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ROBUST SPEED CONTROL OF PMSM USING Mixed NONLINEAR H_∞/SMC Techniques

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Abstract: A new technique for high performance and robust speed control of permanent magnet synchronous motor (PMSM) using a mixed non-linear H_∞ and Sliding Mode Control (SMC) is applied. In spite of non-linear modes, motor parameters variation and uncertainty of load torque, non-linear robust techniques introduce a precise speed control. Since the load torque is an external disturbance and its variation and type is not generally available, H_∞ technique is a suitable approach to minimize its influence on the output. However, motor parameters variations throw away the response from H_∞ response and influence its response, therefore SMC technique is used to conduct the response towards H_∞ response. Combination of these two techniques gives a suitable technique to robust speed control of PMSM. *Copyright © 2008 IFAC*

Keywords: PMSM, Robust, Nonlinear Control, Nonlinear H_∞, SMC.

1. INTRODUCTION

Many advantages of permanent magnet synchronous motors (PMSM) lead to their wide application in industry. They are used in robots and similar industry equipment as well as in many control systems. Quick and precise response, eliminating disturbance, approaching reference speed and low-sensitivity against parameters variation are the criteria of desirable performance of these motors. These characteristics make possible to use them in robotic and instrumentations. Application of vector control technique in these motors improves their performance. see (Senjyu, *et al*., 1996; Senjyu, *et al*., 1997).

However, system non-linearity, motor parameters variation and load torque variation make difficult to control the motor precisely. There are various methods to solve these problems. Some methods use adaptive or neural network techniques (Tengfei and Wang, 2006; Yu, *et al*., 2006; Zhou and Wang, 2002; Elbuluk, *et al*., 2002; Mingji, *et al*., 2004) and others are suitable due to slow change of some parameters such as stator resistance. However, some parameters of motor such as inductance, flux or load torque can quickly change and the above-mentioned methods do not provide a suitable response, see (Soo, *et al*., 2001) ; therefore, robust control techniques are used for this motor. One of most important techniques is SMC which overcomes the parameters variation well (Lee and Shtessel , 1996; Golea, *et al*., 1999).

However, the problem is the load torque variation that is identified by observer in some methods (Baik, *et al*., 2000); this has own problems such as dependency on the model, noise effect etc. In some methods of Sliding Mode Control (SMC) it is treated similar to parameters (Zhang and Panda, 1999), but the value and type of the load as an external disturbance is not defined and such action cannot be true. Other problem with SMC technique is its chattering which may be solve if boundary layer is employed with SMC technique. Unfortunately variation of some motor parameters leads to a steady-state error with this solution.

The H-infinity technique is an efficient method for robustness of non-linear against disturbances. In addition to guarantee the stability, this technique minimizes the disturbance influence at the output. Nonlinear H_∞ has been described by (Isidori and Astolfi, 1992; Van der Shaft, 1992). The technique has no assumption on the disturbance except the energy limitation. In the introduced method, H-infinity is used for robustness of the response against load variation, since load torque variation is an external factor which can be any type, H-infinity technique has no assumption on the disturbance and is a suitable technique for dealing with the proposed problem. In such a case, the response is somehow robust against parameters variation. However, parameters variation throws away the response from H-infinity technique, so SMC technique can be used in order to control this variation and conduct the response to the H-infinity response. Therefore, considering different natures of load and parameters variation, two techniques have been employed to solve the problems. In section II of the paper the mathematical and vector model of PMSM is introduced. In section III, application of H-infinity in PMSM is proposed and it combination with SMC is given in section IV. Simulation results and their comparison with other methods are discussed in section V. Finally section VI concludes the paper.

