

A New Nonlinear Control Methodology for Irrigation Canals Based on a Delayed Input Model

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Abstract: This paper is devoted to nonlinear feedback design for irrigation canals. Such systems are classically described by Saint-Venant nonlinear partial differential equations.

Here instead, an ordinary differential equation model (still nonlinear) with a state-dependent input delay is used, on the basis of a model previously proposed in Litrico et al [2003]. The control design approach is based on a state prediction computation and the state predictor is constructed from a dynamic inversion in the same spirit as in Georges et al [2007]. The proposed methodology is analyzed and tested in simulation, first on the basis of the control model, and then using some "more accurate" model.

1. INTRODUCTION

Irrigation canals are used to conduct water from its upstream source towards downstream users. Managing irrigation canals efficiently (i.e satisfying users needs) and at the same time reducing water waste is an increasingly important issue. For these reasons, control approaches have been more investigated in the last decade (see for example [Mareels et al, 2005, Litrico et al, 2005, Halleux et al, 2003, Georges et al, 2002, ...] and references therein).

Canals belong to the class of transport systems, where the delay plays a major role in the dynamics. Those dynamics are usually mathematically described by Saint-Venant nonlinear hyperbolic partial differential equations. Saint-Venant equations don't have a known analytical solution, unless for special cases with no friction and no slope (Malaterre et al [1998]). To obtain an approximate solution of Saint-Venant equations different techniques have been developed. The most used numerical scheme for hydro-dynamic is the implicit Preissmann scheme (Chaudhry [1987]).

There are two classical politics to control irrigation canals: the local upstream control and the distant downstream control (for more details see Malaterre et al [1998]). In the present paper, we focus on the distant downstream politics which consist in controlling the downstream water level using the upstream control variable. Its main advantage lies in reducing water waste.

Different methodologies have been used to design controllers which are classified from linear to nonlinear ones. Among the most cited linear controllers, we find the classical linear PID approach (Malaterre et al [1998]). Despite of its simple implementation, it does not however take into account the time delay explicitly, and including a Smith predictor makes the control sensitive to modeling errors. In order to better take into account perturbations and modeling errors, robust approaches have been developed (Malaterre et al [2000], Litrico et al [2006]). A predictive approach has also been investigated in Rodellar et al [1989] or in Bogovich et al [2007].

Those controllers on the other hand neglect the nonlinear dynamics of the canal, which might limit the performances obtained with linear controllers. Therefore, a nonlinear approach can be of interest. Dulhoste et al [2004] have developed a nonlinear control law based on an Input/Output linearization method. Interesting results have been obtained in simulation, as well as in real-time experiments performed by Besançon et al [2004]. However, this approach is limited so far to upstream local control and the time delay is not explicitly taken into account in the model. In the approaches cited above, the control law is determined from a finite dimension model. In Coron et al [2007], by means of a Lyapunov approach a stabilizing boundary control laws is proposed from the Saint-Venant equations.

The objective of the present work is to develop a nonlinear control law for irrigation canal based on a nonlinear model where the delay explicitly appears.

To that end, the canal is described by a nonlinear *statedependent* input delay model of the form:

$$\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t - \tau(x(t)))$$
(1)

derived from the recent work of Litrico et al [2003].

The control law is based on a state predictor which is constructed by a dynamic inversion.

In order to illustrate the control performances (transient and steady-state responses), we first test the method on a model (1), and then on a more realistic model given by a Preissmann scheme.

The paper is organized as follows: section 2 focuses on the canal modeling. We will first recall classical Saint-Venant equations and then present the input-delay model that will be considered for control design. In section 3, the proposed control approach is presented on the basis of a state predictor. Section 4 then gives some corresponding simulation results on the basis of a delayed-input simulator for the canal on one hand, and a more accurate Saint-Venant based model on the other one. Finally, some conclusions and extensions of our work are proposed in section 5.

