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A Polyreference Least Square Complex Frequency domain based statistical test for damage detection

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Abstract: Some current approaches in damage detection have led to the implementation of statistical algorithms, based on the so-called ”local approach” assumption. The reference parameter is the modal signature of a state-space model of the studied system, and the tests aim at detecting small deviations in this signature without explicitly computing it. Moreover, the system inputs are assumed to be random white noises, modeling all the various and unknown influences on the system, either environmental or operational. Working with the outputs in time-domain, various tests were designed and experimented - e.g. the subspace-based chi-square test.
On the other hand, efficient system identification algorithms working in the frequency domain have been designed, first dealing with experimental conditions - meaning there are known controlled inputs - but also offering promising perspectives with regards to the operational conditions - without any measurement of the inputs.
In this paper, a frequency-domain statistical test for change detection is proposed, based on this recent frequency-domain modal analysis method and on the local approach to change detection.

Keywords: Statistical methods/signal analysis for FDI; Fault detection and diagnosis; Vibration and modal analysis; Frequency domain identification.

1. INTRODUCTION

Vibration monitoring by identification technics has been been the subject of much research works Zimmerman et al. [1995], Balageas et al. [2006], Doebling et al. [1998], Farrar [2001], Maeck [2003]. Typically, the aim is to monitor small changes in a structure's modal parameters before those can pose a threat to the structure's operation. Ideally this should be done without actually estimating the modal model, which becomes extremely expensive in term of computer resources when dealing with actual structures. This is of importance both in the design process - in aeronautics, plane parts have to undergo numerous fatigue tests - and in operational conditions - monitoring the status of a bridge, or an in-flight plane wing for example. When working in operational conditions, there is an additional problem, that is we usually don't have any control on the system inputs - for example, a bridge is subjected to effects from the traffic on it, the wind, the temperature...

One approach to the vibration-based structural health monitoring problem is the *statistical local approach*. Details may be found in Benveniste et al. [1987], Basseville [1998], Basseville et al. [2004]. This approach relies on the two following assumptions :

- the input forces can be modeled as a non-stationary white noise ;
- let θ be the modal parameter describing the system response to random perturbations. Then given a reference value θ_* of the modal parameter, and a sequence $(Y_i)_{i=1..n}$ of n samples obtained under an unknown value θ, we assume that θ_* and θ are close enough, that is :

$$\theta - \theta_* = \frac{1}{\sqrt{n}}\ \delta\ ,\ \text{with } \delta \text{ independent of } n \qquad (1)$$

It has led to the design of subspace-based damage-detection algorithms, in the form of a statistical χ^2-test, see Basseville et al. [2000], Basseville et al. [2001]. Another application follows from considering the χ^2-test as a function of θ : the model validation consists then in finding a value for θ that minimizes the test . In addition it may also provide confidence intervals on the modal parameters, see Basseville et al. [2006], Mevel and Goursat [2006], Canales and Mevel [2007].

While all this work was done on the time domain, powerful frequency domain modal analysis tools were designed, see Cauberghe [2004], Guillaume [2006]. They present many interesting features, such as working on structures with hundreds or thousands of inputs. This is especially noteworthy in the light of the difficulties encountered by subspace-based methods in this regard. Transposing the monitoring problem from the time domain to the frequency domain had already been suggested in Benveniste and Delyon [2000]. In this paper we extend their work while integrating the algorithms already developed for frequency domain modal analysis.

In the first section, we outline some aspects of frequency-domain system identification algorithms dedicated to modal analysis. The second section shows how we derive a statistical test to monitor changes in the modal structure. In the third section we explain the actual implementation and validation of this test, running it on the model validation problem.

2. FREQUENCY-DOMAIN MODAL ANALYSIS

This section recalls some aspects developed in Cauberghe [2004] Guillaume [2006] that are needed for the later parts of this paper.

