

Planning setpoints for contact force transitions in regrasp tasks of 3D objects^{*}

Patrick Grosch^{*} Raúl Suárez^{*} Raffaella Carloni^{**}
Claudio Melchiorri^{**}

^{*} *Institut d'Organització i Control de Sistemes Industrials (IOC),
Technical University of Catalonia (UPC), Barcelona, Spain.
e-mails: patrick.grosch,raul.suarez@upc.edu.*

^{**} *Center for Research on Complex Automated Systems (CASYS),
Department of Electronics, Computer Science and Systems (DEIS),
University of Bologna, Bologna, Italy.
e-mails: rcarloni,cmelchiorri@deis.unibo.it*

Abstract: This paper presents a simple and fast solution to the problem of finding the time variation of n contact forces that keep an object under equilibrium while one of the n contact forces is removed/added from/to the grasp. The object is under a constant perturbation force, like for instance its own weight. It is assumed no acceleration of the object during the regrasp operation, as well as the knowledge of the starting and ending grasp configurations. The procedure returns the set points of the n contact forces for a feed-forward control system of a manipulator device in a regrasping action. The procedure was implemented and an illustrative numerical example is included in the paper.

1. INTRODUCTION

The search for flexible end-effectors and the development of grasping and manipulation strategies according to different criteria has become a growing research area during the last two decades [2, 3, 7, 10].

One of the issues within this research field lies is the regrasping of an object, i.e. the variation of the contact points on the grasped object while some grasp properties are kept. This particular task implies finding the initial/final grasp contact points, determining the finger movements, and computing the proper forces to be applied by the fingers when a contact is removed or a new contact is established in order to keep the equilibrium conditions and to satisfy the dynamic constraints of the system [12, 11]. Regrasping operations are typically needed when the pick-up grasp configuration is not compatible with the actions to be done with the object or with the object placement itself, for instance due to physical constraints in the environment, due to the non-holonomic constraints of the finger contacts, or due to the limits in the articulation ranges of the grasping device.

Different approaches have been presented in the regrasping problem. A detailed description including a discussion about the use of two manipulators can be found in [6]. Some relevant works are those of Tournassoud et al. [12], who proposed a system based on polyhedral models for manipulators equipped with parallel jaw grippers, and Kerr et al. [5] who used a multi-finger hand (this type of

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end-effectors are expensive and rarely found in industrial manipulators, but are useful in non repetitive tasks in unstructured environments due to their high dexterity). Recent works in regrasp [1, 8, 9] are focused on algorithms to determine the sequence of grasps configurations to go from an initial state to a desired final state, but they did not deal with the forces needed to perform the regrasp, which is the central point of this paper. The computation of optimal grasping forces in a given grasp configuration is presented in [4].

After this brief introduction the paper is organized as follows. In Section 2 the problem to be solved is described and formalized, followed by a particularization of the problem for a planar objects in Section 3. In Section 4 the problem for planar objects is analyzed, the behavior of the system dynamics is characterized, and a graphical tool used to find the solution of the problem is introduced. The proposed solution for planar objects is described in Section 5. In Section 6 the solution for planar object is generalized for 3D objects. An example is presented in Section 7 to illustrate the proposed approach. Finally, the last section of the paper gives some conclusions and describes ongoing and future works.

2. PROBLEM STATEMENT

The problem to be solved can be summarized as follows: Given a n contact point grasp of a 3D object that balances an external perturbation force (it may be the own object weight), we want to remove one of the contacts while keeping, during the action, the balance of the external force, or, as inverse situation, given a $n - 1$ contact point

grasp add a n^{th} contact point such that the additional finger helps in the balance of the external perturbation. Then, the problem to be solved is the determination of the time variation of the force set point functions for the contact forces that allows the n^{th} contact to be removed/added without losing the force equilibrium during the process.

This problem is found in regrasping manipulation of objects, when a finger is removed from one contact point on the object surface to be place in another one. In this particular case the stated problem appears twice, first when retreating the finger and second when replacing it on the desire new contact point.