2. CANAL MODEL

The classical representation of water flow dynamics in a canal is given by so-called Saint-Venant equations of the following form:

$$L\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{1}{L}\frac{\partial [Q2/h]}{\partial x} + gLh\left[\frac{\partial h}{\partial x} - I + J(h,Q)\right] = 0$$
 (2)

with t: time variable, x: space variable, Q: water flow rate, h: water level, L: canal width, g: gravitational acceleration, I: canal slope, and J: friction (in general nonlinearly depending on h, Q).

In the present work, following Litrico et al [2003], we will instead use an ODE model with state-dependent input delay.

In Litrico et al [2003] indeed, the authors have proposed a nonlinear output delay ODE model, where the delay depends on the discharge (water flow). This model is based on the diffusive wave model (equation 3) obtained after simplification of Saint-Venant equations. This model is valid assuming uniform flow and rectangular wide channel.

$$\frac{\partial Q}{\partial t} + \nu(Q)\frac{\partial Q}{\partial x} - \Delta(Q)\frac{\partial 2Q}{\partial x^2} = 0 \tag{3}$$

where Q(x,t) is the water flow at point x and time t, ν is the celerity coefficient, and Δ the diffusion one, given by:

$$\nu(Q) = \frac{5I^{0.3}Q^{0.4}}{3L^{0.4}n^{0.6}}$$
$$\Delta(Q) = \frac{Q}{2LI}$$

with n the Manning friction coefficient (characterizing the friction J).

The boundary conditions are given by Q(x = 0, t) = u(t)where u is the control input, and $\lim_{x\to\infty} \frac{\partial Q}{\partial x} = 0$.

In Litrico et al [2003], from linearization around different equilibrium points, a family of linear ODEs is first obtained. It is then shown that it can be represented by one nonlinear ODE with a delayed output of the form:

$$\dot{x}(t) = A(x_2(t))x(t) + B(x_2(t))u(t) y(t) = x_2(t - \tau(x_2(t)))$$
(4)

where: $x = (x_1 \ x_2)^T$ is the state vector, $x_2 = Q(x = X, t)$ is the downstream water flow (for a canal of length equal to X), x_1 is its time derivative and A, B are recalled bellow:

$$A = \begin{pmatrix} -\frac{S}{P} & -\frac{1}{P} \\ 1 & 0 \end{pmatrix}$$
$$B = \begin{pmatrix} \frac{1}{P} \\ 0 \end{pmatrix}$$
$$\tau = \frac{X}{\nu} - S$$

$$S = 2\sqrt{\frac{2X\Delta}{\nu 3}}\cos(\frac{\phi}{3})$$
$$P = \frac{2X\Delta}{\nu 3}(1 - \frac{3\Delta}{S\nu^2})$$
$$\phi = \frac{\pi}{2} + \arctan\sqrt{\frac{9\Delta}{2X\nu - 9\Delta}}$$

Based on recently developed control methodologies for nonlinear systems with delayed input (Georges et al [2007], ?]), our idea is to derive from equation (4) a nonlinear model with delayed input.

Using the transfer function resulting from the linearization of the diffusive wave equation:

$$F = \frac{Ge^{\tau s}}{1 + Ss + Ps2} \tag{5}$$

and the proof in Litrico et al [2003], it can easily be shown that system (4) can be written as follows:

$$\dot{\xi}(t) = A(\xi_2(t))\xi(t) + B(\xi_2(t))u(t - \tau(\xi_2(t)))$$

$$y(t) = \xi_2(t)$$
(6)

with A, B as above.

In the same spirit as in Georges et al [2007], where the case of *time-varying* input delay has been considered, we will propose a control scheme for system (6).

3. CONTROL SCHEME

There is a wide literature on linear time-delay systems (see Niculescu et al [2004] for a recent picture of this area), and for nonlinear systems (see Maza-Casas et al [2000], Mazenc et al [2004, 2006], Zhang et al [2006]), but very few of them are dedicated to time-varying or even state-dependent delays as in Verriest [2002], and a fortiori for nonlinear systems.

