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Regulation of a Manned Sea-Surface Vehicle via Stochastic Optimal Control. ⋆

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Abstract: A control system is proposed for the regulation problem of the roll-motion of a manned sea-surface vehicle. Taking into account the roll-ship equations coming from the Conolly theory, a novel stochastic model has been proposed for the uncertainties driving the total mechanical torque acting on the vehicle, deriving from the wind and/or the sea-wave action. The proposed model results in a bilinear stochastic system to which a feedback controller is applied, giving linear-optimal performance with respect to a classical quadratic index. Open loop simulations carried out on real data validate the choice of the stochastic model of the uncertainties, producing a ship-roll time evolution which resembles the real data. Closed loop simulations on a *virtual ship* show the effectiveness of the proposed control scheme.

Keywords: Ship roll-motion; optimal control; stochastic control; bilinear systems.

1. INTRODUCTION

In this paper some preliminary - nevertheless meaningful - results are presented, concerning the on-line controller section of a larger control device that is being designed for an high-speed, manned sea-surface vehicle. The system is developed within a project financed by the Italian Ministry of Economic Development under the name of TMS (Trasmissioni Marine di Superficie). The system is currently being implemented and some preliminary results provided in this paper are validated on real data gathered from the prototype vessel realized in the project.

One main component of the project is the design and the realization of a new type of transmission for a surface propeller; the second main component is a software system embedded in the on-board hardware that integrates the standard functionalities (supervision of the devices that are present on board, GPS, engines, fuel, etc.) with new sophisticated control functions. One such function is referred to as Navigation Management, and suggests the optimal courses to reach a destination based on sea condition, comfort requirements, fuel and time available; the second function is the on-line control of the stability of the vessel in order to reduce the pitch and roll to a minimum. Both functions are based on real time measurements of the horizontal and longitudinal oscillations of the vessel obtained by sensors installed in its barycentre. The first function (Navigation Management) is used at given time intervals and strongly relies on data mining techniques applied to the data gathered by the sensors. The second function is based on the new on-line controller that is the specific topic of this paper.

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The vehicle is endowed with a couple of *flaps* that can be used as actuators by the controller in order to stabilize the roll-motion. Moreover the vehicle mount a *trimmer*, i.e. a device able to vary the slope of the surface drive, which can be used as well in order to regulate the pitching of the watercraft. As far as the system's observations are concerned, the vehicle considered mounts real-time measurement devices for the pitch and roll angles.

The problem of stabilizing a watercraft against pitching/rolling has been widely studied in the literature. In this paper a linear second order system is considered, modeling the ship-roll motion, according to the Conolly theory (see Conolly (1968)). Despite of its quite simple structure, it provides an exhaustive model for small roll displacements, and it has been widely adopted in the field of active fin control with the purpose of ship-roll reduction. Reference papers, among the others, areFortuna et al. (1996), where a pair of compensators are proposed: one designed by using classical frequency domain techniques, one obtained as an adaptive LQ compensator; H_∞ and robust adaptive H_∞ controllers are proposed in Sheng et al. (1999) and Haipeng et al. (2003), respectively; in Anantha et al. (2007) a virtual instrumentation-based fin active control is obtained.

A key-role in building the model is played by the external disturbance torque of the sea waves acting on the watercraft. Such an uncertain torque is mainly produced by the wind, but also affected by other atmosphere-conditions, ship moving, earthquake and gravitation of earth and moon. Many models are available in the literature (see Bhattacharyya (1978)). Generally, the sea waves are studied by means of wave energy spectrum (e.g. the Pierson-Moskowitz spectrum, see Pierson et al. (1964) or Lewis et al. (1967) for more details), according to the superposition of a large number of sinusoidal waves (see Price et al. (1974)).

In the present paper a way to represent these external disturbances as stochastic processes is proposed, as well as to cast them together into the Conolly model, thus preserving the simplicity and widely-tested utility of such a kind of watercraft's model. The resulting model will be a stochastic bilinear one, i.e. linear drift and multiplicative state-noise, to which the optimal stochastic control theory is applied in order to perform the regulation task.