2.1 Modal model and parameters

We assume the studied system follows the following equation :

$$\mathbf{M}y''(t) + \mathbf{C}y'(t) + \mathbf{K}y(t) = f(t) \tag{2}$$

with N_m degrees of freedom, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K} \in \mathbb{R}^{N_m \times N_m}$ the mass, damping, and stiffness matrices, respectively, and $y(t)$, $f(t) \in \mathbb{R}^{N_m}$ the response and excitation vectors, respectively. Transforming to the Laplace domain leads to the following transfer function :

$$\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{F}(s) \tag{3}$$

with

$$\mathbf{H}(s) = \left(\mathbf{M}s^2 + \mathbf{C}s + K\right)^{-1} \tag{4}$$

From there we derive the full modal model of size $N_m \times N_m$:

$$\mathbf{H}(s) = \sum_{m=1}^{N_m} \left(Q_m \frac{\Phi_m \Phi_m^T}{s - \lambda_m} + Q_m^* \frac{\Phi_m^* \Phi_m^{*T}}{s - \lambda_m^*} \right) \tag{5}$$

where λ_m is the m-th pole, $\Phi_m \in \mathbb{C}^{N_m}$ is the associated modeshape and Q_m is a scaling factor. The natural frequencies are $\omega_m = \mathrm{Im}(\lambda_m)/2\pi$ with damping ratios $\zeta_m = -\mathrm{Re}(\lambda_m)/|\lambda_m|$. Now, when measuring frequency responses functions, we usually get data only for a limited number of inputs N_i and outputs N_o, with the property that $N_i \ll N_o$. We then introduce a smaller modal model with size $N_o \times N_i$:

$$\mathbf{H}(s) = \sum_{m=1}^{N_m} \left(\frac{\Phi_m L_m^T}{s - \lambda_m} + \frac{\Phi_m^* L_m^{*T}}{s - \lambda_m^*} \right) \tag{6}$$

where the $L_m \in \mathbb{C}^{N_i}$ are the modal participation factors Maia and Silva [1997], Heylen et al. [1998], Ewins [2000].

2.2 Common-denominator transfer function model

Since direct estimation of these modal parameters is hampered by the inherent non-linearity of the model, common-denominator transfer function description are usually favored Guillaume [2006]. There are several such descriptions ; the one we will adopt is the so-called *Polyreference implementation.*

Given $\left(e^{-i\omega_l}\right)_{1\le l\le N_f}$ a basis for the Fourier transformation of the time data to the frequency domain, we define a polynomial basis function $(\Omega_l)_{1\le l\le N_f}$. For a discrete-time model, it is usually $\Omega_l = e^{i\omega_l T_s}$, with T_s the sampling period of the time data. The common-denominator transfer function model is then

$$\mathbf{H}(\Omega_l) = \mathbf{B}(\Omega_l)\left(\mathbf{A}(\Omega_l)\right)^{-1} \tag{7}$$

where $\mathbf{B}(\Omega_l) = \sum_{j=0}^{n} \mathbf{b}_j \Omega_l^j$ is the numerator polynomial for output, with $\mathbf{b}_j \in \mathbb{C}^{N_o \times N_i}$, and $\mathbf{A}(\Omega_l) = \sum_{j=0}^{n} \mathbf{a}_j \Omega_l^j$ is the common-denominator polynomial, with $\mathbf{a}_j \in \mathbb{C}^{N_i \times N_i}$. The frequency response function (FRF) between all the inputs and any given output o is

$$\mathbf{H}_o(\Omega_l) = \mathbf{B}_o(\Omega_l)\left(\mathbf{A}(\Omega_l)\right)^{-1}$$

with $\mathbf{H}_o(\Omega_l)$ (resp. $\mathbf{B}_o(\Omega_l)$) the o-th row of $\mathbf{H}(\Omega_l)$ (resp. $\mathbf{B}(\Omega_l)$).

