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Model Reduction for Switched Linear Discrete-Time Systems with polytopic uncertainties and arbitrary switching

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Abstract: In this paper, the problem of H_∞ model reduction for switched linear discrete-time systems with polytopic uncertainties is investigated. A reduced-order switched model is constructed for a given robustly stable switched system, which has the same structural polytopic uncertainties as the original system such that the resulting error system is robustly asymptotically stable and an H_∞ error performance is guaranteed. A sufficient condition for the existence of the desired reduced-order model is derived and formulated in terms of a set of linear matrix inequalities. By solving the corresponding convex optimization problem in such existence condition, the vertex system of reduced-order model can be obtained, which also provides a suboptimal H_∞ gain for the error system between the original system and the reduced-order model. A numerical example is given to show the effectiveness and the potential of the proposed techniques.

1. INTRODUCTION

Within the past decades, switched systems have received increasing attentions in the hybrid systems field. A hybrid system is meant a dynamic system combining continuous and discrete dynamics, in which corresponding parts are described by differential (or difference) equations and finite automaton (or other discrete event system), respectively. Switched systems, which often ignore the details of corresponding discrete dynamics, assume that the switching signals belong to a certain class and be determined either by time or by system state, or both, or other supervisory decision procedures Daafouz et al. [2002], Morse [1997]. If the switching signals are governed by stochastic processes (for instance, Markovian chains), the corresponding system is termed as jump system, which has been extensively investigated in the past decades, see for example, Shi et al. [1999], Zhang et al. [2003]. The motivation to study switched systems is mainly in twofold. Firstly, many physical systems inherently exhibit multi-models or multi-structure feature, for example, piecewise affine systems (PWA) Ferrari-Trecate et al. [2002], in which mode will change when the system state hits certain "boundaries" and thereby many general nonlinear systems can be modeled by such systems approximately. Secondly, in order to overcome the shortcomings of traditionally used single controller and improve system performance, many intelligent control strategies are designed based on the idea of controllers switching Liberzon and Morse [1999], Morse [1996], and the corresponding closed-loop systems are often described by switched systems. The applications using switched systems theory include modeling of networked control systems (NCS) Lin and Antsaklis [2003], stirred tank reactor El-Farra et al. [2005], wind turbine regulation Leith et al. [2003], etc. On the general topic, recent development and other practical examples in the field of switched systems, we refer readers to Liberzon and Morse [1999], McClamroch and Kolmanovsky [2000], Morse [1997] and the references therein.

In switched systems, the switching signals are often considered as arbitrary in the occurring time of subsystems, and a basic problem is to find non (or less)-conservative conditions to guarantee the stability of the systems under arbitrary switching signals Liberzon and Morse [1999]. One of efficient methods regarding this issue is the multiple Lyapunov functions (MLF) idea Branicky [1998], where an individual Lyapunov function candidate is constructed for each subsystem. As a special kind of MLF, the switched quadratic Lyapunov function (SQLF) approach proposed in Daafouz et al. [2002], attracts the poly-quadratic stability idea for polytopic uncertain system such that the control and filtering problems for a class of discrete-time switched systems under arbitrary switching can be solved Zhang et al. [2006a,b].

On the other hand, it is well known that practical systems are generally characterized by high-order mathematical models, which bring serious difficulties to analysis, synthesis and simulations of the systems concerned. Therefore, model reduction has been a popular research area and has attracted a lot of attention in the last few decades.

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Given a full-order model of some kind of dynamic system, the objective of model reduction is to find a reduced-order model such that these two models are close in some sense, such as the notable H_∞ performance criteria, which is to minimize the energy of the estimation error for all the energy bounded disturbances. In recent years, many important results have been reported for various kinds of nominal systems such as bilinear systems Zhang and Lam [2002], Markovian jump systems Zhang et al. [2003], and two-dimensional systems Gao et al. [2005] by using various efficient approaches, especially the linear matrix inequality (LMI) technique Ebihara and Hagiwara [2004], Zhang et al. [2003]. In addition, model reduction problem of uncertain dynamic systems has also been investigated by some people. A novel idea to approximate the original uncertain system by an uncertain reduced model, has been proposed recently in the literature, see for example, Dolgin and Zeheb [2003], Gao et al. [2005]. However, from the author's best knowledge, up to the date, some issues such as H_∞ model reduction problem on switched systems has not been fully investigated yet, whether with or without modeling uncertainties.