The paper is organized as follows. In §2 the proposed watercraft's model is described in detail, whereas in §3 the control system is developed in both its main parts, i.e. the filtering and the controller ones. Finally in §4 the results of numerical simulations are presented, partially carried out on real data.

2. THE PROPOSED STOCHASTIC MODEL

Denote with $\alpha(t)$ the roll-angle of a sea-surface vehicle. According to the Conolly theory (see Conolly (1968)), the linear roll motion equation in a single degree of freedom can be written as:

$$I_\alpha \ddot{\alpha}(t) + \bar{\beta}\dot{\alpha}(t) + \bar{k}\alpha(t) = \bar{k}\xi(t) + \bar{p}u(t), \tag{1}$$

where I_α is the structural moment of inertia, $\bar{\beta}$ is the damping coefficient relative to the surrounding water, $\bar{k}\alpha(t)$ is the restoring moment, $\xi(t)$ is the effective wave slope and $\bar{p}u(t)$ denotes the control torque produced by the fins: $\bar{p}$ is a coefficient depending of the translation-speed of the ship, and $u(t)$ is the fin angular position, which is the control variable. Despite of its quite simple structure (it is a linear second order system, actually) such a model is effectively exhausting in modeling the ship motion in many framework, see e.g. Price et al. (1974), Lewis et al. (1967), Bhattacharyya (1978). As a matter of fact, model (1) has been widely adopted in the field of active fin control, with the purpose of ship-roll reduction (e.g. Fortuna et al. (1996), Sheng et al. (1999), Haipeng et al. (2003), Anantha et al. (2007)).

By exploiting standard computations, eq.(1) may be written in the following first order ordinary-differential-equation (ODE) model:

$$\dot{\alpha}(t) = \omega(t) \tag{2}$$

$$\dot{\omega}(t) = -k\alpha(t) - \beta\omega(t) + k\xi(t) + pu(t). \tag{3}$$

where $\omega(t)$ is the roll angular speed and the coefficients k, β, p are given by:

$$k = \frac{\bar{k}}{I_\alpha}, \qquad \beta = \frac{\bar{\beta}}{I_\alpha}, \qquad p = \frac{\bar{p}}{I_\alpha}. \tag{4}$$

Note that $\xi(t)$ denotes the *external and uncertain* contribution to the whole torque applied to the ship. Such an external-torque is due to a sea or wind action and cannot be deterministically described, as for the other contributions. Indeed, it depends of the actual sea- and/or weather-conditions the ship is getting to. Differently from the standard frequency-based approaches (Bhattacharyya (1978)), it is assumed here that $\xi(t)$ admits the following linear stochastic representation, for a suitably fixed integer n:

$$dz(t) = \Lambda z(t)dt + FdW(t), \qquad z(t) \in R^n \tag{5}$$

$$\xi(t) = \Gamma z(t) \tag{6}$$

where Λ is an asymptotically stable matrix and $W(t)$ is a standard Wiener process. A possible choice for the matrices (Λ, F, Γ) is the following:

$$dz_1(t) = \lambda_1 z_1(t)dt + z_2(t)dt, \tag{7}$$

$$dz_2(t) = \lambda_2 z_2(t)dt + z_3(t)dt, \tag{8}$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$dz_n(t) = \lambda_n z_n(t)dt + \sigma dW(t), \tag{9}$$

$$\xi(t) = [1 \ 0 \ \cdots \ 0]z(t), \tag{10}$$

for some $\lambda_i < 0$, $i = 1, \ldots, n$. Let us briefly comment on the above equations (7)-(9). Intuitively, any occurrence of the function z shows a somewhat irregular shape but time-correlated: it does reproduce some sea-wave or wind shot action. The probability distribution of such an external 'disturbance' is assumed to be generated by a linear stochastic system driven by white noise, that is the system of equations (7)-(9) provided the white noise is identified, as it is a standard procedure, with the formal derivative of the standard Wiener-process: $dW(t)/dt$. Generally speaking, such a way to represent stochastic occurrences of functions has been used in the literature even in very different settings. For instance in Germani et al. (1988), in an image-processing framework, the occurrence of an image had been stochastically represented as being generated by the brightness-function derivatives up to an (high) fixed order where the derivative is assumed a (two-dimensional) white noise. If the time-evolution of $\xi(t)$ is thought, as it is reasonable, as to a somewhat 'irregular' periodic wave (reproducing indeed the shape of the water motion) then the λ's can be interpreted as parameters related to the wave 'frequency'.