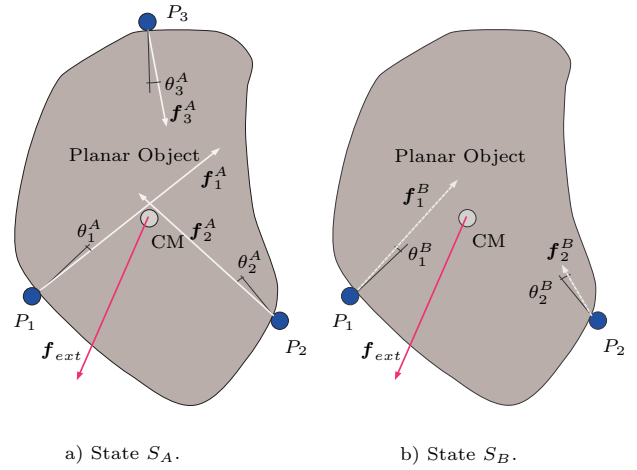


Fig. 1. Grasp states: a) initial configuration, b) final configuration.

3. PARTICULARIZATION OF THE PROBLEM FOR PLANAR OBJECTS

Before solving the general problem, an initial study is done for the case of a planar object in transition from three contact point to two contact point grasp. The following basic nomenclature will be used throughout the paper.

- S_A, S_B : two grasp states in equilibrium (forces applied at the contact points balance any external force)
- CM : center of mass of the object.
- f_{ext} : external force acting on the object (it may be the own object weight).
- P_i : contact point i on the object boundary.
- r_i : location of P_i with respect to CM.
- f_i : force applied on P_i .
- C_i : friction cone at P_i (set of possible forces f_i applicable at P_i).
- θ_i : angle between f_i and the object normal direction at P_i .
- τ_i : torque around CM produced by f_i applied on P_i .
- w_i : generalized force $w_i = (f_i, \tau_i)$.
- Π_0 : force plane in the wrench space (i.e. null torque plane).
- Π_i : plane in the wrench space containing all w_i generated at P_i .
- $S\Pi_i$: subset of Π_i containing w_i generated at P_i due to forces f_i inside C_i .
- $S\Pi'_i$: representation of $S\Pi_i$ with all the force heads on the cone origin.

Let S_A be a grasp with three contact points $P_i, i = 1, 2, 3$, on the object boundary (Figure 1a) and S_B be another grasp with only two contact points, which are points P_1 and P_2 from S_A (Figure 1b). It is assumed that in S_A and S_B the finger forces f_i applied at P_i balance an external perturbation force f_{ext} , i.e. the summations of the forces and moments applied on the object are null.

The problem to be solved can now be stated as the search of the time variation of the finger forces $f_1(t)$ and $f_2(t)$ that balance f_{ext} while $f_3(t)$ varies from its value in S_A to zero in S_B or vice versa. $f_1(t), f_2(t)$ and $f_3(t)$ are the setpoints values for the finger control system during the manipulation action.

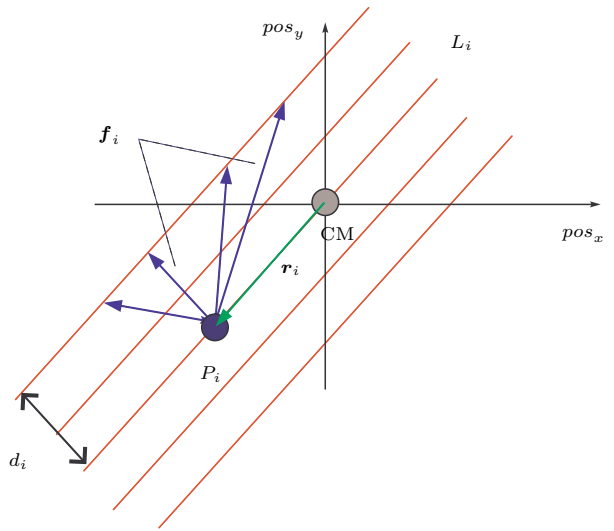


Fig. 2. Lines L_i of constant torque τ_i due to forces f_i applied at P_i .

