Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A design method of modified PID controllers for multiple-input/multiple-output plants

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Abstract: In this paper, we examine a design method of modified PID (Proportional-Integral-Derivative) controllers for multiple-input/multiple-output plants. PID controller structure is the most widely used one in industrial applications. Recently the parametrization of all stabilizing PID controller has been considered. Yamada and Hagiwara proposed a design method of modified PID controllers such that modified PID controllers make the closed-loop system for single-input/single-output unstable plants stable and the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other. However, no method has been published to guarantee the stability of PID control system for multiple-input/multiple-output plants and the admissible sets of P-parameter, I-parameter and D-parameter to guarantee the stability of PID control system are independent from each other. In this paper, we propose a design method of modified PID controllers such that the modified PID controller make the closed-loop system for multiple-input/multiple-output plants stable and the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other.

Keywords: PID control, multiple-input/multiple-output plant, stability, admissible set

1. INTRODUCTION

PID (Proportional-Integral-Derivative) controller is most widely used controller structure in industrial applications (Datta *et al.*, 2000; Suda, 1992; Astrom and Hagglund, 1995). Its structural simplicity and sufficient ability of solving many practical control problems have contributed to this wide acceptance.

Several papers on tuning methods for PID parameters have been considered (Ziegler and Nichols, 1942; Hazebroek and van der Warden, 1950*a*; Hazebroek and van der Warden, 1950*b*; Wolf, 1951; Chien *et al.*, 1952; Cohen and Coon, 1953; Lopez *et al.*, 1967; Miller *et al.*, 1967; Kitamori, 1979; Kitamori, 1980; Cominos and Munro, 2002). However the method in (Ziegler and Nichols, 1942; Hazebroek and van der Warden, 1950*a*; Hazebroek and van der Warden, 1950*b*; Wolf, 1951; Chien *et al.*, 1952; Cohen and Coon, 1953; Lopez *et al.*, 1967; Miller *et al.*, 1967; Kitamori, 1979; Kitamori, 1980; Cominos and Munro, 2002) do not guarantee the stability of closed-loop system. The reference in (Zheng *et al.*, 2002; Lin *et al.*, 2004; Viorel *et al.*, 2005; Tamura and Shimizu, 2006) propose design methods of PID controllers to guarantee the stability of closed-loop system. However, using the method in (Zheng *et al.*, 2002; Lin *et al.*, 2004; Viorel *et al.*, 2005; Tamura and Shimizu, 2006), it is difficult to tune PID parameters to meet control specifications. If admissible sets of PID parameters to guarantee the stability of closed-loop system are obtained, we can easily design stabilizing PID controllers to meet control specifications. The problem to obtain admissible sets of PID parameters to guarantee the stability of closed-loop system is known as a parametrization problem (Yang, 1994; Ho *et al.*, 1997; Datta *et al.*, 2000). If there exists a stabilizing PID controller, the parametrization of all stabilizing PID controller is considered in (Yang, 1994; Ho *et al.*, 1997; Datta *et al.*, 2000). However the method in (Yang, 1994; Ho *et al.*, 1997; Datta *et al.*, 2000) remains a difficulty. The admissible sets of P-parameter, I-parameter and D-parameter in (Yang, 1994; Ho *et al.*, 1997; Datta *et al.*, 2000) are related each other. That is, if P-parameter is changed, then the admissible sets of I-parameter and D-parameter change. From practical point of view, it is desirable that the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other. Yamada and Moki initially tackle this problem and propose a design method for modified PI controllers for any minimum phase systems such that the admissible sets of P-parameter and I-parameter are independent from each other (Yamada and Moki, 2003). Yamada expand the result in (Yamada and Moki, 2003) and propose a design method for modified PID controllers for minimum phase plant such that the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other (Yamada, 2005). Yamada and Hagiwara gave a design method for modified PID controllers for unstable plants (Yamada and Hagiwara, 2006). However the method in (Yamada and Moki, 2003; Yamada, 2005; Yamada and Hagiwara, 2006) cannot apply for multiple-input/multiple-output plants.