In this section, the purpose is to propose a control methodology for nonlinear systems with state-dependent input delay, as in model (6) of irrigation canals presented above. The chosen approach is based on the so-called 'finite spectrum assignment' which already exists for linear delayedinput systems (Manitius et al [1979], Mondie et al [2003]). This approach also follows our recent work presented in Georges et al [2007] for the case of *time-varying* input delays. It can be presented for systems of the form:

$$\dot{x}(t) = F(x(t), u(t - \tau(x(t))))$$
(7)

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $\tau(x(t))$ is a varying delay with known evolution w.r.t. its arguments, assumed to remain larger than 0 for any t, x (for causality). Our approach is based on the following principle: firstly a prediction of the state at an appropriate prediction time δ , denoted by $x_p(t, t + \delta)$, is computed from the available state x(t) at time t and input controls $u(\theta), \ \theta \in [t - \delta, t]$. Then the predicted state is used to compute the control law. The prediction time is chosen so that the effect of the delay vanishes and the closed-loop system is no more a time-delay system. This can be obtained by choosing δ such that:

$\tau(x(t+\delta(t))) = \delta(t)$

The control law is chosen according to the non-delayed system, yielding a state feedback law $u = \Phi(x)$, and from

this we get the control law to be applied to the delayed system as:

$$u(t) = \Phi(x_p(t, t+\delta)).$$

According theorem 1 of Besançon et al [2007], the control law u(t) can be calculated as for nonlinear systems without delay. The next point is to compute the predicted state $x_p(t,t+\delta(t))$: we propose to do this comptuation in an approximate way for instance by using one step of an implicit Euler method as follows:

$$\hat{x}_p(t, t + \delta(t)) = x(t) + \delta(t)F(\hat{x}_p(t, t + \delta(t)), u(t)).$$
(8)

The problem now turns to be the on-line computation of the fixed point $X(t) := \hat{x}_p^T(t, t + \delta(t))$ solution of

$$\hat{x}_p(t,t+\delta(t)) = x(t) + \tau(\hat{x}_p(t,t+\delta(t))) \times F(\hat{x}_p(t,t+\delta(t)), \Phi(\hat{x}_p(t,t+\delta(t))))$$
(9)

which can be written as:

$$X(t) = H(X(t), x(t)).$$
 (10)

This fixed point computation could be performed for example by using a Newton-Raphson method.

This technique being time consuming - and therefore not appropriate for on-line computation - we instead propose another approach based on "dynamic inversion".

Suppose that we seek for the solution of G(x, t) = 0, where G is a nonlinear C1-function : $\mathbb{R}^n \times [0, +\infty) \to \mathbb{R}^n$ and the Jacobian matrix $\frac{\partial G}{\partial x}$ is supposed to be invertible. The main idea is how to compute the solution of the differential equation

$$\dot{G} + \Lambda G = 0 \tag{11}$$

where Λ is any positive definite matrix ensuring the asymptotic stability of this equation. In the coordinates x, the equation (11) is equivalent to

$$\frac{\partial G}{\partial x}\dot{x} + \frac{\partial G}{\partial t} + \Lambda G(x,t) = 0.$$
(12)

Since $\frac{\partial G}{\partial x}$ has full rank, (12) is equivalent to

$$\dot{x} = -\left[\frac{\partial G}{\partial x}\right]^{-1} \left[\frac{\partial G}{\partial t} + \Lambda G(x, t)\right].$$
(13)

The motivation may be found in the fact that if the initial state x_0 is a solution of G(x, t = 0) = 0, then the trajectory x(t) of (11), is a solution of G(x(t), t) = 0, $\forall t > 0$. Since (11) is asymptotically stable, even when the initial state is not a solution of G(x, t = 0) = 0, x(t) will reach asymptotically the manifold G(x,t) = 0, since the solution of (11) is $G(x,t) = e^{-\Lambda t}G(x(0),0)$ and $\lim_{t \to +\infty} G(x(t), t) = 0 \text{ for all } \Lambda > 0. \text{ The coefficient } \Lambda \text{ can}$ be used to control the speed of convergence.

