-loop plant is constructed by combining the linearised model, sensor and actuator models and weighting functions, with 20 states, 10 outputs and 8 outputs. 11 different airspeeds space every 5m/s in the range $[22, 72]$ are considered used as the approximate design grid $\mathcal{P}$. By observing the trajectories of the coefficients X_q, Z_ρ, Z_q, M_u and $Z_{\delta e}$ which are affected significantly by V_t, it was found that a quadratic fitting is enough to mimic the parameter trajectory. Thus, based on the discussion in section 4, a choice of basis function $g_i(V_t)$ for Y are $[g_1(V_t) \;\; g_2(V_t) \;\; g_3(V_t)] = [1 \;\; V_t \;\; V_t^2]$. A single basis function $f_1(V_t) = 1$ is chosen for X. The bound of changing rate of airspeed is $\dot{V}_t \in [-10 \;\; 10]$. The PDLF synthesis problem is implemented with the Matlab LMI toolbox Gahinet [1996]. The total number of LMIs to solve is 44, the number of decision variables is 841. The convex optimization procedure takes 1 hour 12 minutes to solve and the resulted $\mathcal{L}_2$ performance γ is 8.4.

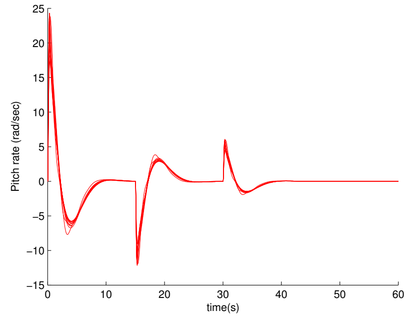

Fig. 2. Pitch rate response (PDLF) with varying γ

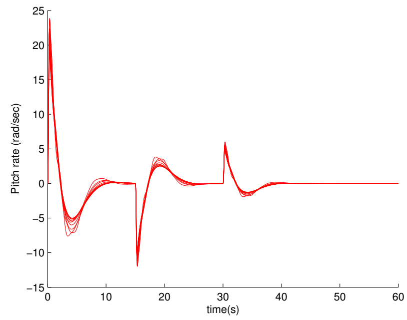

Fig. 3. Pitch rate response (PDLF)

As a comparison, the SQLF synthesis only has to solve 23 LMIs with 421 decision variable and the elapsed computation time is about 5 minutes. However, the final achieved γ is 11.22. The closed-loop system response to a doublet height signal at different airspeeds are shown in Fig.2–7. It can be seen that the decoupled pitch rate and airspeed responses of the PDLF with varying γ are slightly better than normal PDLF. Fig.8–10 show the unit height response of SQLF. Fig.11 is the singular value plots of controller for PDLF. Fig.12 is the difference between frequency response of closed-loop system and matching model W_m. At high and low frequencies the difference is very low, so good tracking is expected. The difference is maximised around the operating bandwidth, which may degrade the overshoot or settling time to step demand.

we chose a parameter γ dependent explicit on V_t

$$\gamma(V_t) = \gamma_1 + \gamma_2 V_t$$

and minimized γ_1, γ_2 subject to (6)–(8) for SQLF synthesis. The optimization procedure yields $\gamma_1 = 0.4745$ and $\gamma_2 = 0.1675$ and the optimal induced-$\mathcal{L}_2$ norm $\gamma = 0.4745 + 0.1675Vt$. The results are shown in the Table 1.

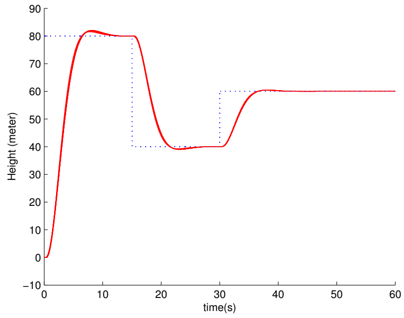

Fig. 4. Height response (PDLF)with varying γ

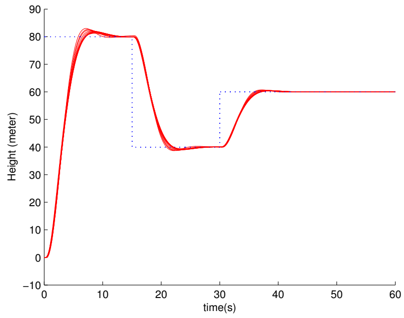

Fig. 5. Height response (PDLF)