2. MATHEMATICAL MODEL OF PMSM

The d-q Equations of PMSM, which are in fact the vector equations of the motor, are as follows:

$$\begin{cases} \frac{d\omega_r}{dt} = \frac{3P}{4J}\lambda_f i_q - \frac{D}{J}\omega_r - \frac{T_l}{J} \\ \frac{di_q}{dt} = \frac{-P\lambda_f}{2L_s}\omega_r - \frac{P}{2}\omega_r i_d - \frac{R}{L_s}i_q + \frac{V_q}{L_s} \\ \frac{di_d}{dt} = -\frac{R}{L_s}i_d + \frac{P}{2}\omega_r i_q + \frac{V_d}{L_s} \end{cases} \quad (1)$$

where ω_r is the angular speed of the motor, i_d and i_q are the d and q currents, V_d and V_q are the input d and q voltages in vector control domain. R and L_s are the stator winding resistance and inductance respectively, P is the number of poles, λ_f is the flux of PM, J is the motor inertia, D is the damping factor, T_l is the load torque that is considered as disturbance. Since H_∞ solution for the above-mentioned system of equations does not provide a tracking response and the aim is to obtain a robust tracking response, a state variable which is the difference between the speed and the reference speed is added to the system. This variable is called q. Therefore, the following equation is added to (1) in order to complete the model of the system:

$$\frac{dq}{dt} = \omega_d - \omega_r \quad (2)$$

where ω_d is the reference speed.

3. DESIGN OF H_∞ CONTROLLER

The reason for using the non-linear H-infinity in place of the linear H-infinity is that the motor is non-linear. If the linear H-infinity is used, there will be no guarantee for stability and it may be unstable when the load torque quickly changes. However, use of the non-linear H-infinity can guarantee the stability over any condition.

In the design of H-infinity controller, the following model for PMSM is defined:

$$\begin{cases} \dot{x} = f(x) + B_1 w + B_2 u \\ z = C_1 x + D_1 u \qquad C_1^t D_1 = 0 \end{cases} \quad (3)$$

where $x^T = [\omega_r \;\; i_q \;\; i_d \;\; q]$, $u^T = [V_q \;\; V_d]$ and w is the disturbance or load torque (T_l) and z is the output, and also $B_1^t = \begin{bmatrix} -\frac{1}{J} & 0 & 0 & 0 \end{bmatrix}$ and $B_2^t = \begin{bmatrix} 0 & \frac{1}{L_s} & 0 & 0 \\ 0 & 0 & \frac{1}{L_s} & 0 \end{bmatrix}$. z is expressed as a combination of inputs and outputs. In H_∞ technique the aim is to minimize the influence of disturbance (w) on output (z). A larger C_1 compared to D_1 has more disturbing influence in the inputs and less in the states. Since the goal is the tracking speed control, variables ω and q are more important, therefore their corresponding elements in matrix C_1 will be non-zero. It is noted that i_d has no influence on the torque directly, and it raises the amplitude of the current, so it is controlled toward zero in some designs. However, in some precise controls with lower importance of losses, i_d can be also be used to control speed or position. In the presented method, effect of load change on i_d can be set by changing the value of its corresponding element in C_1.

The non-linear H_∞ controller for system (3) will be as follows (Van der Shaft, 1992):

$$u = -(D_1^T D_1)^{-1} B_2^T V_x^T(x) \quad (4)$$

where $V_x(x) = \frac{\partial V}{\partial x}$, and $V(x)$ is the solution of HJI inequality, i.e.,:

$$\frac{\partial V}{\partial x}(x) f(x) + \frac{1}{2}\frac{\partial V}{\partial x}(x)\left(\frac{1}{\gamma^2}B_1.B_1^t - B_2.(D_1^T.D_1)^{-1}.B_2^T\right) \quad (5)$$
$$\frac{\partial V^T}{\partial x}(x) + \frac{1}{2}x^T C_1^T C_1 x \le 0$$

In this case:

$$\|z\|_2^2 < \gamma^2 \|w\|_2^2 \quad (6)$$

where γ is attenuation level of the effect of w on z. Since (5) in general does not have a close form solution, an approximate solution is considered. Let $V(x)$ be approximated as follows:

$$V(x) = V^{(2)}(x) + V^{(3)}(x) + V^{(4)}(x) + \ldots \quad (7)$$

where $V^{(i)}(x)$ is an ith-order polynomial of x. In this way, one has:

$$f(x) = f^{(1)}(x) + f^{(2)}(x) \quad (8)$$

Other coefficients in PMSM are linear and there is no need to split-up them.