4. PROBLEM ANALYSIS

4.1 Torques generated by contact forces

A force f_i applied at P_i produces, with respect to the object center of mass CM, a torque $\tau_i = f_i \times r_i$, where r_i describes the position of P_i with respect to CM.

Consider a line L_i parallel to r_i (see Figure 2). Any f_i applied at P_i such that the vector f_i represented with the tail at P_i has its head on L_i produces the same torque τ_i , thus we refer to the lines L_i as iso-torque lines. The value of τ_i associated to a given L_i is the product of $\|r_i\|$ (which is constant for a given point P_i) times the distance d_i between L_i and P_i , thus τ_i linearly varies with respect to d_i . This linearity means that, in the wrench space, all the wrenches $w_i = (f_i, \tau_i)$ (i.e. the wrenches produced by a force f_i applied at P_i) define a plane Π_i (see Figure 3). Since P_i is a contact point on the object boundary, f_i cannot have any direction, it is constrained to lie inside the friction cone C_i , and therefore only a subset of Π_i , called $S\Pi_i$, can be actually generated. $S\Pi_i$ is the projection along the τ -axis of C_i over Π_i (Figure 3).

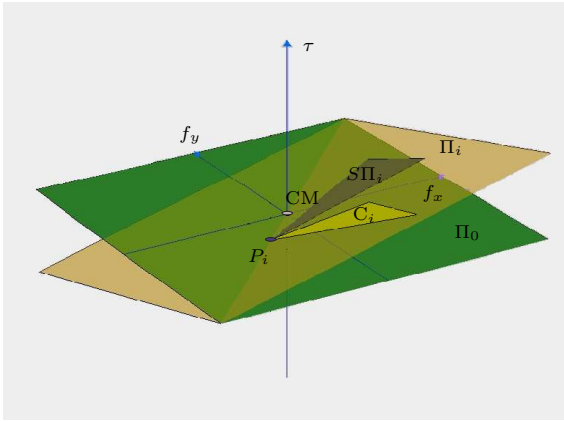


Fig. 3. C_i (light gray cone), and subset SII_i (dark gray cone) of Π_i (C_i and SII_i stretch out from P_i to infinity).

4.2 Wrench loops

The system equilibrium under wrenches w_i in the 3D space due to forces f_i applied on P_i , would be graphically analyzed and characterized. The equilibrium condition is that $\sum w_i + w_{ext} = 0$, being w_{ext} the wrench produced by f_{ext} ; note that if f_{ext} is the object weight then $w_{ext} = (f_{ext}, 0)$. Graphically, this condition can be seen as a closed loop path in the 3D wrench space drawing all the vectors w_i and w_{ext} with the tail attached to the head of another one. From now on, this loop will be called “wrench loop”, and the set of all the possible wrench loops produced by the possible wrenches generated at the contact points will be called “Generic Wrench Loop” (GWL). Since the grasp states S_A and S_B are assumed to be in equilibrium, the GWL is always non null, and it can be graphically constructed as follows (remind that w_i can be represented as free vectors so they can be translated in the wrench space with no loss of significance).

- (1) Consider first the vector representing the external wrench $w_{ext} = (f_{ext_x} \ f_{ext_y} \ 0)$ (the vector with the tail at the origin in Figure 4).
- (2) The second vector to be considered is the wrench w_1 due to f_1 applied on P_1 . Since $f_1 \in C_1$ then $w_1 \in SII_1$, thus the entire SII_1 is represented with its vertex on the head of f_{ext} (Figure 4).
- (3) The third vector to be considered in the path loop is the wrench w_2 due to f_2 applied on P_2 . As in the previous step, $f_2 \in C_2$ then $w_2 \in SII_2$, and the entire SII_2 can be represented with its vertex on the tail of f_{ext} (i.e. the origin of the wrench space)(Figure 4); this links the tail of the vectors w_2 with the tail of f_{ext} , in order to properly link the wrench vectors (i.e. make the head of w_2 matching the tail of f_{ext}), the vectors in SII_2 are graphically represented with their heads on the vertex of SII_2 , defining in this way the cone SII'_2 symmetrical of SII_2 with respect to the vertex, as it is illustrated in Figure 5 (for clarity purpose, from now on the plane Π_0 is not represented in the figures).