Thus, in this paper, the problem of H_∞ model reduction for switched linear discrete-time systems with polytopic uncertainties is investigated. For a given robustly stable switched system, our objective is focused on the construction of a reduced-order switched model, which also resides in a polytope and approximates the original system well in an H_∞ norm sense, that is, utilize a polytopic uncertain reduced switched model to approximate the original uncertain switched system. Based on the SQLF approach and parameter-dependent stability idea, a sufficient condition for the existence of desired reduced order model is derived and formulated in terms of a set of linear matrix inequalities (LMIs). By solving the corresponding convex optimization problem in such existence condition, the vertex systems of reduced-order model can be obtained, which also provides a suboptimal H_∞ gain for the error system between original system and reduced-order system. A numerical example is given to show the effectiveness and the potential of the proposed techniques.

Notation: The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $|\cdot|$ refers to the Euclidean vector norm; $l_2[0,\infty)$ is the space of square summable infinite sequence and for $\omega = \{\omega(k)\} \in l_2[0,\infty)$, its norm is given by $\|\omega\|_2 = \sqrt{\sum_{k=0}^{\infty} |\omega(k)|^2}$. In addition, in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. A symmetric matrix $P > 0$ (≥ 0) means P is positive (semi-positive) definite. I and 0 represent identity matrix and zero matrix, respectively.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of uncertain switched linear discrete-time systems given by

$$(\Sigma):\ x(k+1) = A_i(\lambda)x(k) + B_i(\lambda)\omega(k) \tag{1}$$

$$y(k) = C_i(\lambda)x(k) \tag{2}$$

where $x(k) \in \mathbb{R}^p$ is the state vector, $\omega(k) \in \mathbb{R}^l$ is the input vector which belongs to $l_2[0,\infty)$, $y(k) \in \mathbb{R}^q$ is the measurement output vector, i denoting $i(k)$ for simplicity, is a piecewise constant function of time, called a switching signal, which takes its values randomly in the finite set $\mathcal{I} = \{1,\ldots,N\}$, $N > 1$ is the number of subsystems, and the matrices $(A_i(\lambda),\ B_i(\lambda),\ C_i(\lambda))$ denote the ith subsystem. At an arbitrary discrete time k, the switching signal i is dependent on k or $x(k)$, or both, or other switching rules.

The matrices of each subsystem have appropriate dimensions with partially unknown parameters. It is assumed that $(A_i(\lambda), B_i(\lambda), C_i(\lambda)) \in \Re_i$, where $\Re_i$ is a given convex bounded polyhedral domain described by s vertices in the ith subsystem

$$\Re_i \triangleq \left\{ [A_i(\lambda), B_i(\lambda), C_i(\lambda)] = \sum_{m=1}^{s} \lambda_m [A_{i,m}, B_{i,m}, C_{i,m}] \right. \\ \left. \sum_{m=1}^{s} \lambda_m = 1,\ \lambda_m \geq 0,\ i \in \mathcal{I} \right\} \tag{3}$$

Remark 1. The polytopic type uncertainty considered in (3) is generally studied in recent literature, which is well recognized to describe the parametric uncertainty in engineering practice more precisely than the norm-bounded uncertainty Jin and Park [2001]. In addition, the parameters and structure of the uncertainties in practice are usually the same throughout either the multi-models or switched control systems De Koning [2003], McClamroch and Kolmanovsky [2000], thus we assume both the number of vertices and uncertain parameter λ_m in each subsystem to be equal here without loss of generality.