Now, given measured FRFs $\left(\widehat{\mathbf{H}}_o(\omega_l)\right)_{o,k}$, modal analysis algorithms aim at estimating the coefficients $(a_{i_1 i_2 j})_{i_1,i_2,j}$ and $(b_{oij})_{o,i,j}$. It is then easy to compute the modal parameters $(\lambda_m, \Phi_m, L_m)_{m=1\ldots N_m}$ of the system. In order to do so, we define

$$\mathbf{E}_o(\omega_l) = \widehat{\mathbf{H}}_o(\omega_l)\mathbf{A}(\Omega_l) - \mathbf{B}_o(\Omega_l) \tag{8}$$

Then the cost function

$$C = \sum_o \sum_l |\mathbf{E}_o(\omega_l)|^2 \tag{9}$$

$$= \sum_o \sum_l \mathrm{trace}(\mathbf{E}_o^H(\omega_l)\mathbf{E}_o(\omega_l)) \tag{10}$$

is minimized in a least-square sense, with $.^H$ denoting the Hermitian transconjugate (for details see Guillaume [2006]).

In the next section, we explain how we use these quantities to design a statistical test that detects changes in the system modes without actually computing them.

3. A FREQUENCY-DOMAIN LOCAL TEST FOR CHANGE DETECTION

3.1 The scalar frequency-domain local test

In Benveniste and Delyon [2000], a frequency-domain test for change detection in transfer functions is presented. The model is

$$y_n = G(z)u_n + v_n$$

with $(u_n)_n$, $(y_n)_n$ the measured scalar input and output sequences, $(v_n)_n$ a random white noise sequence, and $G(z)$ the unknown transfer function. The test aims at deciding if, given a *nominal* transfer function G_0, and the sequence of input/output couples $(u_n, y_n)_n$, the hypothesis $\mathbf{H}_0 : G = G_0$ is acceptable. This test is built using the so-called local point of view, developed in Benveniste et al. [1987] : it is assumed that

$$G - G_0 = \frac{1}{\sqrt{K}}\widetilde{G} \tag{11}$$

with $\widetilde{G}$ unknown but fixed. The inputs and outputs are transformed to the frequency domain by Discrete Fourier

Transform (DFT) Ljung [1999] ; however, instead of transforming the whole data set in one pass, the DFT is performed on K successive blocks each of length N, resulting in $\left(U_k^N(\omega)\right)_{k=1\ldots K}$ and $\left(Y_k^N(\omega)\right)_{k=1\ldots K}$. Without performing this manipulation, we wouldn't get the limit theorems we need. The considered statistic is :

$$\zeta_K^N(G_0,\omega) = \frac{1}{\sqrt{K}}\sum_{k=1}^{K} U_k^N(-\omega)\left(Y_k^N(\omega) - G_0(e^{i\omega})U_k^N(\omega)\right) \tag{12}$$

Then, assuming that the input and noise spectra $S^{uu}(e^{i\omega})$ and $S^{vv}(e^{i\omega})$ are both non-zero, the following result holds Benveniste and Delyon [2000] for $K, N \to \infty$ and $\frac{\sqrt{K}}{N} \to 0$:

$$\zeta_K^N(G_0,\omega) \sim \mathcal{N}\left(S^{uu}(e^{i\omega})\widetilde{G}(e^{i\omega}), S^{uu}(e^{i\omega})S^{vv}(e^{i\omega})\right) \tag{13}$$

This result in the following χ^2-test that decides between $\mathbf{Hyp}_0 : \widetilde{G}(e^{i\omega}) = 0$ and $\mathbf{Hyp}_1 : \widetilde{G}(e^{i\omega}) \neq 0$

$$\chi_K^N(G_0,\omega) = \frac{|\zeta_K^N(G_0,\omega)|^2}{\widehat{S}^{uu}(e^{i\omega})\widehat{S}^{vv}(e^{i\omega})} \tag{14}$$

with the following estimates for the spectra :

$$\widehat{S}^{uu}(e^{i\omega}) = \frac{1}{K}\sum_{k=1}^{K}|U_{0,k}^N(\omega)|^2 \tag{15}$$

$$\widehat{S}^{vv}(e^{i\omega}) = \frac{1}{K}\sum_{k=1}^{K}|Y_{0,k}^N(\omega) - G_0(e^{i\omega})U_{0,k}^N(\omega)|^2 \tag{16}$$