The application of this approach to (10) leads to the stateprediction-based control law given by:

$$\dot{X} = (I_d - \frac{\partial H}{\partial X})^{-1} [F(x(t), u(t - \tau(x(t)))) - \Lambda(X - H(X, x(t)))]$$
(14)

$$u(t) = \Phi(X(t)) \tag{15}$$

with

$$\frac{\partial H}{\partial X} = \frac{\partial \tau}{\partial x}F + \tau \frac{\partial F}{\partial x} + \tau \frac{\partial F}{\partial x} \frac{\partial \Phi}{\partial x}$$

Notice that the stability of the closed-loop system resulting from control (14)-(15) can now be analyzed in a similar spirit as in Georges et al [2007], but taking into account the fact that the delay now depends on the state.

To that end, it can be noticed that this practical realization of the control law introduces an additive perturbation term depending on the state x(t) for the closed-loop dynamics expressed at time $t + \delta(t)$.

From the fixed point problem (10) indeed it is first clear that there exists a function ψ such that $\hat{x}_p(t, t + \delta) =$ $\psi(x(t))$ (using the implicit function theorem).

With the notation $z(t) = x(t + \delta(t))$, the closed-loop system can be expressed as

$$\begin{aligned} \dot{z}(t) &= (1+\delta)F(z(t), \Phi(\psi(x(t)))) \\ &= (1+\dot{\delta})F(z(t), \Phi(z(t))) \\ &+ (1+\dot{\delta})[F(z(t), \Phi(\psi(z(t-\delta)))) - F(z(t), \Phi(z(t)))] \end{aligned} (16) \\ &= (1+\dot{\delta})\{F(z(t), \Phi(z(t))) + P(z(t), z(t-\delta))\} \end{aligned}$$

Notice that from a Taylor series expansion of z(t) at time $t-\delta$, we get an expression of z(t) w.r.t. $z(t-\delta)$ and clearly P vanishes when its arguments are zero. Consequently, on some neighborhood of the origin small enough, we can get: $\|P(z(t), z(t-\delta))\| \leq \gamma \|z(t-\delta)\|$ for some γ . Considering first the control law obtained by on-line solving of the fixed point equation (10), this motivates the following statement:

Theorem 1. If the following conditions hold:

• There exists a smooth control law Φ and a Lyapunov function V(x) such that the following assumptions hold. , $\forall x \in D$, where $D \subset \mathbb{R}^n$ is a domain that contains the origin:

(1)
$$c_1 ||x||^2 \le V(x) \le c_2 ||x||^2$$

(2)
$$\frac{\partial V(x)}{\partial x} F(x, \Phi(x)) \leq -c_3 ||x||^2$$

(3) $||\frac{\partial V(x)}{\partial x}|| \leq c_4 ||x||$

where c_1 , c_2 , c_3 and c_4 are some positive scalar numbers.(Khalil [1996])

The perturbation term $P(z(t), z(t - \delta))$ satisfies $\begin{aligned} \|P(z(t), z(t-\delta))\| &\leq \gamma \|z(t-\gamma)\|, \ \forall z(t) \in D \ \text{and} \ \gamma \end{aligned}$ "small enough", e.g. $\gamma \leq \frac{c_3}{4c_4},$

then the closed-loop system (16) is locally asymptotically stable.

Proof. Let us consider the Lyapunov-Krasovskii function candidate $W(z_t) = V(z) + \mu \int_{t-\delta}^{t} ||z(\theta)||^2 d\theta$, where $z_t =$

 $z(t + \theta), \theta \in [-\delta, 0]$ as usual in Lyapunov-Krasovskii formalism (see e.g. Hale et al [1993]), and $\mu > 0$ is to be specified later on.