Here, we are interested in constructing a reduced-order switched system with the following form

$$(\hat{\Sigma}) : \hat{x}(k+1) = \hat{A}_i(\lambda)\hat{x}(k) + \hat{B}_i(\lambda)\omega(k) \tag{4}$$

$$\hat{y}(k) = \hat{C}_i(\lambda)\hat{x}(k) \tag{5}$$

where $\hat{x}(k) \in \mathbb{R}^v$ is the state vector of the reduced-order system with $v < p$, and $(\hat{A}_i(\lambda), \hat{B}_i(\lambda), \hat{C}_i(\lambda), i \in \mathcal{I})$ are matrices with compatible dimensions to be determined, and belong to a convex polytope with the same structure as described in (3), that is

$$[\hat{A}_i(\lambda), \hat{B}_i(\lambda), \hat{C}_i(\lambda)] = \sum_{m=1}^{s} \lambda_m [\hat{A}_{i,m},\ \hat{B}_{i,m},\ \hat{C}_{i,m}] \tag{6}$$

In addition, the switching signal i in (4)-(5) is also assumed to be available in real-time and homogeneous with the one in system (Σ).

Augmenting the model of system (Σ) to include the states of system $(\hat{\Sigma})$, we obtain the following error system

$$(\bar{\Sigma}) : \xi(k+1) = \bar{A}_i(\lambda)\xi(k) + \bar{B}_i(\lambda)\omega(k) \tag{7}$$

$$e(k) = \bar{C}_i(\lambda)\xi(k) \tag{8}$$

where $e(k) = y(k) - \hat{y}(k)$ and

$$\xi(k)=\begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix},\ \bar{A}_i(\lambda)=\begin{bmatrix} A_i(\lambda) & 0 \\ 0 & \hat{A}_i(\lambda) \end{bmatrix},$$
$$\bar{B}_i(\lambda)=\begin{bmatrix} B_i(\lambda) \\ \hat{B}_i(\lambda) \end{bmatrix},\ \bar{C}_i(\lambda)=\begin{bmatrix} C_i(\lambda) & -\hat{C}_i(\lambda) \end{bmatrix} \quad (9)$$

Note that the developed error system also is a switched linear system, which contains the same structure polytopic uncertainties with uncertain switched system (Σ) in (1)-(2).

To present the main objective of this paper more clearly, we first introduce the following definition for the error system $(\bar{\Sigma})$ in (7)-(8), which will be essential for our derivation.

Definition 1. Given a scalar $\gamma > 0$, error system $(\bar{\Sigma})$ is said to be robustly asymptotically stable with an H_∞ error performance γ if it is robustly asymptotically stable and under zero initial condition, $\|e\|_2 < \gamma \|\omega\|_2$ for all nonzero $\omega(k) \in l_2[0,\infty)$.

Thus, the main objective of this paper is to find the matrices $((\hat{A}_i(\lambda), \hat{B}_i(\lambda), \hat{C}_i(\lambda)) : i \in \mathcal{I})$ in system (4)-(5) such that the error system $(\bar{\Sigma})$ in (7)-(8) is asymptotically stable and has a guaranteed H_∞ error performance γ.

Remark 2. From (6) and similar discussion in Dolgin and Zeheb [2003] or Gao et al. [2005], the reduced-order model system in (4)-(5) is an uncertain switched system with the same structural polytope, however, one can still make some analysis and synthesis using the resulting uncertain reduced-order model, such as robust control and robust filtering. Moreover, the problem of approximating an uncertain system by using a fixed reduced-order system without uncertainties is a different model reduction problem beyond what we considered in this paper. Therefore, we shall find the vertices $((\hat{A}_{i,m}, \hat{B}_{i,m}, \hat{C}_{i,m}) : i \in \mathcal{I},\ 1 \le m \le s)$ in (4)-(5) to solve the underlying model reduction problem for the considered uncertain switched system. Notice that when there is only one vertex, the model reduction problem for switched systems with polytopic uncertainties will reduce to the determinate one without parameter uncertainties.

In addition, the following lemma, which presents the H_∞ performance criterion for the error system $(\bar{\Sigma})$, is required for later development. The Lemma can be proved by similar proof techniques as those used in Zhang et al. [2006b] and thus omitted here.