The actual use of this approach is hampered by an important drawback : it cannot detect changes in the transfer functions of a structure under operational conditions when the inputs are unknown. To overcome this drawback, the idea of the new test proposed here is the following : instead of using the input and output data sets, we use the numerator and denominator of the common-denominator transfer function model. For this we have to rewrite the statistic, and verify that the convergence result still holds true in the multidimensional framework.

3.2 The multidimensional frequency-domain local test

Now we explain how to link the identification methods of the first section and the statistical test of the second section. We can already see a strong similarity between the quantities involved in (8) and (12). This similarity inspired the design of the new statistic.

From now on, we are working in the framework defined in the first section for frequency-domain modal analysis. We consider the following model

$$\mathbf{B}(\omega) = \mathbf{H}(\omega)\mathbf{A}(\omega) + \mathbf{V}(\omega) \tag{17}$$

with $\mathbf{V}(\omega) \in \mathbb{C}^{N_o \times N_i}$ a random white noise matrix. Actually this is simply the common-denominator model with an additive perturbation. We assume we have reference data obtained in a nominal state, from which we derive $\left(\mathbf{A}_{0,k}^N(\omega)\right)_{k=1\ldots K} \in \mathbb{C}^{N_i \times N_i}$ and $\left(\mathbf{B}_{k,0}^N(\omega)\right)_{k=1\ldots K} \in \mathbb{C}^{N_o \times N_i}$, with $k = 1\ldots K$ labeling successive blocks of time data with length N. The $\left(\mathbf{A}_{0,k}^N(\omega)\right)_{k=1\ldots K}$ and $\left(\mathbf{B}_{0,k}^N(\omega)\right)_{k=1\ldots K}$ will be our reference variables. With $\mathbf{H}_0(\omega)$ the corresponding nominal transfer function they satisfy

$$\mathbf{B}_{k,0}^N(\omega) = \mathbf{H}_0(\omega)\mathbf{A}_{k,0}^N(\omega) + \mathbf{V}_{k,0}^N(\omega) \tag{18}$$

Let $(\mathbf{H}(\omega_l))_{l=1..N_f}$ be the new FRFs, measured on data recorded later. We assume the local alternative hypothesis holds, that is

$$\mathbf{H} = \mathbf{H}_0 + \frac{\widetilde{\mathbf{H}}}{\sqrt{K}} \text{ for an unknown but fixed } \widetilde{\mathbf{H}} \tag{19}$$

For all $1 \leq l \leq N_f$, we want to test between the two hypotheses $\mathbf{Hyp}_0 : \widetilde{H}(\omega_l) = 0$ and $\mathbf{Hyp}_1 : \widetilde{H}(\omega_l) \neq 0$.

We now define the following statistic :

$$\zeta_K^N(\mathbf{B}_{k,0}^N, \mathbf{A}_{k,0}^N, \omega) = \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\left(\mathbf{B}_{k,0}^N(\omega) - \mathbf{H}(\omega)\mathbf{A}_{k,0}^N(\omega)\right)\left(\mathbf{A}_{k,0}^N(\omega)\right)^H \tag{20}$$

Then the following theorem holds :

Theorem 1. Let $S_0^{aa}(\omega)$ and $S_0^{vv}(\omega)$ be the power spectra for the common denominator and the perturbation in the reference state. Assuming $S_0^{aa}(\omega)S_0^{vv}(\omega) \neq 0$ and $\mathbf{A}_{k,0}^N(\omega)\mathbf{V}_{k,0}^N(\omega)$ has zero mean (at least up to $O(1/N)$), we have for $K, N \to \infty$ and $\frac{\sqrt{K}}{N} \to 0$:

$$\zeta_K^N(\mathbf{B}_{k,0}^N, \mathbf{A}_{k,0}^N, \omega) \sim \mathcal{N}\left(-\widetilde{\mathbf{H}}(\omega)S_0^{aa}(\omega), S_0^{aa}(\omega)S_0^{vv}(\omega)\right) \tag{21}$$