Considering $V^{(2)}(x) = x^t P x$ in which P is symmetric and positive definite and $f^{(1)}(x) = F.x$, P is obtained by solving the following Riccati equation:

$$F^T P + PF + P\left(\frac{1}{\gamma^2}B_1.B_1^T - B_2.(D_1^T.D_1)^{-1}.B_2^T\right)P + C_1^T C_1 = 0 \quad (9)$$

Since the model is linear in respect to the inputs and disturbance, higher-order norms of V are obtained as follows:

$$V_x^{(m)}(x)(F_{cl})x = -\sum_{d=2}^{m-2}\left[V_x^{(d+1)}(x)\left(B_1 u_*^{(m-d)}(x) + B_2 w_*^{(m-d)}\right)\right] \quad (10)$$
$$-\sum_{d=1}^{m-2} V_x^{(d+1)}(x) f^{(m-d)}(x, u_*, w_*) - \sum_{d=2}^{m-2}\left[u_*^{t(d)}(D_1^T D_1)u_*^{(m-d)} + \gamma^2 w_*^{T(d)} w_*^{(m-d)}\right]$$

where:

$$F_{cl} = F + \frac{1}{\gamma^2} B_1 B_1^T P - B_2 \left(D_1^T D_1\right)^{-1} B_2^T \tag{11}$$

$$u_*^{(k)}(x) = -(D_1^T D_1)^{-1} B_2^T (V_x^{(k+1)}(x))^T \tag{12}$$

$$w_*^{(k)}(x) = \frac{1}{\gamma^2} B_2^T (V_x^{(k+1)}(x))^T \tag{13}$$

4. APPLICATION OF SMC FOR CONDUCTING RESPONSE TOWARDS H_∞

In H_∞ technique, the effect of motor parameters variations has not been included. Although this technique is somehow robust against motor parameters variations, the variations throws away the response from H-infinity response. Therefore, the effect of motor parameters variations can be eliminated using SMC. In this case, the SMC solution conducts the response of the system towards solution of H-infinity.

In such a case the sliding level is considered as follows:

$$S = Cx - C\int \left(f(x) - B_2 . \left(D_1^T D_1\right)^{-1} B_2^T . V_x^T(x) \right) dt \tag{14}$$

To obtain the control rule $\dot{S} = 0$:

$$\dot{S} = C\dot{x} - C\left(f(x) - B_2 . \left(D_1^T D_1\right)^{-1} B_2^T . V_x^T(x) \right) \tag{15}$$

Considering $C = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$:

$$\dot{S} = \dot{x}_1 + \dot{x}_2 - f_1(x) - f_2(x) + C . B_2 \left(D_1^T D_1\right)^{-1} B_2^T . V_x^T(x) = 0 \tag{16}$$

Substituting $\dot{x}_1$ and $\dot{x}_2$ leads to:

$$\dot{S} = \frac{1}{L} V_{q-eq} + C B_2 . \left(D_1^t D_1\right)^{-1} B_2^T . V_x^T(x) = 0 \tag{17}$$

and finally:

$$V_{q-eq} = -\begin{bmatrix} 1 & 0 \end{bmatrix} \left(D_1^T D_1\right)^{-1} B_2^T . V_x^T(x) \tag{18}$$

which in fact is the same as the resulting control obtained through H_∞ technique. Therefore this control rule of SMC will be as follows:

$$V_q = V_{q-eq} - \beta \, \mathrm{sgn}(S) \tag{19}$$

Where V_{q-eq} and S are determined by (12) and (14), β is a positive integer and *sgn* is the sign function. Also to improve the response and reduce the chattering, the SMC with boundary layer can be used.

$$V_q = V_{q-eq} - \beta \, \mathrm{sat}(S/\varphi) \tag{20}$$

where φ is the boundary layer width and sat is the saturation function. It is noted that the use of SMC with boundary layer leads to a steady-state error in the speed response. The general block diagram of the control system has been shown in Fig. 1.