Lemma 1. Zhang et al. [2006b] Consider the error system $(\bar{\Sigma})$ and let $\gamma > 0$ be a given scalar. If for the arbitrary switching signal $i \in \mathcal{I}$ and all admissible polytopic uncertainties satisfying (3) there exist matrix functions $\mathcal{P}_i(\lambda) > 0$ and $\mathcal{R}_i(\lambda)$ such that

$$\Xi^{ij}(\lambda) \triangleq \begin{bmatrix} \mathcal{P}_j(\lambda)-\mathcal{R}_i(\lambda)-\mathcal{R}_i^T(\lambda) & 0 & \mathcal{R}_i(\lambda)\bar{A}_i(\lambda) & \mathcal{R}_i(\lambda)\bar{B}_i(\lambda) \\ * & -I & \bar{C}_i(\lambda) & 0 \\ * & * & -\mathcal{P}_i(\lambda) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (10)$$

then, system $(\bar{\Sigma})$ is robustly asymptotically stable with an H_∞ error performance γ.

Remark 3. Note that Lemma 1 presents a fundamental performance criterion for the error system $(\bar{\Sigma})$. It is worth mentioning that the introduction of the matrix function $\mathcal{R}_i(\lambda)$ in above LMI overcomes the difficulty of the cross coupling of product terms among different subsystems, which transfers the interaction among subsystems to the form $\mathcal{P}_j(\lambda) - \mathcal{R}_i(\lambda) - \mathcal{R}_i^T(\lambda)$, thus the resulting terms $\mathcal{R}_i(\lambda)A_i(\lambda)$ and $\mathcal{R}_i(\lambda)B_i(\lambda)$ can be easily dealt with. More details on this technique can be found in Zhang et al. [2006b].

3. MAIN RESULTS

The following theorem gives a sufficient condition for the existence of an admissible H_∞ reduced-order model in the form of (4)-(5).

Theorem 2. Consider the uncertain switched linear system $(\bar{\Sigma})$ and let $\gamma > 0$ be a given scalar. Then, an admissible H_∞ reduced-order model in the form of (4)-(5) exists if for $i \in \mathcal{I}$, $1 \le m \le s$ there exist matrices $\mathcal{R}_{1i,m}$, $\mathcal{R}_{2i}$, $\mathcal{R}_{3i,m}$, $\mathcal{P}_{2i,m}$, $\check{A}_{i,m}$, $\check{B}_{i,m}$, $\check{C}_{i,m}$, positive definite matrices $\mathcal{P}_{1i,m}$, $\mathcal{P}_{3i,m}$ such that

$$\Xi^{ij}_{m,n} + \Xi^{ij}_{n,m} < 0,\ \ (1 \le m \le n \le s)\,,\ \forall (i,j) \in \mathcal{I} \times \mathcal{I}, \quad (11)$$

where,

$$\Xi^{ij}_{m,n} \triangleq \begin{bmatrix} \mathcal{P}_{1j,n}-\mathcal{R}_{1i,n}-\mathcal{R}_{1i,n}^T & \mathcal{P}_{2j,n}-\varepsilon_i E\mathcal{R}_{2i}-\mathcal{R}_{3i,n}^T & 0 & \mathcal{R}_{1i,n}A_{i,m} & \varepsilon_i E\check{A}_{i,m} & \mathcal{R}_{1i,n}B_{i,m}+\varepsilon_i E\check{B}_{i,m} \\ * & \mathcal{P}_{3j,n}-\mathcal{R}_{2i}-\mathcal{R}_{2i}^T & 0 & \mathcal{R}_{3i,n}A_{i,m} & \check{A}_{i,m} & \mathcal{R}_{3i,n}B_{i,m}+\check{B}_{i,m} \\ * & * & -I & C_{i,m} & -\check{C}_{i,m} & 0 \\ * & * & * & -\mathcal{P}_{1i,m} & -\mathcal{P}_{2i,m} & 0 \\ * & * & * & * & -\mathcal{P}_{3i,m} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$E \triangleq [\,I\ \ 0\,]^T, I \in \mathbb{R}^v$$

Furthermore, if a feasible solution to above LMIs exists, then the system matrices of an admissible H_∞ reduced-order model in the form of (4)-(5) are given by $(1 \le n \le s)$

$$\hat{A}_{i,m} = \mathcal{R}_{2i}^{-1}\check{A}_{i,m},\ \hat{B}_{i,m} = \mathcal{R}_{2i}^{-1}\check{B}_{i,m},\ \hat{C}_{i,m} = \check{C}_{i,m} \quad (12)$$