$LS_B = SII_1 \cap SII'_2$ is the set of points that define all the combinations of w_1 and w_2 that balance f_{ext} (see the enlargement in Figure 5), i.e. they indicate

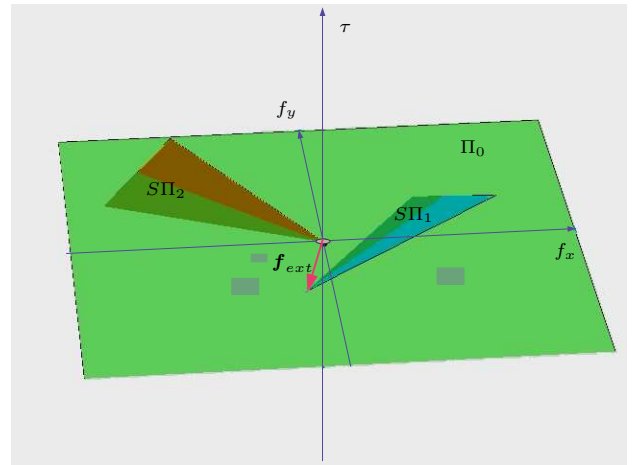


Fig. 4. f_{ext} and two friction cones SII_1 and SII_2 .

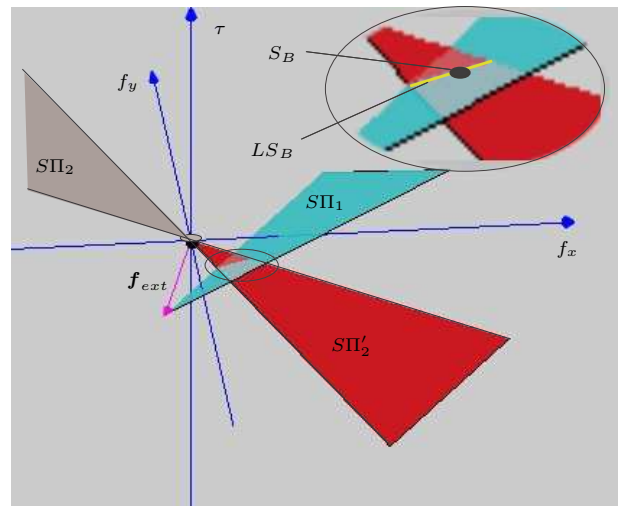


Fig. 5. GWL for S_B , the enlargement shows LS_B .

the combinations of forces f_1 and f_2 applied at P_1 and P_2 that balance f_{ext} and therefore a valid set of forces to reach the equilibrium in S_B . We refer to LS_B as the equilibrium loci for S_B . Note that f_2 could be considered in the Step 2 and then SII'_1 would be considered in this step.

- (4) Finally, the vector w_3 due to the f_3 applied at P_3 is added. Assuming that the value of w_1 is known (it is a point inside SII_1), SII_3 can be represented with its vertex on to the head of the given value of w_1 inside SII_1 . Doing this, $LS_A = SII_3 \cap SII'_2$ is the set of points that define all the combinations of w_2 and w_3 that balance f_{ext} for the given w_1 , generating a wrench loop and allowing therefore the equilibrium of S_A (see Figure 6).