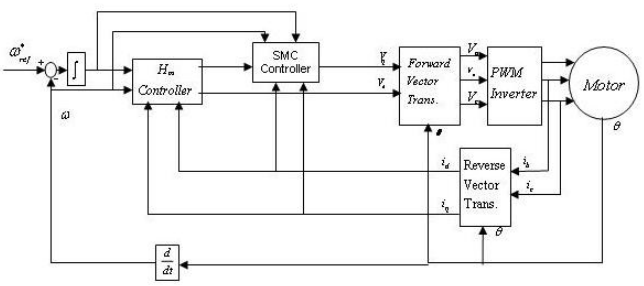


Fig 1. General block diagram of the control system

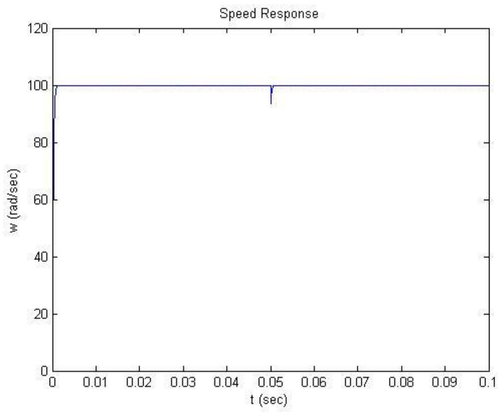
5. SIMULATION RESULTS

Simulations have been carried out using MATLAB software. Parameters of the simulated motor have been given in Table 1. To present the quality of combined controller, This technique is compared with non-linear H_∞ and SMC.

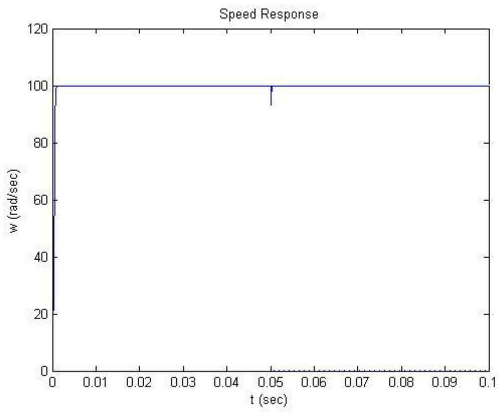
Table 1. Parameters of simulated motor

Parameter	Symbol	Value
Stator resistance	R	5.26 (Ω)
Stator inductance	L_s	0.46×10^{-3} (H)
PM flux	λ_f	24.9×10^{-3} (Wb)
Motor inertia	J	10^{-3} (kg.m^2)
Damping factor	D	1.32×10^{-6} (Nms)
No. of poles	P	4

First the influence of the load upon the output is studied. The non linear H_∞ control for $C_1 = \begin{bmatrix} 1 & 0 & 0 & 5000 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, up to the third mode is designed. For a nonlinear system, the maximum attenuation of the disturbance that corresponds to the smallest γ, has the same value as for its linearization (Van der Shaft, 1992). Therefore, the smallest γ is selected such that the Riccati Eqn. (9) has a positive definite solution for *P*. The minimum value for γ in this simulation was obtained as γ=500. The value of γ is large because the disturbance coefficient is very small. Reference speed is chosen to be 100 rad/s. Fig. 2a shows the speed response of the system with the non linear H_∞ when the load T_l=0.2 Nm is applied at t=0.5 s. As shown, the response is fully tracked and this tracking continues after applying the load. Fig. 2b shows the SMC technique response. In this case, the boundary layer width is taken to be *φ=1000, λ=10000* and *β=10*. Fig. 2c presents the mixed response (H_∞ and SMC with boundary layer) for C_1 and D_1 the same as H_∞ and $\gamma = 500$ and for H_∞, $\varphi = 100$ and $\beta = -5000$ for SMC with boundary layer.