Different setting of parameters n, λ_i, σ provide different time-correlation function of the disturbance $\xi(t)$. For instance, according to the choice of $n = 1$, the farther is λ from the imaginary axis, the more the process $\xi(t)$ will be like to a white noise process. Parameter σ in eq. (9) refers to the variance of the 'white noise', and is generally time-varying as far as the sea- and/or weather-conditions change. Nevertheless, since these changes in the overall weather-conditions reasonably are not frequent in the control time-horizon considered, it is assumed here σ to be a constant, and in order to cast it in the model the following further equation is considered:

$$\dot{\sigma}(t) = 0, \tag{11}$$

so keeping in mind to estimate it later as a system-state variable of the control system. Similarly as before for the λ's, σ is related to the external-torque 'wave', but now to the amplitude of the wave. Thus σ may be thought as being a sea-strength related parameter: the bigger the latter the larger is σ.

The sensor-data available on board of the ship are the measurements of the angular-position $\alpha(t)$:

$$dY(t) = \alpha(t)dt + \sigma' dW'(t), \tag{12}$$

where W' is the scalar Wiener process. Thus the measurement error of the sensor is represented as a Gaussian white noise, with (known) variance σ': $\sigma' \cdot (dW'/dt)$, whereas dY/dt is the measurement available for processing.

By taking into account the deterministic eqs.(2-3), and (11) in the stochastic-processes-theoretic formalism of (7)-(9) and then casting all equations together in a single vector equation:

$$dX(t) = AX(t)dt + Hu(t)dt + BX(t)dW(t), \qquad (13)$$

$$dY(t) = CX(t)dt + \sigma' dW'(t), \qquad (14)$$

with

$$X(t) = [\,\alpha(t) \;\; \omega(t) \;\; z_1(t) \;\; z_2(t) \;\; \cdots \;\; z_n(t) \;\; \sigma(t)]^T \qquad (15)$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ O_{(n+1)\times 2} & A_{22} \end{bmatrix}, \qquad H = \begin{bmatrix} H_1 \\ O_{(n+1)\times 1} \end{bmatrix} \qquad (16)$$

$$B = \begin{bmatrix} O_{2\times 2} & O_{2\times(n+1)} \\ O_{(n+1)\times 2} & B_{22} \end{bmatrix}, \qquad C = [\,C_1 \;\; O_{1\times(n+1)}] \qquad (17)$$

and:

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -k & -\beta \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 0 \\ p \end{bmatrix} \qquad (18)$$

$$A_{12}(i,j) = \begin{cases} k, & \text{if } (i,j) = (2,1) \\ 0, & \text{otherwise.} \end{cases} \qquad (19)$$

$$A_{22} = \begin{bmatrix} \Lambda & O_{n\times 1} \\ O_{1\times n} & 0 \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \lambda_1 & 1 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}, \qquad (20)$$

$$B_{22} = \begin{cases} 1, & \text{if } (i,j) = (n, n+1), \\ 0, & \text{otherwise,} \end{cases} \qquad C_1 = [\,1 \;\; 0\,] \qquad (21)$$

where $O_{r\times l}$ denotes the null-block in $\mathbb{R}^{r\times l}$. Such a partition of the matrices will be useful in the sequel. The ship-roll regulation problem is stated as the following stochastic optimal control problem on the finite-horizon $[0, T]$:

$$\min_{u \in \mathcal{U}(Y)} J(u), \qquad (22)$$

$$J(u) = \frac{1}{2}\mathbf{E}\Big\{ X_1(T)^T F_{11} X_1(T) + \int_0^T \Big(X_1^T(t) Q_{11} X_1(t) + u^T(t) R u(t) \Big) dt \Big\}, \qquad (23)$$