5. PROPOSED SOLUTION

The graphical representation of GWL is used now to determine the temporal evolution of w_1 , w_2 , and w_3 , to change from S_A to S_B . Since the sets SII_i are convex, the simplest way to change the wrenches w_i from their value in S_A to their value in S_B assuring that $w_i \in SII_i$ is to make them follow a straight line, while keeping a closed wrench

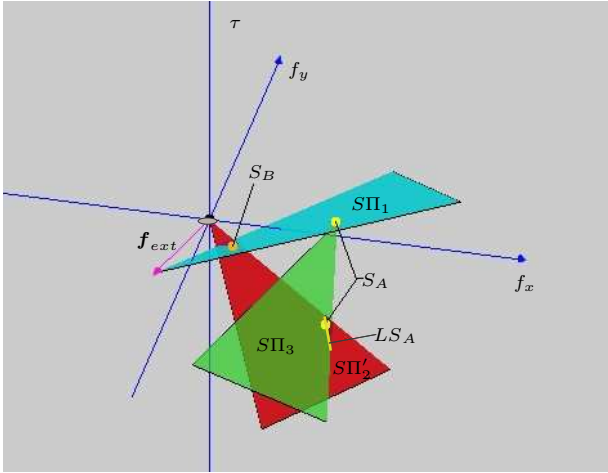


Fig. 6. GWL for S_A showing the three friction cones SII_1 , SII_2 and SII_3 .

loop. Thus, consider that w_1 varies on a straight segment $Path_1 \in SII_1$ and w_2 on a straight segment $Path_2 \in SII_2$.

The plane defined by $Path_1$ and $Path_2$, constrains w_3 to lie on the intersection of this plane with SII_3 while the vertex of SII_3 slide on $Path_1$ when w_1 change from S_A to S_B . This intersection defines $Path_3$, which fix a constant direction for w_3 while its module decreases from the initial value in S_A to zero in S_B . In order to keep a closed wrench loop (i.e. the equilibrium), the three paths can be followed changing in a synchronized way the magnitudes of w_1 , w_2 and w_3 , this makes the triangle defined by the three paths to decrease from the initial state S_A up to disappear in S_B keeping the same shape. Figure 7 shows an example of the vectors w_1 , w_2 and w_3 in an intermediate state (white vectors) while changing from S_A to S_B , the final vectors w_1 and w_2 in S_B (white dashed line vectors), and the $Path_1$, $Path_2$ and $Path_3$.

Using the supraindex A and B to indicated the values of w_i in states S_A and S_B respectively, and letting $T(t)$ be a function that smoothly varies in time between one and zero, the time variations of w_i according to this behavior can be expressed as,

$$w_1(t) = w_1^B + (w_1^A - w_1^B) T(t) \quad (1)$$

$$w_2(t) = w_2^B + (w_2^A - w_2^B) T(t) \quad (2)$$

$$w_3(t) = w_3^A T(t) \quad (3)$$

Note that w_1 and w_2 move, respectively, along the straight segments $Path_1$ and $Path_2$ as linear functions of $T(t)$ while w_3 decreases to zero keeping always the same direction.

6. GENERALIZATION TO 3D OBJECTS

The proposed solution can be generalized for the case of 3D objects considering 3-dimensional forces in

$$f_i(t) = f_i^B + (f_i^A - f_i^B) T(t) \quad i = 1..n \quad (4)$$

or considering 6-dimensional wrenches in

$$w_i(t) = \begin{pmatrix} f_i(t) \\ r_i \times f_i(t) \end{pmatrix} = w_i^B + (w_i^A - w_i^B) T(t) \quad (5)$$

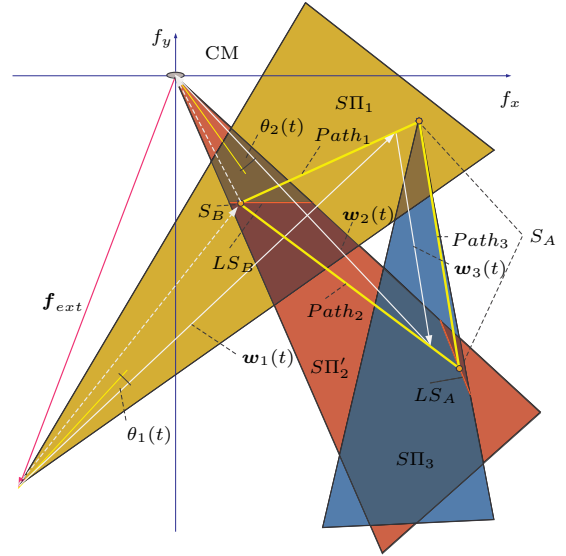


Fig. 7. GWL for S_A and S_B , including the initial forces in S_A , the final forces in S_B , and the paths $Path_i$ for the three wrenches w_i .