For ease of notation, we will omit the N and ω in the proof. We have :

$$\begin{aligned}
\zeta_K(\mathbf{B}_{k,0}, \mathbf{A}_{k,0}) &= \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\left(\mathbf{B}_{k,0} - \mathbf{H}\mathbf{A}_{k,0}\right)\left(\mathbf{A}_{k,0}\right)^H \\
&= \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\left(\mathbf{V}_{k,0} - \frac{\widetilde{\mathbf{H}}}{\sqrt{K}}\mathbf{A}_{k,0}\right)\left(\mathbf{A}_{k,0}\right)^H \\
&= \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\mathbf{V}_{k,0}\left(\mathbf{A}_{k,0}\right)^H - \widetilde{\mathbf{H}}\frac{1}{K}\sum_{k=1}^{K}\mathbf{A}_{k,0}\left(\mathbf{A}_{k,0}\right)^H
\end{aligned}$$

The first term behavior is deduced from the Central Limit Theorem for triangular arrays of random variables Ibragimov and Kash'minskii [1981] :

$$\frac{1}{\sqrt{K}}\sum_{k=1}^{K}\mathbf{V}_{k,0}\left(\mathbf{A}_{k,0}\right)^H \sim \mathcal{N}\left(0, S_0^{aa}S_0^{vv}\right)$$

The second term is readily recognizable :

$$\widetilde{\mathbf{H}}\frac{1}{K}\sum_{k=1}^{K}\mathbf{A}_{k,0}\left(\mathbf{A}_{k,0}\right)^H \longrightarrow \widetilde{\mathbf{H}}S_0^{aa}$$

Putting these two terms together brings the announced result Q.E.D.

From this result, we derive the following χ^2-test that decides between $\mathbf{Hyp}_0$ and $\mathbf{Hyp}_1$:

$$\chi_K^N(\mathbf{B}_{k,0}^N, \mathbf{A}_{k,0}^N, \omega) = \frac{\zeta_K^N(\mathbf{B}_{k,0}^N, \mathbf{A}_{k,0}^N, \omega) \left(\zeta_K^N(\mathbf{B}_{k,0}^N, \mathbf{A}_{k,0}^N, \omega)\right)^H}{\widehat{S}_0^{aa}(\omega)\widehat{S}_0^{vv}(\omega)} \quad (22)$$

It is computed for all $\omega = \omega_l$ and for all $1 \leq l \leq N_f$, with the following estimates for the spectra :

$$\widehat{S}_0^{aa}(\omega) = \frac{1}{K}\sum_{k=1}^{K} |\mathbf{A}_{0,k}^N(\omega)|^2 \quad (23)$$

$$\widehat{S}_0^{vv}(\omega) = \frac{1}{K}\sum_{k=1}^{K} |\mathbf{B}_{0,k}^N(\omega) - \mathbf{H}_0(\omega)\mathbf{A}_{0,k}^N(\omega)|^2 \quad (24)$$

Regarding the choice of K and N, we follow the same rule of thumb as in Benveniste and Delyon [2000]. The proof of (21) has shown that there are two terms contributing to the asymptotic behavior of our statistic. The difference between the first term and its asymptotic behavior is of order $O(\frac{1}{\sqrt{K}} + \frac{1}{N})$, while for the second term this difference is of order $O(\sqrt{\frac{K}{N}})$. Then the total difference between our statistic $\zeta_K^N(\mathbf{B}_{k,0}^N, \mathbf{A}_{k,0}^N, \omega)$ and its asymptotic behavior is of order $O(\frac{1}{\sqrt{K}} + \frac{1}{N} + \sqrt{\frac{K}{N}})$. Minimizing this difference in K for given N leads to $K \sim \sqrt{N}$.