In this paper, we expand the result in (Yamada and Moki, 2003; Yamada, 2005; Yamada and Hagiwara, 2006) and propose a design method of modified PID controllers such that the modified PID controller makes the closed-loop system stable for any multiple-input/multiple-output plants and the admissible sets of P-parameter, I-parameter and D-parameter to guarantee the stability of closed-loop system are independent from each other.

2. PROBLEM FORMULATION

Consider the closed-loop system written by

$$\begin{cases} y = G(s)u \\ u = C(s)(r-y) \end{cases}, \tag{1}$$

where $G(s) \in R^{p\times p}(s)$ is the multiple-input/multiple-output strictly proper plant. $G(s)$ is assumed to have no zero on the origin and to be coprime. $C(s) \in R^{p\times p}(s)$ is the controller, $u \in R^p$ is the control input, $y \in R^p$ is the output and $r \in R^p$ is the reference input.

When the controller $C(s)$ has the form written by

$$C(s) = A_P + A_I\frac{1}{s} + A_D s, \tag{2}$$

then the controller $C(s)$ is called the PID controller (Yang, 1994; Ho *et al.*, 1997; Datta *et al.*, 2000; Suda, 1992), where $A_P \in R^{p\times p}$ is the P-parameter, $A_I \in R^{p\times p}$ is the I-parameter and $A_D \in R^{p\times p}$ is the D-parameter. A_P, A_I and A_D are settled so that the closed-loop system in (1) has desirable control characteristics such as steady state characteristic and transient characteristic. For easy explanation, we call $C(s)$ in (2) the conventional PID controller. The transfer function from r to y in (1) is written by

$$y = \left\{ I + G(s)\left(A_P + A_I\frac{1}{s} + A_D s\right)\right\}^{-1} G(s) \left(A_P + A_I\frac{1}{s} + A_D s\right) r. \tag{3}$$

It is obvious that when A_P, A_I and A_D are settled at random, the stability of the closed-loop system in (1) does not guaranteed. In addition, there exists $G(s)$ that cannot be stabilized using the conventional PID controllers. In addition, even if there exists stabilizing conventional PID controller, the admissible sets of A_P, A_I and A_D are related each other. From practical point of view, it is desirable that the admissible sets of A_P, A_I and A_D are independent from each other.

The purpose of this paper is to overcome these problems and to propose a design method of modified PID controllers $C(s)$ to make the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ such that the admissible sets of P-parameter A_P, I-parameter A_I and D-parameter A_D to guarantee the stability of closed-loop system are independent from each other.

3. THE BASIC IDEA

In this section, we describe the basic idea to design for modified PID controllers $C(s)$ to make the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ such that the admissible sets of P-parameter A_P, I-parameter A_I and D-parameter A_D to guarantee the stability of closed-loop system are independent from each other.

In order to design of modified PID controllers $C(s)$ that can be applied to any multiple-input/multiple-output plants, we adopt the parametrization of all stabilizing controllers for multiple-input/multiple-output plants. According to (Vidyasagar, 1985; Morari and Zafiriou, 1989), the parametrization of all proper internally stabilizing controllers $C(s)$ for multiple-input/multiple-output plants $G(s)$ is written by

$$\begin{aligned} &C(s) \\ &= (Y(s) - Q(s)\tilde{N}(s))^{-1}(X(s) + Q(s)\tilde{D}(s)) \\ &= (\tilde{X}(s) + Q(s)D(s))(\tilde{Y}(s) - Q(s)N(s))^{-1}, \end{aligned} \tag{4}$$

where $\tilde{N}(s) \in RH_\infty^{p\times p}$, $\tilde{D}(s) \in RH_\infty^{p\times p}$, $N(s) \in RH_\infty^{p\times p}$ and $D(s) \in RH_\infty^{p\times p}$ are coprime factors of $G(s)$ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s) = N(s)D^{-1}(s) \tag{5}$$

$X(s) \in RH_\infty^{p\times p}$, $Y(s) \in RH_\infty^{p\times p}$, $\tilde{X}(s) \in RH_\infty^{p\times p}$ and $\tilde{Y}(s) \in RH_\infty^{p\times p}$ are functions satisfying

$$\begin{aligned} &\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix} \begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} \begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix} \end{aligned} \tag{6}$$

and $Q(s) \in RH_\infty^{p\times p}$ is any function.