which are equivalent (applying the distributive property of the cross product). In order to prove that the solution given by equations (4) and (5) can be used as a general solution for n contact points on a 3D object it must be proved that: **first**, it satisfies the general equilibrium condition

$$\sum_{i=1}^n w_i(t) + \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} = 0 \quad \text{for } t_A \leq t \leq t_B \quad (6)$$

and **second**, that $f_i(t)$, $i = 1..n$, lie inside the friction cone at the respective contact points.

To prove the first statement the equilibrium conditions at the initial grasp state S_A and the final grasp state S_B are used. These conditions can be written as

$$\sum_{i=1}^n w_i(t_A) + \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} = \sum_{i=1}^n w_i^A + \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} = 0 \quad (7)$$

and

$$\sum_{i=1}^n w_i(t_B) + \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} = \sum_{i=1}^n w_i^B + \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} = 0 \quad (8)$$

which can be rewritten as

$$\sum_{i=1}^n w_i^A = \sum_{i=1}^n w_i^B = - \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} \quad (9)$$

Replacing $w_i(t)$ in equation (6) by the expression given in equation (5),

$$\sum_{i=1}^n [w_i^B + (w_i^A - w_i^B) T(t)] + \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} = 0 \quad (10)$$

rearranging equation (10)

$$\sum_{i=1}^n w_i^B + \left(\sum_{i=1}^n w_i^A - \sum_{i=1}^n w_i^B \right) T(t) + \begin{pmatrix} f_{ext} \\ 0 \end{pmatrix} = 0 \quad (11)$$

and replacing the summations in equation (11) using equation (9)

$$-\begin{pmatrix} \mathbf{f}_{ext} \\ 0 \end{pmatrix} + (-\begin{pmatrix} \mathbf{f}_{ext} \\ 0 \end{pmatrix} + \begin{pmatrix} \mathbf{f}_{ext} \\ 0 \end{pmatrix}) T(t) + \begin{pmatrix} \mathbf{f}_{ext} \\ 0 \end{pmatrix} = \mathbf{0} \quad (12)$$

that gives zero $\forall t$, which is the first prove needed.

For the second proof consider that $\mathbf{f}_i(t)$ is a vector function defining in the force space points on the straight line defined by \mathbf{f}_i^A and \mathbf{f}_i^B , now, since \mathbf{f}_i^A and \mathbf{f}_i^B belongs to the friction cone and the friction cone is a convex space, all the points defined by $\mathbf{f}_i(t)$ lie inside the friction cone.

7. EXAMPLE

The proposed approach has been implemented and we describe here an example in 2D to illustrate how it works. The problem to be solved is the force transition for the object and the states S_A and S_B shown in Figure 1.

Given the external force $\mathbf{f}_{ext} = [-1.5 \ -3.5]$, and the contact points $P_1 = [-4 \ -4]$, $P_2 = [4 \ -5]$ and $P_3 = [0 \ 8]$, the applied forces that produce equilibrium at S_A and S_B are:

$$\begin{aligned} \mathbf{f}_1^A &= [3.7897 \ 3.0034] \\ \mathbf{f}_2^A &= [-3.0096 \ 4.4156] \\ \mathbf{f}_3^A &= [0.7199 \ -3.9190] \\ \mathbf{f}_1^B &= [1.8557 \ 2.4555] \\ \mathbf{f}_2^B &= [-0.3557 \ 1.0445] \end{aligned}$$