Proof. According to Lemma 1, the error system $(\bar{\Sigma})$ is robustly asymptotically stable and guarantees a prescribed H_∞ error performance if there exist $\mathcal{P}_i(\lambda) > 0$ and $\mathcal{R}_i(\lambda)$ satisfying (10). Then, by defining matrix functions

$$\mathcal{P}_i(\lambda) \triangleq \begin{bmatrix} \mathcal{P}_{1i}(\lambda) & \mathcal{P}_{2i}(\lambda) \\ * & \mathcal{P}_{3i}(\lambda) \end{bmatrix},\ \mathcal{R}_i(\lambda) \triangleq \begin{bmatrix} \mathcal{R}_{1i}(\lambda) & E\mathcal{R}_{2i} \\ \mathcal{R}_{3i}(\lambda) & \mathcal{R}_{2i} \end{bmatrix}$$

and by considering (9) and some basic matrix manipulations, it can be readily established that (10) is equivalent to: $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$

$$\Xi^{ij}(\lambda) \triangleq$$

$$\begin{bmatrix} \mathcal{P}_{1j}(\lambda) - \mathcal{R}_{1i}(\lambda) - \mathcal{R}_{1i}^T(\lambda) & \mathcal{P}_{2j}(\lambda) - \mathcal{R}_{2i} - \mathcal{R}_{3i}^T(\lambda) & 0 & \mathcal{R}_{1i}(\lambda)A_i(\lambda) & E\mathcal{R}_{2i}\hat{A}_i(\lambda) & \begin{matrix}[\mathcal{R}_{1i}(\lambda)B_i(\lambda) \\ +E\mathcal{R}_{2i}\hat{B}_i(\lambda)]\end{matrix} \\ * & \mathcal{P}_{3j}(\lambda) - \mathcal{R}_{2i} - \mathcal{R}_{2i}^T & 0 & \mathcal{R}_{3i}(\lambda)A_i(\lambda) & \mathcal{R}_{2i}\hat{A}_i(\lambda) & \begin{matrix}[\mathcal{R}_{3i}(\lambda)B_i(\lambda) \\ +\mathcal{R}_{2i}\hat{B}_i(\lambda)]\end{matrix} \\ * & * & -I & C_i(\lambda) & -\hat{C}_i(\lambda) & 0 \\ * & * & * & -\mathcal{P}_{1i}(\lambda) & -\mathcal{P}_{2i}(\lambda) & 0 \\ * & * & * & * & -\mathcal{P}_{3i}(\lambda) & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (13)$$

Here, we further use the efficient parameter dependent stability idea in coping with uncertainties for general dynamic systems de Oliveira et al. [1999], Feron et al. [1996], and assume the matrix functions $\mathcal{P}_i(\lambda)$ and $\mathcal{R}_i(\lambda)$ to be the following forms:

$$\mathcal{P}_i(\lambda) \triangleq \sum_{m=1}^{s} \lambda_m \mathcal{P}_{i,m} = \sum_{m=1}^{s} \lambda_m \begin{bmatrix} \mathcal{P}_{1i,m} & \mathcal{P}_{2i,m} \\ * & \mathcal{P}_{3i,m} \end{bmatrix}, \quad (14)$$

$$\mathcal{R}_i(\lambda) \triangleq \sum_{m=1}^{s} \lambda_m \mathcal{R}_{i,m} = \sum_{m=1}^{s} \lambda_m \begin{bmatrix} \mathcal{R}_{1i,m} & E\mathcal{R}_{2i} \\ \mathcal{R}_{3i,m} & \mathcal{R}_{2i} \end{bmatrix}, \quad (15)$$

defining matrix variables ($1 \leq m \leq s$)

$$\check{A}_{i,m} \triangleq \mathcal{R}_{2i}\hat{A}_{i,m},\ \check{B}_{i,m} \triangleq \mathcal{R}_{2i}\hat{B}_{i,m},\ \check{C}_{i,m} \triangleq \hat{C}_{i,m} \quad (16)$$

and taking (3) and (13)-(16) into account, we have

$$\Xi^{ij}(\lambda) = \sum_{m=1}^{s}\sum_{n=1}^{s} \lambda_m \lambda_n \Xi^{ij}_{m,n}$$
$$= \sum_{m=1}^{s} \lambda_m^2 \Xi^{ij}_{m,m} + \sum_{m=1}^{s-1}\sum_{n=m+1}^{s} \lambda_m \lambda_n (\Xi^{ij}_{m,n} + \Xi^{ij}_{n,m}).$$