On the parametrization of all stabilizing controllers $C(s)$ in (4) for $G(s)$, the controller $C(s)$ in (4) includes free-parameter $Q(s)$. Using free-parameter $Q(s)$ in (4), we propose a design method of modified PID controllers $C(s)$ to make the closed-loop system in (1) stable and to be able to apply to any unstable plant $G(s)$. In order to design the modified PID controllers $C(s)$, the free parameter $Q(s)$ in (4) is settled for $C(s)$ in (4) to have the same characteristics to conventional PID controller $C(s)$ in (2). Therefore, next, we describe the role of conventional PID controller $C(s)$ in (2) in order to clarify the condition that the modified PID controller $C(s)$ must be satisfied. From (2), using $C(s)$, the P-parameter A_P, the I-parameter A_I and the D-parameter A_D are decided by

$$A_P = \lim_{s\to\infty} \left\{ -s^2 \frac{d}{ds}\left(\frac{1}{s}C(s)\right)\right\}, \tag{7}$$

$$A_I = \lim_{s\to 0}\{sC(s)\} \tag{8}$$

and

$$A_D = \lim_{s\to\infty} \frac{d}{ds}\{C(s)\}, \tag{9}$$

respectively. Therefore, if the controller $C(s)$ in (4) holds (7), (8) and (9), the role of controller $C(s)$ in (4) is equivalent to the conventional PID controller $C(s)$ in (2). That is, we can design stabilizing modified PID controllers such that the role of controller $C(s)$ in (4) is equivalent to the conventional PID controller $C(s)$ in (2).

Next, we describe a design method for the free parameter $Q(s)$ in (4) to make the controller $C(s)$ in (4) works as a modified PID controller. In the following, we call $C(s)$

(1) the modified P controller if $C(s)$ in (4) satisfies (7),
(2) the modified I controller if $C(s)$ in (4) satisfies (8),
(3) the modified D controller if $C(s)$ in (4) satisfies (9),
(4) the modified PI controller if $C(s)$ in (4) satisfies (7) and (8),
(5) the modified PD controller if $C(s)$ in (4) satisfies (7) and (9)
(6) and the modified PID controller if $C(s)$ in (4) satisfies (7), (8) and (9).

4. MODIFIED PID CONTROLLER

In this section, we describe a design method of the free parameter $Q(s)$ in (4) to makes the controller $C(s)$ in (4) works as a modified PID controller.

4.1 Modified P controller

In this subsection, we mention a design method of modified P controller $C(s)$ that holds (7), makes the closed-loop system in (1) stable and is able to apply to any stable plant $G(s)$.

The modified P controller $C(s)$ satisfying (7) is written by (4), where

$$Q(s) = Q_0 (= \text{const}) \tag{10}$$

and

$$Q_0 = \lim_{s\to\infty} \left\{ (Y(s)A_P - X(s))(\tilde{N}(s)A_P + \tilde{D}(s))^{-1} \right\} \tag{11}$$

Since $Q(s)$ in (10) is included in RH_∞, the controller $C(s)$ in (4) with (10) makes the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ independent from A_P.

4.2 Modified I controller

In this subsection, we mention a design method of modified I controller $C(s)$ that holds (8), makes the closed-loop system in (1) stable and is able to apply to any stable plant $G(s)$.