(a)



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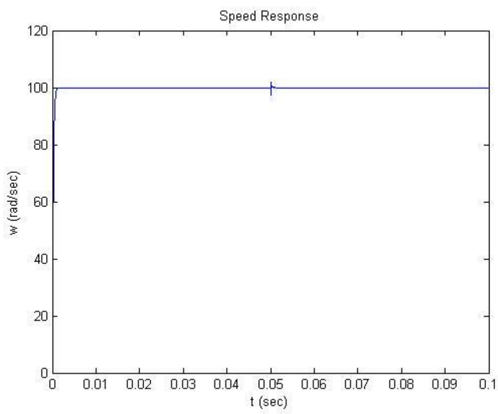

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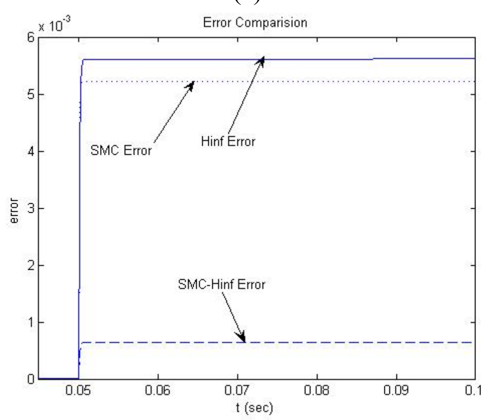
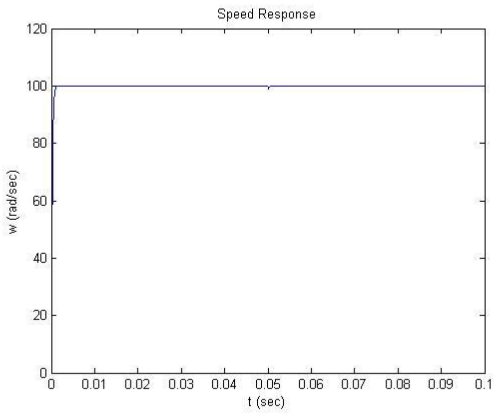


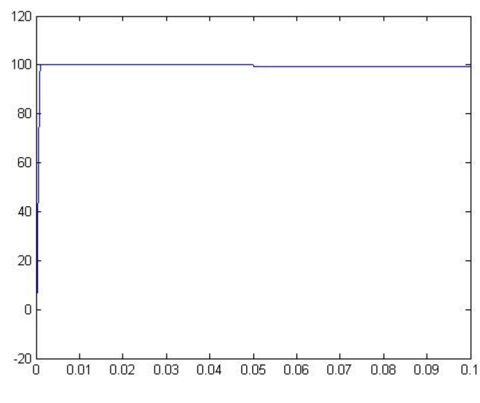
(d)

Fig. 2. Speed response by applying the load: (a) H_∞, (b) SMC, (c) Mixed and (d) comparison of errors

As seen in the Fig. 2.c, SMC also help H_∞ in the control of load disturbance and the mixed method response does better job for load variation. Fig. 2d shows the integral of the square of speed error from the reference speed at the instant of applying the load. As seen, the response of the mixed method has a sensible difference with other two techniques. In this section the influence of the parameter variation is shown. At t=0.5 s, the stator resistance is increased 50%. Fig. 3 shows the responses of different techniques. In this case load T_l=0.2 Nm is applied to the system at t=0. As seen in the figure, stator resistance change leads to a steady-state error in the SMC response. The integral of the square of error is highly rising. Also it is clear that the response of the mixed method is better than that of the H_∞ and the error is lower.