Thus, if condition (11) holds, then $\Xi^{ij}(\lambda) < 0$, which implies (10) holds, i.e. the error system $(\bar{\Sigma})$ is robustly asymptotically stable with a guaranteed H_∞ error performance γ, meanwhile, if a solution exists, the vertex matrices of reduced-order model system are given by (12). This completes the proof. □

Condition (11) in Theorem 1 is formulated in terms of a set of LMIs by letting $\nu = \gamma^2$, which can be solved by means of numerically efficient convex programming algorithms Boyd et al. [1994]. Moreover, the minimal performance index γ based on Theorem 1 can be obtained by the following convex optimization procedure:

$$\mathit{min}\ \nu\ \mathit{subject\ to}\ (11)$$

Note that the above minimal $\gamma = \sqrt{\nu}$ will be suboptimal for the error system $(\bar{\Sigma})$ because the condition (11) is sufficient.

4. ILLUSTRATIVE EXAMPLE

In this section, we will present a numerical example to demonstrate the validity and applicability of the developed theoretic result.

Consider the switched linear discrete-time system (1)-(2) consisting of two uncertain subsystems, where there are two groups of vertex matrices in subsystem 1:

$$A_{11} = \rho \begin{bmatrix} 1.3 & 2.2 & -1.3 & 0.8 \\ 0.5 & -0.3 & 1.9 & -0.6 \\ -0.7 & -0.5 & -0.4 & -1.2 \\ -1.7 & 2.1 & 0.3 & 2.8 \end{bmatrix}, B_{11} = \rho \begin{bmatrix} 1.9 \\ -1.8 \\ 1.6 \\ -0.8 \end{bmatrix},$$

$$C_{11} = \rho\,[\,12.0\ 5.0\ 1.3\ 0.61\,],$$

$$A_{12} = \rho \begin{bmatrix} -1.3 & 2.2 & -1.3 & 0.8 \\ 0.5 & -0.3 & 1.9 & -0.6 \\ 0.7 & -0.5 & -0.4 & -1.2 \\ -1.7 & 2.1 & 0.3 & -2.8 \end{bmatrix}, B_{12} = \rho \begin{bmatrix} 1.9 \\ 1.8 \\ -1.6 \\ 0.8 \end{bmatrix},$$

$$C_{12} = \rho\,[\,12.0\ -5.0\ 1.3\ -0.61\,],$$

and two groups of vertex matrices in subsystem 2:

$$A_{21} = \rho \begin{bmatrix} 1.1 & 2.2 & -1.3 & 0.8 \\ 0.5 & -0.3 & 1.5 & -0.6 \\ -0.7 & -0.3 & -0.4 & -1.2 \\ -1.7 & 2.1 & 0.3 & 2.0 \end{bmatrix}, B_{21} = \rho \begin{bmatrix} 2.3 \\ -1.3 \\ 1.6 \\ -0.4 \end{bmatrix},$$

$$C_{21} = \rho\,[\,12.0\ 5.0\ 1.3\ -0.41\,],$$

$$A_{22} = \rho \begin{bmatrix} -1.1 & 2.2 & -1.3 & 0.8 \\ 0.5 & -0.3 & 1.5 & -0.6 \\ 0.7 & -0.3 & -0.4 & -1.2 \\ -1.7 & 2.1 & 0.3 & -2.0 \end{bmatrix}, B_{22} = \rho \begin{bmatrix} 2.3 \\ 1.3 \\ -1.6 \\ 0.4 \end{bmatrix},$$

$$C_{22} = \rho\,[\,12.0\ -5.0\ 1.3\ 0.41\,],$$

where ρ is a scalar parameter denoting the size of convex polytope each uncertain subsystem can be expanded into. The arbitrary switching signal can be generated by Matlab and a possible case is shown in Figure 1, the corresponding algorithm can be referred to Zhang et al. [2006b]. Note that the switching instants are random in Figure 1, and the dwell time in each mode, which is coined in Morse [1996] and detailed in Liberzon and Morse [1999], might be one sampling instant or longer.