Then the time derivative of W is given by:

$$\dot{W} = (1+\dot{\delta})\frac{\partial V(z)}{\partial z}F(z,\Phi(z)) + (1+\dot{\delta})\frac{\partial V(z)}{\partial z}P(z(t),z(t-\tau)) + \mu[\|z(t)\|^2 - (1-\dot{\delta}(t))\|z(t-\delta)\|^2].$$
(17)

Now notice that from the definition of δ we have:

$$\left[1 - \frac{\partial \tau}{\partial x}(z)f(z)\right]\dot{\delta} = \frac{\partial \tau}{\partial x}(z)f(z) \tag{18}$$

From this we get that: $1-\dot{\delta} \geq \varepsilon$ whenever $\left|\frac{\partial \tau}{\partial x}F(z,\Phi(z))\right| \leq \frac{1-\varepsilon}{2-\varepsilon}$ for any $0 < \varepsilon < 1$. Moreover, this also guarantees that $1+\dot{\delta} > \tilde{\varepsilon}$ with $\tilde{\varepsilon} = \frac{2-\varepsilon}{3-2\varepsilon} > 0$.

Finally, it is also clear that in this case, $1 + \dot{\delta} < 2$.

Hence, given such an ε , and using conditions (1) and (2) of the theorem, we can obtain on some small enough neighborhood of the origin:

$$\dot{W} \leq -c_3 \tilde{\varepsilon} \|z(t)\|^2 + 2c_4 \gamma \|z(t)\| \|z(t-\delta)\| + \mu [\|z(t)\|^2 - \varepsilon \|z(t-\delta)\|^2]$$
(19)

$$\dot{W} \leq -\left(\|x(t)\| \|x(t-\tau)\| \right) \\
\times \begin{pmatrix} c_3 \tilde{\varepsilon} - \mu & -c_4 \gamma \\ -c_4 \gamma \varepsilon \mu \end{pmatrix} \begin{pmatrix} \|x(t)\| \\ \|x(t-\tau)\| \end{pmatrix}.$$
(20)

Hence choosing $\mu < c_3 \tilde{\varepsilon}$, we get that for:

$$\gamma < \frac{\sqrt{(c_3\tilde{\varepsilon} - \mu)\varepsilon\mu}}{c4} \tag{21}$$

the right-hand side of the above inequality is negative definite, and thus $\dot{W} \leq -\rho ||z(t)||$ for some $\rho > 0$ which gives the local asymptotical stability of z = 0 by the Lyapunov-Krasovskii stability result, and finally that of x = 0 for the system in time t.

Notice that choosing e.g. $\mu = \frac{c_3\tilde{\varepsilon}}{2}$, condition (21) becomes $\gamma < \frac{c_3\tilde{\varepsilon}\sqrt{\varepsilon}}{2c4}$. Since the right-hand side can be made arbitrarily close to $\frac{c_3}{2c_4}$ by choosing ε close enough to 1 (by lower values), this can in turn make (21) to be satisfied whenever $\gamma \leq \frac{c_3}{4c_4}$, which ends the proof.

Now if we consider the control law with the "dynamic inversion" for the fixed point resolution, then the stability can still be guaranteed by invoking Tikhonov's theorem Khalil [1996], in a similar way as in Georges et al [2007] for constant input delays.

We are now ready to provide a new control scheme for irrigation canals, by following this approach for model (6).

4. SIMULATION RESULTS

In this section, let us present some simulation results obtained with the here above proposed control design when applied to model (6).

The considered control purpose will be that of controlling the downstream flow $y = \xi_2$ by acting on the upstream one u.

The simulated model for the canal is first chosen as in (6), as an illustration of theorem 1, and then as a more accurate model given by a classical Preissmann scheme for Saint-Venant equations (2), for further validation of the proposed control.

The numerical values are also taken from the case study of Litrico et al [2003], namely a 10km-long and 8m-wide channel, with a 0.04% slope.