With these forces and contact points the following wrenches are generated:

$$\begin{aligned} \mathbf{w}_1^A &= [3.7897 \ 3.0034 \ 3.1448] \\ \mathbf{w}_2^A &= [-3.0096 \ 4.4156 \ 2.6145] \\ \mathbf{w}_3^A &= [0.7199 \ -3.9190 \ -5.7593] \\ \mathbf{w}_1^B &= [1.8557 \ 2.4555 \ -2.3930] \\ \mathbf{w}_1^B &= [-0.3557 \ 1.0445 \ 2.3930] \end{aligned}$$

In order to produce a smooth transition at the beginning and at the end of the finger remove action the function $T(t)$ was defined by a spline with five control points (Figure 8), which assures $dT(t)/dt = 0$ at the initial time ($t = 0$) and at the desired final time ($t = 4$).

Using equations (1), (2) and (3), the functions $\mathbf{w}_1(t)$, $\mathbf{w}_2(t)$ and $\mathbf{w}_3(t)$ that allow the object equilibrium were obtained; the results are graphically shown in Figure 9 that shows the variation in the magnitude of $\mathbf{f}_i(t)$, $i = 1, 2, 3$, and Figure 10 that shows the variation in the angles θ_i between the object normal direction and $\mathbf{f}_i(t)$. Note that the direction of $\mathbf{f}_3(t)$ is constant while its module decreases to zero, and that the directions of $\mathbf{f}_1(t)$ and $\mathbf{f}_2(t)$ remains all the time inside the friction cone limits. Figure 11 shows the physical object with the forces \mathbf{f}_i in an intermediate situation between the states S_A and S_B and the Path₁, Path₂ and Path₃ being followed.

As an additional verification of the system equilibrium, it was checked whether $\mathbf{f}_{ext}^T - \mathbb{G}\mathbf{f}_g^T = 0$ is satisfied, being \mathbb{G} the grasp matrix and $\mathbf{f}_g = [\mathbf{f}_1^{P_1}, \mathbf{f}_2^{P_2}, \mathbf{f}_3^{P_3}]^T$ with $\mathbf{f}_i^{P_i}$ the force \mathbf{f}_i expressed in a coordinate system fixed at P_i ; the condition was satisfied $\forall t$.

8. CONCLUSIONS AND FUTURE WORKS

A fast non iterative solution to the problem of finding the force variations that keep the object equilibrium when a finger is removed from a n contact point grasp (or added to a $n - 1$ contact point grasp) has been proposed and implemented. The approach is simple and efficient.

The ongoing work includes the determination of a procedure to change from a grasp with n contacts to another grasp with n different contacts (doing in this way a full regrasp of the object), automatically solving intermediate consecutive grasps S_j that differ in only one contact point

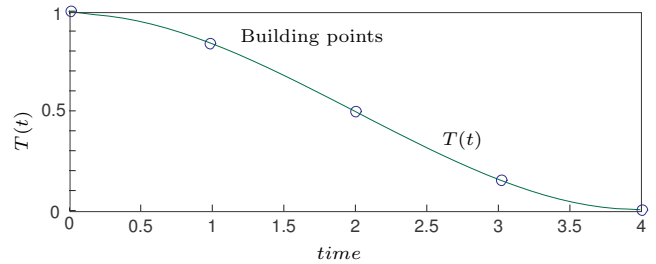


Fig. 8. Time function $T(t)$.

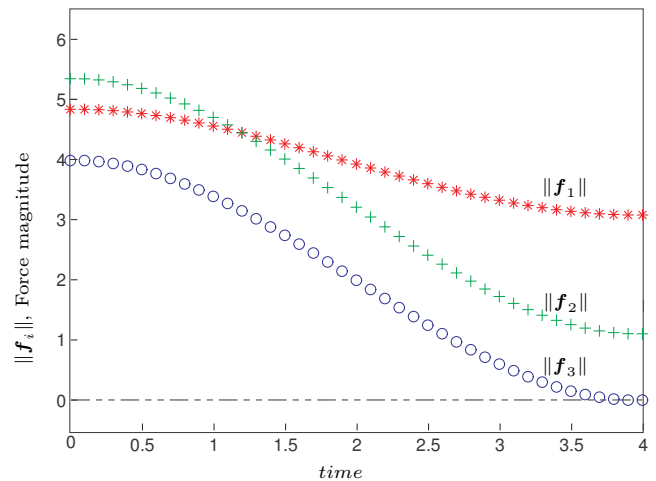


Fig. 9. Variation of the magnitude of \mathbf{f}_i .