It remains to validate this proposed test on actual data. This is done in the next section.

4. IMPLEMENTATION AND VALIDATION OF THE PROPOSED TEST FOR THE MODEL VALIDATION PROBLEM

4.1 Model validation

Damage detection and model validation may be viewed as two instances of a common broader question : does a given data set $(Y_n)_n$ match a given signature θ_0 ?

In the case of damage detection, we want to know if the fresh data recorded are still coherent with the reference structural parameter. To tackle this problem, a large number of independent recording is needed to obtain information about the distribution of the damage detection test and decide wether there has been significant changes in the parameter.

In the case of model validation, we have just one data set, and we want to know if it matches the reference signature, or if some slight modifications of this parameter leads to a better fit. This result in testing many different signatures to minimize a relevant statistical criterion.

It is quite common for damage detection tests to stem from modal identification procedures. The subspace-based identification algorithm Van Overschee and De Moor [1996], Peeters and De Roeck [1999] led to the subspace-based χ^2-test for damage detection, see Basseville [1998], Basseville et al. [2004]. This test has also been adapted to the model validation problem, see Mevel and Goursat [2006], Basseville et al. [2006], Canales and Mevel [2007].

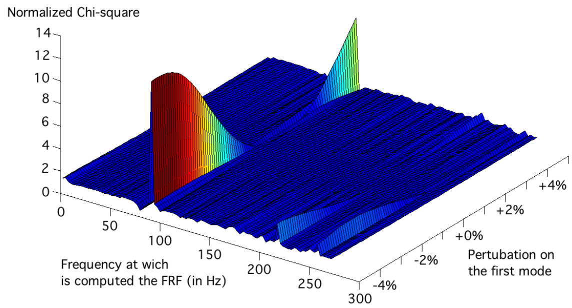

Fig. 1. The χ^2-test on the whole frequency band, with varying perturbation on the first mode.

We now implement the new frequency domain statistical test developed in the previous section, and run it on a model validation problem.

4.2 Implementation and results

We used time-domain measures taken during an airplane in-flight test, see Cauberghe [2004]. During the experiment, the airplane was artificially excited by the flaps by injecting an excitation signal. Accelerometers were placed in 7 different locations. This provides us with a dataset of $N_i = 1$ input and $N_o = 7$ outputs. The actual implementation of our algorithm runs then as follows.

- First we perform frequency-domain modal analysis on the full dataset of length $n = 24000$, and we get the estimated modal parameters. This will provide us with reference values of the modes. On our example, the identified modal frequencies are as follows :

First mode	:	98.7 Hz
Second mode	:	201.3 Hz
Third mode	:	275.7 Hz

- Next, we divide the dataset in $K = 28$ successive blocks of length $N = 784$: since we wanted $K \sim \sqrt{N}$, we took $K = n^{\frac{1}{3}}$ and $N = K^2$. On each block we perform again the modal analysis to get the corresponding common-denominator model. This provides the reference values $\mathbf{A}_{k,0}^N$ and $\mathbf{B}_{k,0}^N$ needed for building the residual (20).
- Then, we artificially change the first mode of the system, from 95% to 105% of its nominal value. The modal analysis tools developed in Cauberghe [2004], Guillaume [2006] were designed to estimate modal parameters from time data, but also to build a common-denominator transfer function model from the modal parameters alone. For each value we thus rebuild the FRFs under the changed modal conditions. This provides the $\mathbf{H}$ term, simulating the changed conditions in the residual (20).
- The value of the χ^2-test is computed.
- Last, the test is normalized along the frequency band : this result in the normalized χ^2-test value to be 1 when the first mode is equal to its identified value.

The results are displayed in Figures 1 to 4.