where $X_1(t) = [\,\alpha(t) \;\; \omega(t)\,]$ and $Q_{11} = Q_{11}^T \geq 0$, $R = R^T > 0$, and $F_{11} = F_{11}^T \geq 0$. Clearly, the index J may be rewritten in the more general form:

$$J(u) = \frac{1}{2}\mathbf{E}\Big\{ X(T)^T F X(T) + \int_0^T \Big(X^T(t) Q X(t) + u^T(t) R u(t) \Big) dt \Big\}, \qquad (24)$$

under the differential constraints represented by system (13), (14). Matrices $Q = Q^T \geq 0$, $F = F^T \geq 0$, are computed according to the partitions:

$$F = \begin{bmatrix} F_{11} & O_{2\times(n+1)} \\ O_{(n+1)\times 2} & O_{(n+1)\times(n+1)} \end{bmatrix}, \qquad Q = \begin{bmatrix} Q_{11} & O_{2\times(n+1)} \\ O_{(n+1)\times 2} & O_{(n+1)\times(n+1)} \end{bmatrix}. \qquad (25)$$

An interval $[0, T] \subset \mathbb{R}$ has been fixed to define the above index $J(u)$, which represents the control-horizon, whereas $\mathcal{U}(Y)$ is a set of admissible control functions, anyway included in the set of all output-feedbacks having finite variance. The subset $\mathcal{U}(Y)$ will be precisely defined in the next section.

3. THE CONTROL SYSTEM

The state-equation (13), is a linear one with multiplicative state-noise (such an equation is often referred to as a *bilinear* one in the literature). Moreover it is a feedback equation, in that it includes the additive (output-measurable) term $Hu(t)$. Even thought the cost-index (24) is the classical finite-horizon quadratic one, the bilinearity of the state equation makes the overall control problem different from the classical LQG one, with incomplete information. Nevertheless this problem has been studied and solved back in the years by McLane (1971), for an even more general class of bilinear systems (including control-dependent noise) but in the complete information case, that is within the state-feedbacks control functions. Recently these results have been generalized in Carravetta et al. (2007) to the incomplete-information case (which is the case of the present paper) within the linear output-feedbacks, that is a solution $u(t)$ is sought to the minimum of the index (24), which belongs to the space $\mathcal{L}_t(Y)$ of all the $\mathbb{R}$-valued square-integrable linear transformations of the random variables $\{Y(\tau),\ 0 \leq \tau \leq t \leq T\}$:

$$\min_{u(t) \in \mathcal{L}_t(Y)} J(u), \quad \text{with} \quad u, X, Y \quad \text{subject to (13--14).}$$

In particular it has been shown that a separation property holds, and the optimal linear output-feedback control is given by a linear map of the *linear-optimal* (in the mean-square sense) state-estimate, namely $\widehat{X}$. In the same paper Carravetta et al. (2007) derived a system of output-driven recursive equations providing $\widehat{X}$ (that is: the linear-optimal filter for the considered class of feedback-systems), thus endowing the overall control-system. The following Theorem provides the solution to the optimal control problem proposed, by suitably exploiting the above mentioned results.

Theorem. *Denote u^o the control minimizing (24) within an $\mathcal{U}(Y)$ constituted by all the linear (progressively measurable) maps from the output paths $\{Y(\tau), 0 \leq \tau \leq T\}$. Then:*

$$u(t)^o = L^o(t)\widehat{X}(t), \qquad (26)$$

with:

$$L^o(t) = -R^{-1} H_1^T [\,V_{11}(t) \;\; V_{12}(t)\,]; \qquad (27)$$

$\widehat{X}(t)$ is the optimal linear estimate of $X(t)$, eq.(15), and the block matrices $V_{11}(t)$, $V_{12}(t)$ come from the solution of the following Riccati-like backward equations

$$\begin{aligned}
\dot{V}_{11}(t) &= -A_{11}^T V_{11}(t) - V_{11}(t)A_{11} - Q_{11} \\
&\quad + V_{11}(t)H_1R^{-1}H_1^T V_{11}(t), \qquad (28)\\
V_{11}(T) &= F_{11},\\
\dot{V}_{12}(t) &= -A_{11}^T V_{12}(t) - V_{11}(t)A_{12} - V_{12}(t)A_{22}\\
&\quad + V_{11}(t)H_1R^{-1}H_1^T V_{12}(t), \qquad (29)\\
V_{12}(T) &= O_{2\times(n+1)}.
\end{aligned}$$