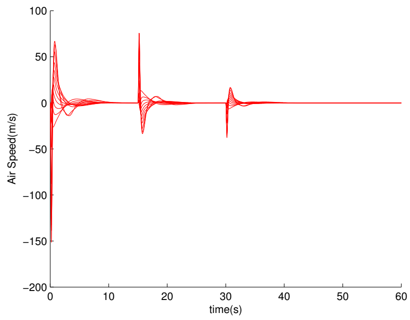

Fig. 6. Airspeed response (PDLF)with varying γ

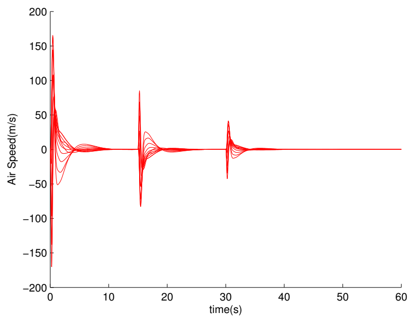

Fig. 7. Airspeed response (PDLF)

Compared to the constant γ of 11.22 in worst case measurement, the optimal $\mathcal{L}_2$ gains have been considerably reduced at lower velocity, thus robust performance can be achieved at those operating points without degrade the overall performance for the operating point at higher velocity.

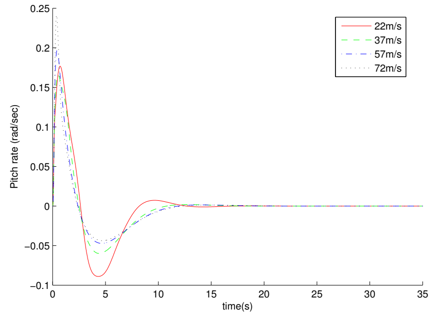

Fig. 8. Pitch rate response (SQLF)

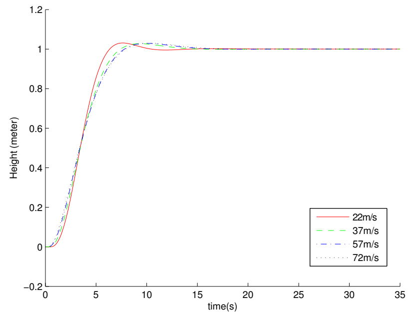

Fig. 9. Height response (SQLF)

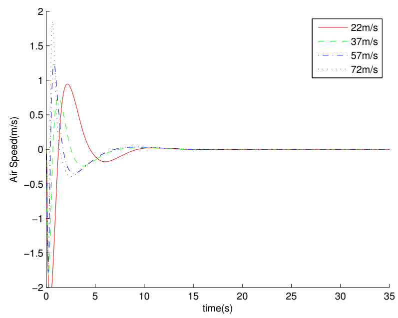

Fig. 10. Airspeed response (SQLF)

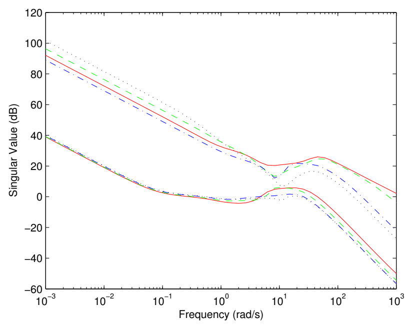

Fig. 11. Controller frequency response (PDLF)

A similar simulation on PDLF synthesis yields the results shown in Table 2 with induced-$\mathcal{L}_2$ performance $\gamma = 0.4094 + 0.1178Vt$, which is compared to constant $\gamma = 8.4$ of PDLF synthesis.

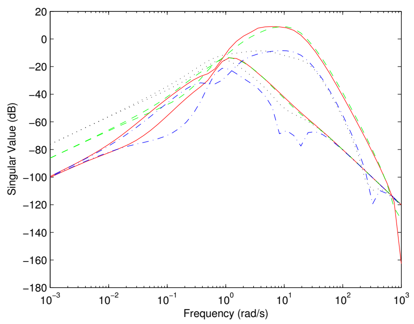

Fig. 12. Response of difference between closed-loop system and W_m (PDLF)

Table 1. Different γ at different grid points (SQLF)

V_t	22	27	32	37	42	47
γ	4.20	5.04	5.89	6.74	7.58	8.43
V_t	52	57	62	67	72	
γ	9.28	10.12	10.97	11.82	12.66	

Table 2. Different γ at different grid points (PDLF)

V_t	22	27	32	37	42	47
γ	3.00	3.59	4.18	4.77	5.36	5.95
V_t	52	57	62	67	72	
γ	6.54	7.12	7.71	8.30	8.89	

6. CONCLUSION

In this paper we have designed the LPV controller for longitudinal dynamics of UAV using single quadratic Lyapunov function synthesis and parameter-dependent Lyapunov function synthesis, it was found that PDLF approach with varying γ performance has better performance compare to the PDLF approach with constant γ. And it also provides more robust γ performance at benign operating point. The constant induced-$\mathcal{L}_2$ performance bound used in the existing theory is generalized to a function of parameters; The resulting performance criterion implies a norm bound that depends naturally on the particular real time trajectory of the parameters.

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