Figure 1 displays the whole computation of the χ^2-test. One immediately notices that the FRF corresponding to the first mode is strongly excited by its own perturbations ; it looks as if the second and third modes also respond to

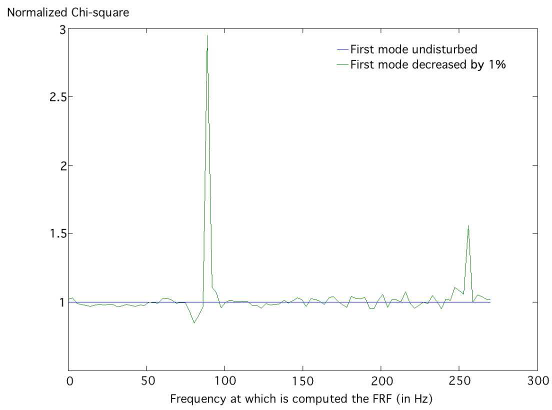

Fig. 2. The χ^2-test on the whole frequency band, with -1 % perturbation on the first mode (this is a section of Figure 1 along the frequency axis).

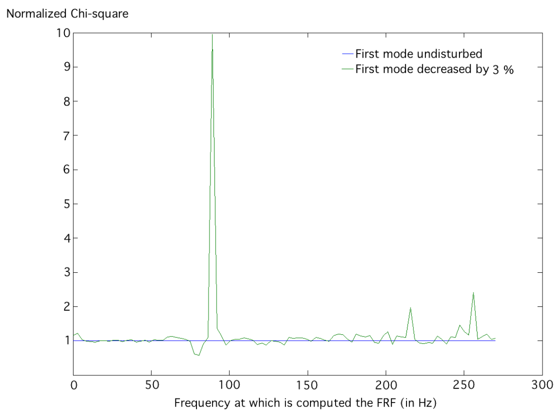

Fig. 3. The χ^2-test on the whole frequency band, with -3 % perturbation on the first mode (this is a section of Figure 1 along the frequency axis).

the perturbations, this may be due to correlation between the modes.

It is worthy to mention that this curve was computed in less than a minute, actually in 33 seconds. This mean that it should be possible to have this algorithm work efficiently on huge data sets, and structures with many more outputs.

Figures 2 and 3 display the χ^2-test on the whole frequency band for a -1% and -3% perturbation on the first mode respectively. We indeed see that the first mode presents a strong response to its own perturbations. The second mode apparently doesn't respond, while the third mode displays a response not as marked as the first but quite apparent nevertheless.

For the Figure 4 we focus the FRF on the frequency associated to the first mode, and we look at the variations of the χ^2-test with the perturbations on the first mode. We see that the minimum of the test is 0.36, attained for a perturbation of +1.2% of the first mode, corresponding to a frequency of 99.9 Hz.

5. CONCLUSION

We have presented a new algorithm for damage detection, working in the frequency domain. It has been tested on model validation of an experimental system, with encouraging results.

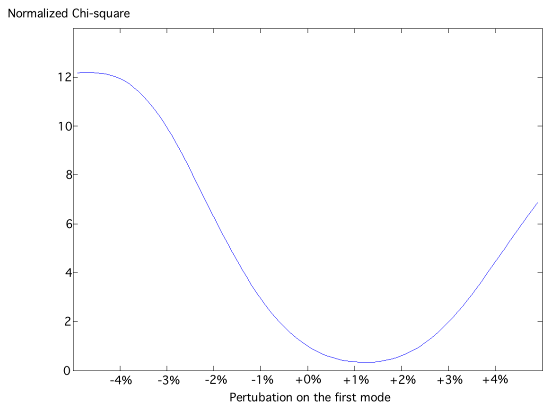

Fig. 4. The χ^2-test at the frequency of the first mode, with varying perturbation on the first mode (this is a section of Figure 1 along the perturbation axis).

Future work will involve running the algorithm on the change detection problem ; transforming the procedure from input/output to output-only ; adding a Jacobian to address the localization problem ; and testing the procedure on huge data sets obtained on systems with hundreds or thousands of outputs.

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