The control is thus designed as in (14)-(15), where the non-delayed law Φ is chosen as a classical state feedback linearization law, ensuring a time response to set-point changes of about 5*h*. In this simulation, Λ is chosen:

$$\Lambda = \left(\begin{array}{cc} 10 & 0\\ 0 & 10 \end{array}\right)$$

This control law is then tested under set-point changes from $y = 0.93[m^3/s]$ - which corresponds to a downstream water level h = 0.5[m] - to $y = 1.24[m^3/s]$ (i.e. h = 0.6[m]) at time $t = 51 \ min$.

In order to validate the proposed approach, let us first present simulation results obtained with the approximated model (6). For the sake of comparison, the results obtained by an IMC controlled as proposed in Litrico et al [2003] are given altogether. From the corresponding figure (1), it can be seen how our controller can perform very similarly to the IMC one (and can actually be tuned a little bit faster), when facing a step change of set-point. It can further be observed how the delay indeed varies during operation on figure (2).



Fig. 1. Comparaison between IMC controller and instantaneous predictor



Fig. 2. State-dependent delay

In view of those promising results, and in order to better validate the approach, we have then performed the same simulations with a canal simulated by a Preissmann implementation of Saint-Venant equations instead of model (6). The obtained results in this case are illustrated on figure 3. It can here be seen that the control still achieves its target in a similar way as in the simulations based on the simplified model.

It can also be checked that the prediction performs pretty well on figure (3).

Now a purpose of higher interest in irrigation system is the disturbance rejection. In such systems indeed, there are various kinds of perturbations (leak, withdrawal,...etc). The controller as it is developed does not allow the perturbation rejection, which brought us back to add an integrator into the controller. After 31 hours, one applies



Fig. 3. Downstream water flow step response



Fig. 4. Prediction error

a withdrawal perturbation (10% of the downstream water flow, located at the downstream of the canal). The corresponding simulation results are then presented on figure (5) for the tracking result, and figure (6) for the prediction error.



Fig. 5. Downstream water flow step response with withdrawal perturbation



Fig. 6. Prediction error

It can be noted that the modified controller rejects the perturbation.

Noting however that x_1 , the downstream flow derivative which is to be used in the control, is not measured, the implementation of an observer to estimate it becomes judicious. In view of the model structure (6), one can think of a high gain observer in the spirit of Gauthier et al [1992]:

$$\begin{pmatrix} \hat{\xi}_1\\ \hat{\xi}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0\\ 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{\xi}_1\\ \hat{\xi}_2 \end{pmatrix} + \begin{pmatrix} \phi(\hat{\xi}, u(t - \tau(\hat{\xi}_2)))\\ 0 \end{pmatrix} - (22)$$
$$S_{\theta}^{-1}C^T(y - \hat{y})$$

where ϕ follows from the expressions of A, B, and S is given by:

$$\theta S_{\theta} = A^T S_{\theta} + S_{\theta} A - C^T C$$

for θ large enough.

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Notice that the "non-standard" nonlinearity $u(t - \tau(\hat{\xi}_2))$ in (22) could be removed from the observation error equations by injecting the measured value of ξ_2 in u instead of its estimate.

Notice also that the stability of the resulting observerbased control law is not here completely analyzed and just checked in simulation. The corresponding results when performing simulations on the basis of a Preissmann scheme are shown in figures 3 (for the step response) and 4 for the observation error.

Clearly the observer-based control law still fairly achieves the expected performances.



Fig. 7. Downstream water flow step response under observer-based control with a full Saint-Venant model



Fig. 8. Observation error for x_1

5. CONCLUSION

In this paper, a new nonlinear control methodology for irrigation canals has been proposed, taking into account the diffusive nature of the canal dynamics and the presence of a state-dependent delay. This control scheme is based on some state prediction and exactly linearizing design, relying on an appropriate nonlinear model. The approach has been validated by successful simulations both on the control model and on a 'more accurate' Saint-Venant model. Further validation studies, hopefully including experimentations, will be part of future developments.

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