The modified I controller $C(s)$ satisfying (8) is written by (4), where

$$Q(s) = (Q_0 + Q_1 s) \frac{1}{\tau_0 + \tau_1 s}, \tag{12}$$

$$Q_0 = Y(0)\tilde{N}^{-1}(0)\tau_0, \tag{13}$$

$$Q_1 = Q_0 \frac{\tau_1}{\tau_0} + \tau_0 \left[\left(\frac{d}{ds} \{Y(s)\} \Big|_{s=0} - Q(0) \frac{d}{ds} \left\{ \tilde{N}(s) \right\} \Big|_{s=0} \right) A_I - X(0) - Q(0)\tilde{N}(0) \right] \left(\tilde{N}(0) A_I \right)^{-1}, \tag{14}$$

$\tau_i \in R > 0$ $(i = 0, 1)$. From $\tau_i > 0$ $(i = 0, 1)$, $Q(s)$ in (10) is included in RH_∞. This implies that the controller $C(s)$ in (4) with (12) makes the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ independent from A_I.

4.3 Modified D controller

In this subsection, we mention a design method of modified D controller $C(s)$ that holds (9), makes the closed-loop system in (1) stable and is able to apply to any stable plant $G(s)$.

The modified D controller $C(s)$ satisfying (9) is written by (4), where

$$Q(s) = Q_0 s \tag{15}$$

and

$$Q_0 = \lim_{s\to\infty} \left\{ Y(s)A_D(\tilde{D}(s) + s\tilde{N}(s)A_D)^{-1} \right\}. \tag{16}$$

Since $Q(s)$ in (15) is improper, $Q(s)$ in (15) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (15) is modified as

$$Q(s) = Q_0 \frac{s}{1 + \tau_D s}, \tag{17}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ in (17), $Q(s)$ in (17) is included in RH_∞. This implies that the controller $C(s)$ in (4) with (17) makes the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ independent from A_D.

4.4 Modified PI controller

In this subsection, we mention a design method of modified PI controller $C(s)$ that holds (7) and (8), makes the closed-loop system in (1) stable and is able to apply to any stable plant $G(s)$.

The modified PI controller $C(s)$ satisfying (7) and (8) is written by (4), where

$$Q(s) = \left(Q_0 + Q_1 s + Q_2 s^2 \right) \frac{1}{\tau_0 + \tau_1 s + \tau_2 s^2}, \tag{18}$$

$$Q_0 = Y(0)\tilde{N}^{-1}(0)\tau_0, \tag{19}$$

$$Q_1 = Q_0 \frac{\tau_1}{\tau_0} + \tau_0 \left[\left(\frac{d}{ds} \{Y(s)\} \Big|_{s=0} - Q(0) \frac{d}{ds} \left\{ \tilde{N}(s) \right\} \Big|_{s=0} \right) A_I - X(0) - Q(0)\tilde{N}(0) \right] \left(\tilde{N}(0) A_I \right)^{-1}, \tag{20}$$

$$Q_2 = \lim_{s\to\infty} \left\{ (Y(s)A_P - X(s))(\tilde{N}(s)A_P + \tilde{D}(s))^{-1} \right\} \tau_2 \tag{21}$$

and $\tau_i \in R > 0$ $(i = 0, 1, 2)$. From $\tau_i > 0$ $(i = 0, 1, 2)$, $Q(s)$ in (18) is included in RH_∞. This implies that the controller $C(s)$ in (4) with (18) makes the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ independent from A_P and A_I.

4.5 Modified PD controller

In this subsection, we mention a design method of modified PD controller $C(s)$ that holds (7) and (9), makes the closed-loop system in (1) stable and is able to apply to any stable plant $G(s)$.

The modified PD controller $C(s)$ satisfying (7) and (9) is written by (4), where

$$Q(s) = Q_0 + Q_1 s, \tag{22}$$

$$\begin{aligned} Q_0 = \lim_{s\to\infty} \Bigg[\Bigg[&(Y(s) - Q_1 s\tilde{N}(s))A_P - X(s) \\ &-s^2 \frac{d}{ds}\{Y(s) - Q_1 s\tilde{N}(s)\} \\ &\cdot(Y(s) - Q_1 s\tilde{N}(s))^{-1} Q_1 \tilde{D}(s) \\ &+s^2 Q_1 \frac{d}{ds}\left\{\tilde{D}(s)\right\}\Bigg] \tilde{D}^{-1}(s)\Bigg], \end{aligned} \tag{23}$$

$$Q_1 = \lim_{s\to\infty} \left\{ Y(s)A_D(\tilde{D}(s) + s\tilde{N}(s)A_D)^{-1} \right\}, \tag{24}$$