Here, we are interested in finding a second-order H_∞ reduced model in the form of (4)-(5) to approximate the above switched system such that the resulting error system are asymptotically stable with a suboptimal H_∞ error performance. By solving the corresponding convex optimization procedure in Theorem 1, the different error performance for given different ρ are calculated and listed in Table 1.

ρ	0.1	0.15	0.2
γ	0.0193	0.0975	0.3258

Table 1 : H_∞ approximation performance indexes by second-order reduced model

In addition, for given $\rho = 0.15$ and consider the input signal $\omega(k) = 0.8\exp(-0.4k)$, Figure 2 shows the output trajectories of the original system and second-order H_∞ reduced model by randomly giving different uncertain parameters λ in (3). Figure 3 presents the output errors between original system and the reduced-order system. It can be observed from simulations that the obtained reduced model approximate the original system very well under the given arbitrary switching signal and various parameter uncertainties.

5. CONCLUSIONS

The problem of H_∞ model reduction for switched linear discrete-time systems with polytopic uncertainties is investigated. A sufficient condition for the existence of desired reduced-order models is derived and formulated in terms of a set of LMIs. By solving the corresponding convex optimization problem in such sufficient condition, the expected reduced-order model is obtained and a suboptimal H_∞ gain for the error system between original system and reduced-order model is designed. A numerical example is included to show the potential and effectiveness of the developed theoretic result. It is noted that in this paper, the model reduction problem is solved by obtaining an uncertain reduced-order model to approximate the original system, thus it is believed that approximating an uncertain switched system by a fixed reduced model is also worth further investigation in the future.

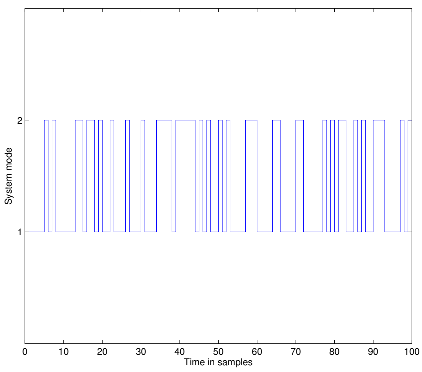

Fig. 1. Switching Signal

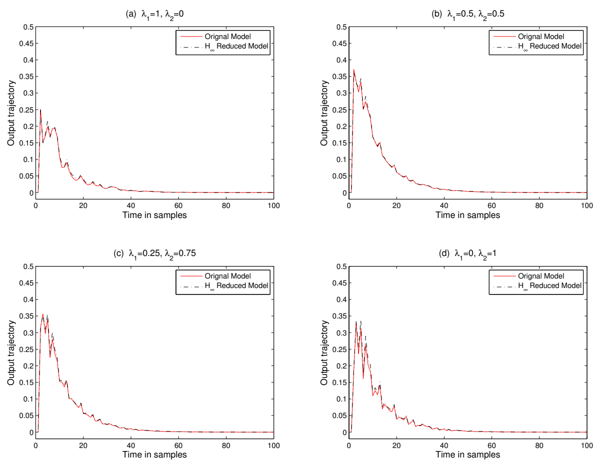

Fig. 2. Output trajectories of original system and second-order reduced model

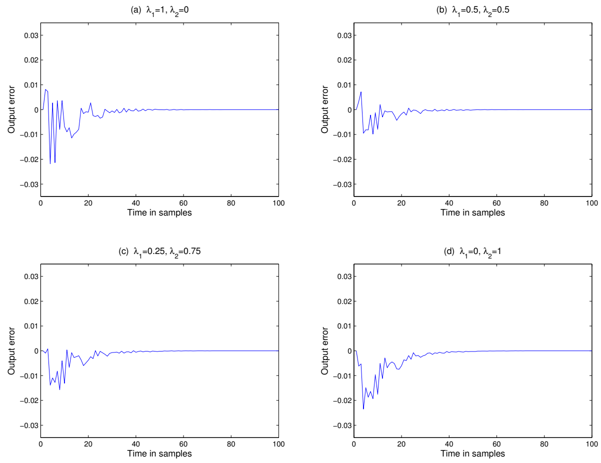

Fig. 3. Output errors between original system and second-order reduced model

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