Proof. According to Carravetta et al. (2007), the solution to the proposed optimal control problem is:

$$u(t)^o = L^o(t)\widehat{X}(t), \tag{30}$$

with

$$L^o(t) = -R^{-1}H^T V(t), \tag{31}$$

and $V(t)$ is the symmetric, positive-semidefinite solution of the following Riccati-like backward equation

$$\begin{aligned}
\dot{V}(t) &= -A^T V(t) - V(t)A - Q - B^T V(t)B\\
&\quad + V(t)HR^{-1}H^T V(t), \qquad (32)\\
V(T) &= F.
\end{aligned}$$

By partitioning $V(t)$ as:

$$V(t) = \begin{bmatrix} V_{11}(t) & V_{12}(t) \\ V_{12}^T(t) & V_{22}(t) \end{bmatrix}, \qquad V_{11}(t) \in \mathbb{R}^{2\times 2}, \tag{33}$$

and according to the partition of matrices A, H, B in (16)-(17), one can immediately derive eq. (27) from (31). Then, by exploiting the partitions in (32):

$$\begin{aligned}
\begin{bmatrix} \dot{V}_{11}(t) & \dot{V}_{12}(t) \\ \dot{V}_{12}^T(t) & \dot{V}_{22}(t) \end{bmatrix} &= -\begin{bmatrix} A_{11}^T & O \\ A_{12}^T & A_{22}^T \end{bmatrix}\begin{bmatrix} V_{11}(t) & V_{12}(t) \\ V_{12}^T(t) & V_{22}(t) \end{bmatrix}\\
&- \begin{bmatrix} V_{11}(t) & V_{12}(t) \\ V_{12}^T(t) & V_{22}(t) \end{bmatrix}\begin{bmatrix} A_{11} & A_{12} \\ O & A_{22} \end{bmatrix} - \begin{bmatrix} Q_{11} & O \\ O & O \end{bmatrix}\\
&+ \begin{bmatrix} V_{11}(t) & V_{12}(t) \\ V_{12}^T(t) & V_{22}(t) \end{bmatrix}\begin{bmatrix} H_1R^{-1}H_1^T & O \\ O & O \end{bmatrix}\begin{bmatrix} V_{11}(t) & V_{12}(t) \\ V_{12}^T(t) & V_{22}(t) \end{bmatrix}\\
&- \begin{bmatrix} O & O \\ O & B_{22}^T \end{bmatrix}\begin{bmatrix} V_{11}(t) & V_{12}(t) \\ V_{12}^T(t) & V_{22}(t) \end{bmatrix}\begin{bmatrix} O & O \\ O & B_{22} \end{bmatrix}, \qquad (34)
\end{aligned}$$

with:

$$\begin{bmatrix} V_{11}(T) & V_{12}(T) \\ V_{12}^T(T) & V_{22}(T) \end{bmatrix} = \begin{bmatrix} F_{11} & O_{2\times(n+1)} \\ O_{(n+1)\times 2} & O_{(n+1)\times(n+1)} \end{bmatrix}; \tag{35}$$

the dimensions of the zero-blocks in (34) are straightforward and are omitted to make more readable the above computation. By taking into account in (34-35) only the block-matrix equations concerning $\dot{V}_{11}(t)$ and $\dot{V}_{12}(t)$, equations (28), (29) are readily satisfied. •