Since $Q(s)$ in (22) is improper, $Q(s)$ in (22) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (22) is modified as

$$Q(s) = Q_0 + Q_1 \frac{s}{1 + \tau_D s}, \tag{25}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ in (25), $Q(s)$ in (25) is included in RH_∞. This implies that the controller $C(s)$ in (4) with (25) makes the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ independent from A_P and A_D.

4.6 Modified PID controller

In this subsection, we mention a design method of modified PID controller $C(s)$ that holds (7), (8) and (9), makes the closed-loop system in (1) stable and is able to apply to any stable plant $G(s)$.

The modified PID controller $C(s)$ satisfying (7), (8) and (9) is written by (4), where

$$\begin{aligned} &Q(s) \\ &= \left(Q_0 + Q_1 s + Q_2 s^2\right) \frac{1}{\tau_0 + \tau_1 s + \tau_2 s^2} + Q_3 s, \end{aligned} \tag{26}$$

$$Q_0 = Y(0)\tilde{N}^{-1}(0)\tau_0, \tag{27}$$

$$\begin{aligned} Q_1 &= Q_0 \frac{\tau_1}{\tau_0} - Q_3\tau_0 + \tau_0 \left[\left(\frac{d}{ds}\{Y(s)\}\Big|_{s=0} - Q(0)\right.\right. \\ &\quad \left.\left. \frac{d}{ds}\left\{\tilde{N}(s)\right\}\Big|_{s=0}\right) A_I - X(0) - Q(0)\tilde{N}(0)\right] \\ &\quad \left(\tilde{N}(0)A_I\right)^{-1}, \end{aligned} \tag{28}$$

$$\begin{aligned} Q_2 = \lim_{s\to\infty} \Bigg[\Bigg[&(Y(s) - Q_3 s\tilde{N}(s))A_P - X(s) \\ &-s^2 \frac{d}{ds}\{Y(s) - Q_3 s\tilde{N}(s)\} \\ &(Y(s) - Q_3 s\tilde{N}(s))^{-1} Q_3 \tilde{D}(s) \\ &+s^2 Q_3 \frac{d}{ds}\left\{\tilde{D}(s)\right\}\Bigg] \tilde{D}^{-1}(s)\tau_2\Bigg], \end{aligned} \tag{29}$$

$$Q_3 = \lim_{s\to\infty} \left\{ Y(s)A_D(\tilde{D}(s) + s\tilde{N}(s)A_D)^{-1} \right\}, \tag{30}$$

and $\tau_i \in R > 0$ $(i = 0, 1, 2)$. Since $Q(s)$ in (26) is improper, $Q(s)$ in (26) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (26) is modified as

$$\begin{aligned} Q(s) = &\left(Q_0 + Q_1 s + Q_2 s^2\right) \frac{1}{\tau_0 + \tau_1 s + \tau_2 s^2} \\ &+ Q_3 \frac{s}{1 + \tau_D s}, \end{aligned} \tag{31}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ and $\tau_i > 0$ $(i = 0, 1, 2)$ in (31), $Q(s)$ in (31) is included in RH_∞. This implies that the controller $C(s)$ in (4) with (31) makes the closed-loop system in (1) stable for any multiple-input/multiple-output plant $G(s)$ independent from A_P, A_I and A_D.

5. NUMERICAL EXAMPLE

In this section, a numerical example is illustrated to show the effectiveness of the proposed method.