(a)



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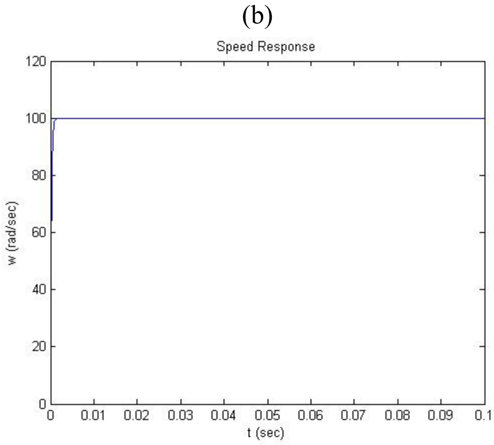

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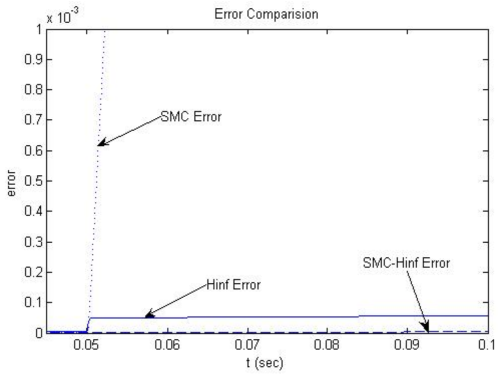
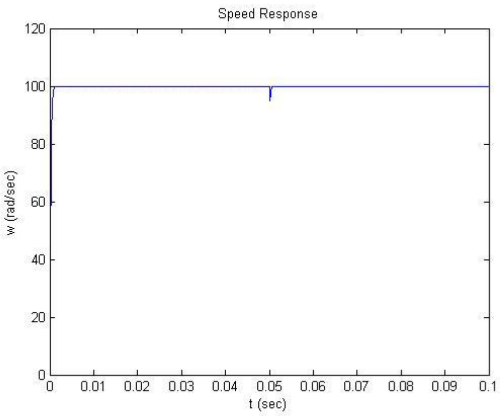
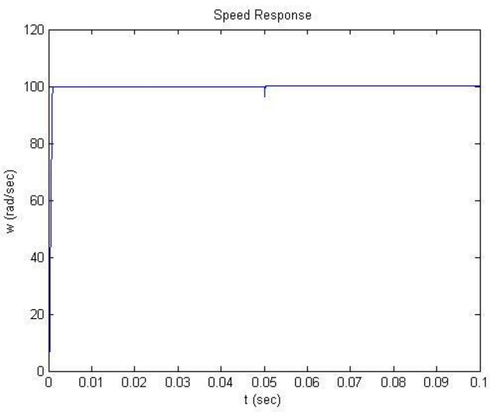


(d)

Fig. 3. Speed response by changing stator resistance t=0.5 s: (a) H_∞, (b) SMC, (c) Mixed and (d) comparison of errors



(a)



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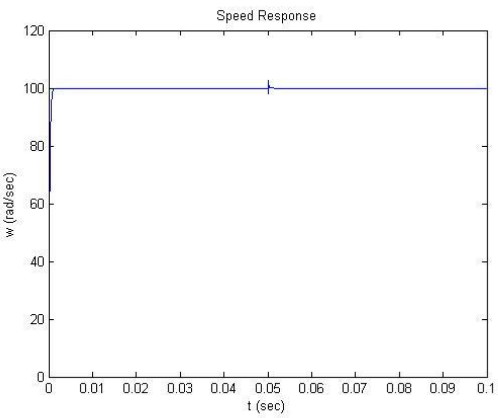