Equation (26) tells us that the linear optimal map of the output's paths up to the current time t always has the form of a finite-dimensional linear map of the *mean-square linear-optimal* state-estimate $\widehat{X}(t)$ (separation property). Therefore the overall control system is endowed with the filter equations for $\widehat{X}$, which by Carravetta et al. (2007) are given by

$$\begin{aligned}
d\widehat{X}(t) &= A\widehat{X}(t)dt + S(t)\widehat{X}(t)dt \qquad (36)\\
&\quad + \frac{1}{\sigma'^2}P(t)C^T\left(dY(t) - C\widehat{X}(t)dt\right), \qquad (37)\\
\widehat{X}(0) &= \mathbf{E}\{\overline{X}\}, \qquad (38)\\
S(t) &= HL^o(t), \qquad (39)\\
P(t) &= \Psi_X(t) - \Psi_{\widehat{X}}(t), \qquad (40)\\
\dot{\Psi}_X(t) &= A\Psi_X(t) + \Psi_X(t)A^T\\
&\quad + S(t)\Psi_{\widehat{X}}(t) + \Psi_{\widehat{X}}(t)S^T(t) \qquad (41)\\
&\quad + B\big(\Psi_X(t) + \mu\mu^T(t)\big)B^T, \quad \Psi_X(0) = \mathrm{Cov}\big(X(0)\big),\\
\dot{\Psi}_{\widehat{X}}(t) &= (A+S(t))\Psi_{\widehat{X}}(t) + \Psi_{\widehat{X}}(t)(A+S(t))^T\\
&\quad + \frac{1}{\sigma'^2}P(t)C^TCP(t), \qquad \Psi_{\widehat{X}}(0) = 0, \qquad (42)\\
\dot{\mu}(t) &= (A+S(t))\,\mu(t), \qquad \mu(0) = \mathbf{E}\{\overline{X}\}. \qquad (43)
\end{aligned}$$

As shown in Carravetta et al. (2004) under suitable assumptions, including existence of a steady-state filter-gain and system-stability (this latter one indeed satisfied in the present case), the incomplete-information linear optimal control for bilinear systems admits a steady-state solution, and the value of the infinite-horizon optimal cost functional can be a-priori calculated. For this purpose, consider the following modified version of the cost functional of eq.(24):

$$J(u) = \lim_{T\mapsto+\infty}\frac{1}{2T}\mathbf{E}\left\{\int_0^T\left(X^T(t)QX(t) + u^T(t)Ru(t)\right)dt\right\}, \tag{44}$$

Such an optimal cost is achieved by applying Theorem 4.1 in Carravetta et al. (2004) to the present case, so that:

$$J^o = J(u^o) = \frac{1}{2}\mathrm{tr}\{L_\infty^o{}^T R L_\infty^o \Phi\} \tag{45}$$

where L_∞^o is the steady-state value of the controller, and

$$\Phi = \lim_{T\to+\infty}\frac{1}{2T}\int_0^T P(t)dt,$$

with $P(t)$ the covariance of the estimation error, solution of the filter-Riccati-equation (40-43).

4. SIMULATION RESULTS

Simulations have been produced in order to pursue the following two aims:

- validate the proposed stochastic model of the sea-wave uncertainties affecting the watercraft, according to real data measurements available from the prototype vessel realized in the project mentioned in the Introduction;

- test "*in silico*" the performances of the proposed optimal stochastic regulator, according to a closed loop simulation framework. In this case, the ship parameters are taken from the literature.