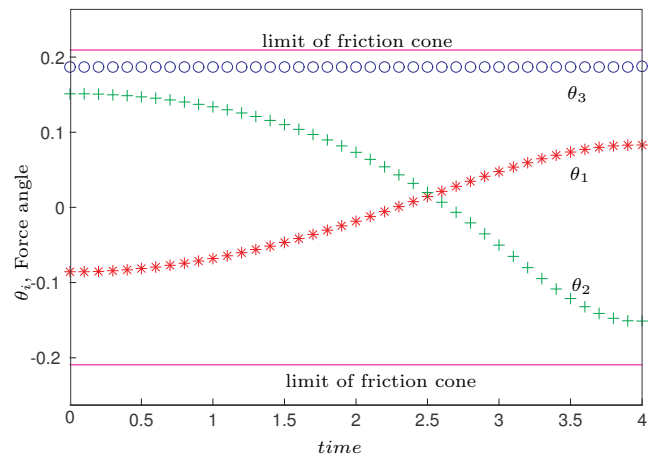


Fig. 10. Variation of the direction of \mathbf{f}_i (angle between the normal direction and \mathbf{f}_i).

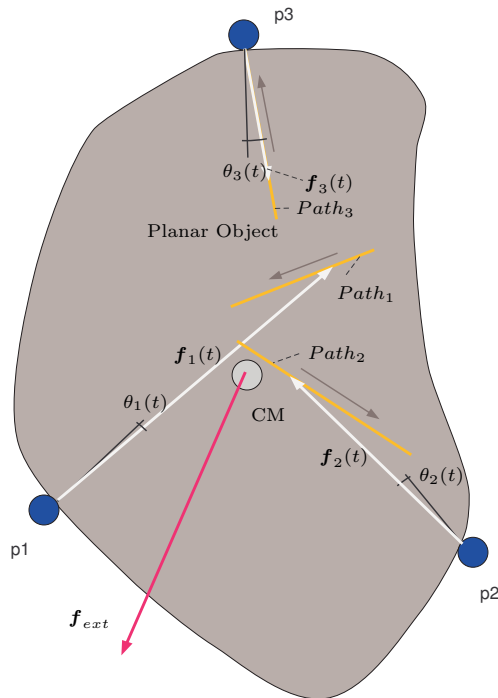


Fig. 11. Physical object, Path₁, Path₂ and Path₃, and forces f_1 , f_2 and f_3 during the state transition.

and changing the object orientation when necessary using wrist movements. The whole procedure would generate position and force set points for the control system of the grasping device. Given the initial and final grasp with n contact points (fingers) the approach includes the following subproblems:

- (1) Automatic determination of a sequence of grasp states that balance the external force (object weight), alternately considering grasps with $n-1$ and n fingers (i.e. repositioning one finger at a time), which is equivalent to automatically and alternately determine for each step the grasp states S_A and S_B in this paper (other than the given initial and final states in the sequence). The search can be done using a "Generic Wrench Loop" (GWL) that describes the forces of the fingers that do not change and selecting a proper point on the corresponding region LS (equivalent to the region LS_B in Figure 5).
- (2) Automatic determination, if necessary, of wrist movements to change the object orientation and the force variations to keep the equilibrium when these movements are performed. Again, this can be done using the GWL representation.

Besides, some dynamic considerations could be addressed in future works, as well as some strategies to assure robustness in front of different sources of uncertainty.

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