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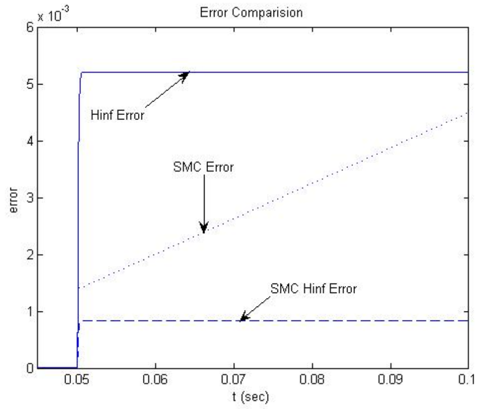
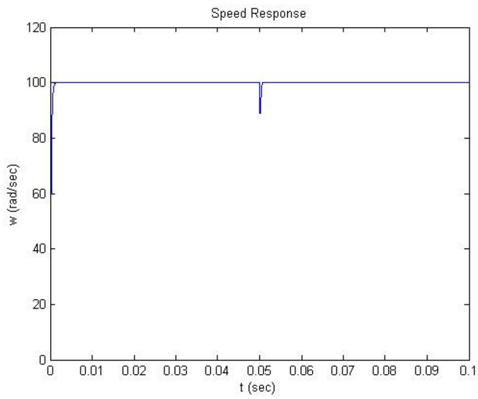


(d)

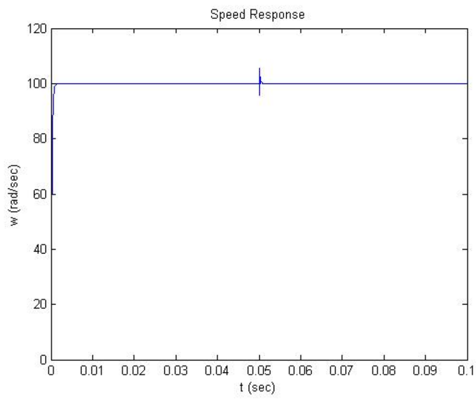
Fig. 4. Speed response by varying stator rotor flux at t=0.5 s: (a) H_∞, (b) SMC, (c) Mixed and (d) comparison of errors

Now if the flux of the motor is reduced by 50% and the load T_l=0.2 Nm is applied to the system, the response shown in Fig. 4 is obtained. In this case it is observed in the steady-state error of SMC technique. Fig. 4b indicates that the mixed technique leads to a more suitable response compared with other techniques and the integral of the error is smaller than that of the other techniques.

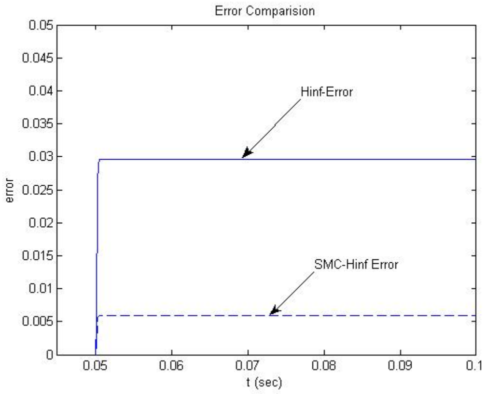
Now all parameters are changed simultaneously, it means that the stator resistance is increased and motor flux is decreased 50% at t=0.5 s and at the same time the load T_l=0.2 Nm is applied to the system. Fig. 5 presents the response of the system.



(a)



(b)



(c)

Fig. 5. Speed response by parameters and load varying at t=0.5 s: (a) H_∞, (b) Mixed and (c) comparison of errors

Fig. 5a shows the response of H_∞. It shows that the control and tracking has been carried out. The SMC technique is unstable in this case. Of course if β increases, the system becomes stable. However, increase of β leads to increase of chattering. In order to reduce the chattering the width of the boundary layer must be increased which leads to the increase of the steady-state error. Fig. 5b presents the system response using mixed control. Fig. 5c shows that the response of the mixed system is better than that of the H_∞.

6. CONCLUSIONS

A novel method using non-linear H_∞ was used to minimize the effect of the varying load in the output with SMC method to minimize the influence of motor parameters variation and system output. Considering the non-linearity of the motor and load torque variation as an external disturbance, H_∞ method is a suitable one for system control. Since the design of H_∞ controller leads to the solution of inequality HJI, its precise solution is very difficult; therefore an approximate solution was used. Also in order to reduce the effect of the parameters, the non-linear controller SMC was employed. Finally, simulations showed the high quality of the control method.

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