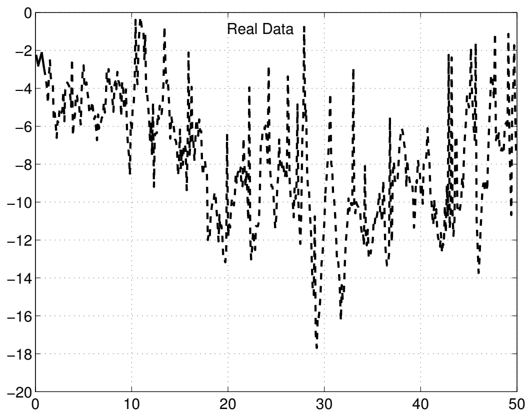

Fig. 1. Roll angle evolutions: real data

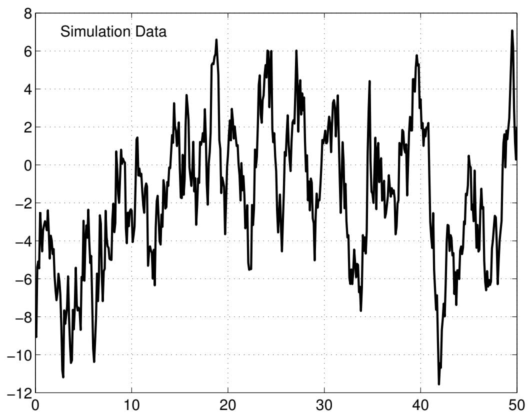

Fig. 2. Roll angle evolutions: simulation data

Numerical simulations are obtained according to the Euler-Maruyama algorithm (see Higham (2001)) on the time interval [0,150], with discretization time $\Delta = 0.01$.

The first aim has been pursued by using a stream of roll-angle's data coming from a sensor actually placed on the watercraft. The time evolution coming from real data is reported in Fig.1. Axis are in time versus degrees in all the figures, unless differently specified. A specific identification procedure to properly choose the model/disturbance parameters is, at present, a work in progress by the same authors: different sets of parameters would produce different patterns of disturbances (e.g. by increasing the dimension n a smoother profile is obtained, while by increasing the absolute value of λ the correlation of the noise reduces). In Fig.2, for example, the roll-angle evolution coming from simulations is reported, according to the following parameters:

$$k = 100, \quad \beta = 20, \quad n = 1, \quad \lambda = -0.8, \quad \sigma = 0.08.$$

Both real and simulation data are reported on time interval of 50sec. It is apparent from trivial *visual inspection* that, in this case, the time evolution coming from simulation data resemble the time evolution coming from real data.

In order to test the proposed optimal algorithm, the ship parameters (k, β, namely) have been taken from reference Sheng et al. (1999), coming from real data:

$$k = 0.4874, \qquad \beta = 0.1850$$

with the control parameter p set equal to 1. The sea-wave uncertainties parameters are set as follows:

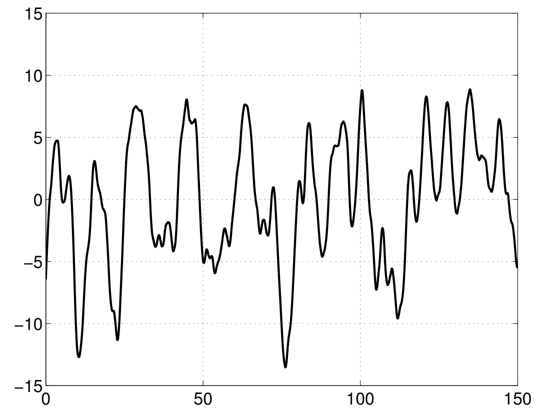

Fig. 3. Time evolution of the roll angle disturbance ξ

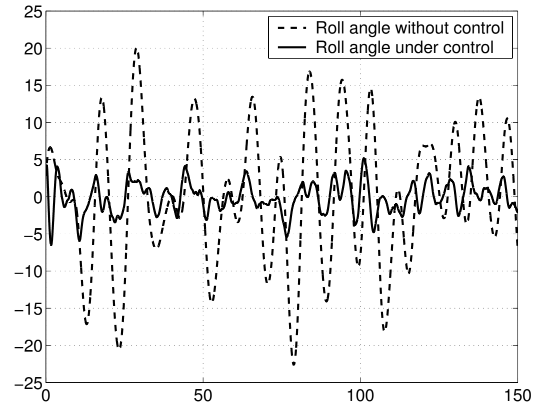

Fig. 4. Time evolution of the roll angle

$$n = 3, \quad \sigma = 0.15, \quad \lambda_i = -0.8, \quad i = 1, 2, 3.$$

The roll-angle disturbance evolution coming from this choice is shown in Fig.3, and provides a roll-angle evolution compatible with the one reported in Sheng et al. (1999).