Consider the problem to design a modified PID controller $C(s)$ for an unstable non-minimum phase plant $G(s)$ written by

$$G(s) = \begin{bmatrix} \dfrac{s-1}{s^4 - 27s^2 - 14s + 120} & \dfrac{2}{s^4 - 27s^2 - 14s + 120} \\ \dfrac{-3}{s^4 - 27s^2 - 14s + 120} & \dfrac{s-1}{s^4 - 27s^2 - 14s + 120} \end{bmatrix}. \tag{32}$$

Since poles of $G(s)$ are in $(2, 0)$, $(-3, 0)$, $(-4, 0)$, $(5, 0)$ and zeros of $G(s)$ are in $(1 + 2.45i)$, $(1 - 2.45i)$, $G(s)$ in (32) is an unstable and of non-minimum phase.

A_P, A_I and A_D are settled by

$$\begin{cases} A_P = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \\ A_I = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \\ A_D = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix} \end{cases}. \tag{33}$$

Using above-mentioned parameter, the modified PID controller $C(s)$ is designed by (4) with (31), where

$$\begin{cases} \tau_0 = 1260 \\ \tau_1 = 71 \\ \tau_2 = 1 \end{cases} \quad (34)$$

and τ_D is selected by $\tau_D = 0.1$.

The response of y written by

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (35)$$

for the reference input r

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad (36)$$

is shown in Fig. 1. Here, the solid line shows the response of y_1 and the dotted line shows that of y_2. Figure 1 shows

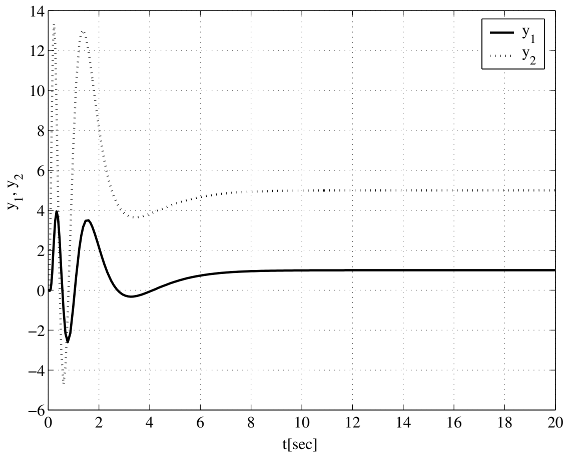

Fig. 1. Step response y of the control system using modified PID controller

that the modified PID controller $C(s)$ makes the closed-loop system stable.

On the other hand, using conventional PID controller with (33), the step responses y_1, y_2 of the control system are shown in Fig. 2. Here, the solid line shows the response of y_1 and the dotted line shows that of y_2. Figure 2 shows that the conventional PID control system is unstable. The reason why the conventional PID control system is unstable is the stability of the conventional PID control system depends on A_P, A_I and A_D. Therefore, when A_P, A_I and A_D are settled by (33), the conventional PID control system is unstable. Contrary to this, the stability of modified PID control system is guaranteed independence of A_P, A_I and A_D.

6. CONCLUSIONS

In this paper, we proposed a design method of modified PID controllers such that modified PID controllers make the closed-loop system for any multiple-input/multiple-output plants stable and the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other. Numerical examples was shown to illustrate the effectiveness of the proposed method.

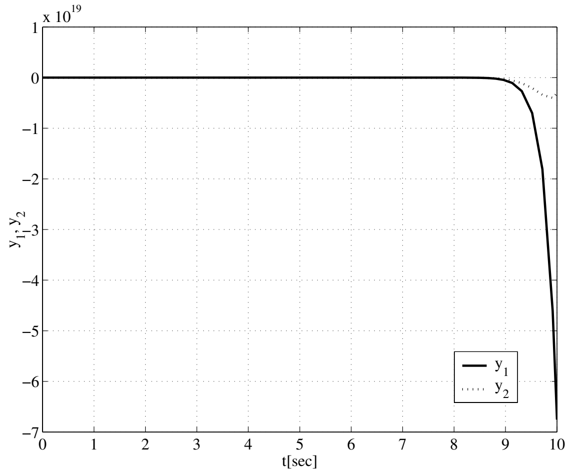

Fig. 2. Step response y of the control system using conventional PID controller

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