The coefficient σ', concerning the output noise variance of eq.(12), has been set equal to 0.01. The control action pursues the double aim to stabilize the ship-roll angle without enhancing its pulsatility. Such a purpose is achieved by suitably setting the weight matrices as follows:

$$F_{11} = Q_{11} = \mathrm{diag}\{10, 10\}, \qquad R = 1.$$

Below are reported the indexes obtained with and without the application of the proposed control. They are actually the mean values obtained on a set of 1,000 simulations:

$$J_{\text{free evolution}} = 26.2282, \qquad J_{\text{controlled evolution}} = 2.0204.$$

The improvements are apparent, and the good performances may be also appreciated by looking at fig.4, which shows the controlled roll angle evolution (red solid line) compared with the free evolution (black dotted line). To best appreciate the performances of the algorithm, the following figures have also been reported: figs.5-7 show the true (red solid line) and estimated (blue dotted line) roll angle, roll speed angle (time versus degree/s) and roll angle disturbance, respectively.

5. CONCLUSION

A general stochastic model has been derived (eqs.(13-14)) for a manned sea-surface vehicle and stochastic control

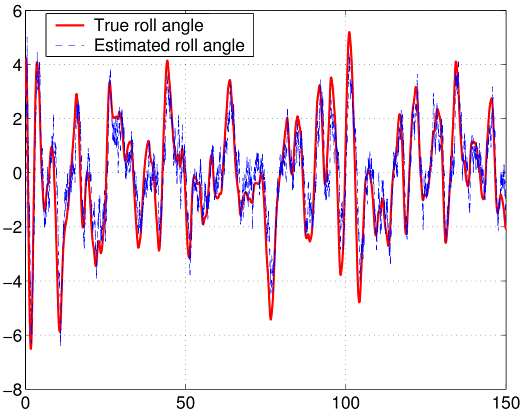

Fig. 5. Roll angle estimate

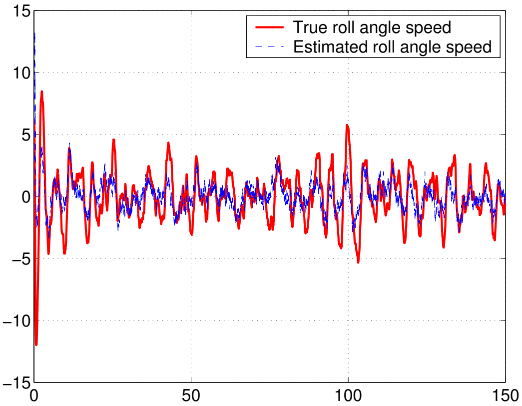

Fig. 6. Roll speed angle estimate

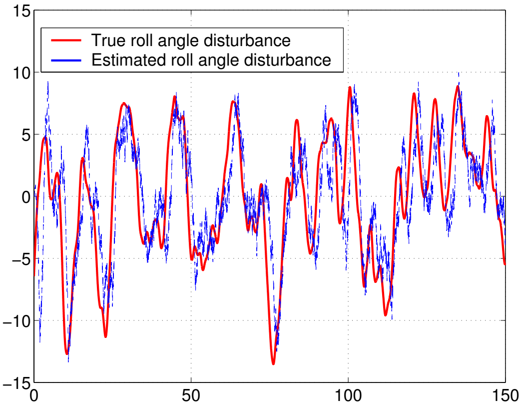

Fig. 7. Roll angle disturbance estimate

(e.g. the optimization problem formalized by the index (24) with $u(t) \in \mathcal{L}_t(Y)$) has been applied in order to solve the roll-regulation problem of the watercraft. Based on the Conolly theory, the proposed model aims to give a representation of the external unpredictable, persistent disturbances acting on the watercraft with a variable strength depending on sea/wind conditions. The result is a general control-system that can be adopted for any kind of watercraft (possibly changing a few set of parameters). A first set of simulations has been carried out from real data, in order to validate the proposed stochastic model of the sea-wave uncertainties. More specific validation tests based on the spectral analysis of the real/simulated data are a work in progress by the authors. A second set of simulations has been carried out using parameters taken from the literature. These results show the high performance of the overall control-system in terms of roll-amplitude damping. These results (in particular, in fig.4 it is apparent the high level of roll active damping) have been evaluated extremely promising within the scope of the TMS project and the on-line controller is now being implemented in the on-board hardware for complete testing. The whole system is planned to be released at the end of the project's activity.

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