Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A Novel Recursive Terminal Sliding Mode with Finite-Time Convergence

Shuanghe Yu*. Jialu Du*. Xinghuo Yu. He Xu*****

**School of Information Science and Technology, Dalian Maritime University,Dalian, China, 116026 (Tel: 86-411-84725191; e-mail:shuanghe@ newmail.dlmu.edu.cn, dujl@ newmail.dlmu.edu.cn).*
*** School of Electrical and Computer Engineering, Royal Melbourne Institute of Technology University Melbourne, VIC 3001, Australia, (e-mail:x.yu@rmit.edu.au)*
****College of Mechanic & Electric Engineering, Harbin Engineering University, Harbin, China, 150001 (e-mail: railway_dragon@163.com)*

Abstract: A novel recursive framework for designing terminal sliding mode (TSM) and fast terminal sliding mode (FTSM) with finite-time convergence is developed in this paper. The principle of finite-time convergence is investigated under these new formulations. The singularity problem around the origin with the previous TSM control can be resolved.

1. INTRODUCTION

It is often desirable to stabilize the motion in finite time. Several approaches have been considered for this problem including the classical minimal-time and minimal energy controllers (Ryan, 1991). The problem of finite time stabilization of a dynamic system has been studied by quite a few people from different perspectives (Haimo, 1986; Bhat & Bernstein, 1998; Hong & Jiang, 2006; Hong, Huang & Xu, 2001;Yu & Man, 2002;). It appears that it is not so easy to achieve such a goal. Besides the purely system theoretic interest in the question of finite time convergence, the theory also gives rise to the high-precision control performance, such as terminal sliding mode (TSM) (Feng, Yu & Man, 2002; Yu, Yu & Man; Yu, Yu, Shirinzadeh & Man, 2005)

Unlike conventional sliding mode design, TSM is based on a class of nonlinear differential equations with the finite time solution. Recently there has been increased interest in the use of TSM. The philosophy of design of TSM control is basically a conventional sliding mode controller with a nonlipschitz sliding surface, where the dynamics of this surface exhibits an attractor and thus tracking error converges in finite time.

The conventional TSM control usually has negative fractional power because of the fractional power sliding mode and its derivative. Thus, the algorithms are very sensitive around the origin and can take unexpectedly large values, leading to the singularity problem. It is obvious that the TSM structure leads to this great difficulty. In some sense, our proposed approach takes the opposite point of view in that we seek to achieve the positive fractional power in each step of derivative. We propose two new recursive algorithms to recover the original TSM and FTSM. In a similar way, we also recover the finite-time convergence property.

This paper is organized as follows. Some basic notions of TSM and finite-time convergence are briefly recalled in the next section, and the necessity of reforming it to deal with the common singularity is stressed. A novel recursive version of TSM is formally introduced in Section 3, where our solution for finite-time convergence and non-singular property, which is the contribution of the present work, is described in details. Some conclusions and further work are discussed in Section 4.

2. PRELIMINARIES

In this section, the old versions of TSM are reviewed and some of their properties are analyzed, especially the singularity problem.

Definition 1. The original expressions of TSM and FTSM (Yu & Man, 2002) are

$$\begin{aligned} s &= \dot{x} + \beta x^{q/p} = 0 \\ s &= \dot{x} + \alpha x + \beta x^{q/p} = 0 \end{aligned} \tag{1}$$

where $\alpha, \beta > 0$, $p > q > 0$ are integers, p and q are odd. However for $x < 0$, the fractional power q/p may lead to the item $x^{q/p} \notin R$, which means $\dot{x} \notin R$ contradicting with the system we are considering.

Definition 2. The TSM and fast TSM can be described by the following first-order nonlinear differential equations (Yu et al., 2005)

$$\begin{aligned} s &= \dot{x} + \beta |x|^{\gamma} sign(x) = 0 \\ s &= \dot{x} + \alpha x + \beta |x|^{\gamma} sign(x) = 0 \end{aligned} \tag{2}$$

where $x \in R, \alpha, \beta > 0, \quad 0 < \gamma < 1$. The equation (1) should be the exact expression of TSM in spite that we have been suggesting only real solution for (1) is considered because this suggestion has been involved in (2).

Definition 3. The so-called non-singular TSM (Feng et al., 2002) can be expressed as

$$s = x + \beta |\dot{x}|^{\gamma} sign(\dot{x}) = 0, \quad \beta > 0, 1 < \gamma < 2 \tag{3}$$

which is equivalent to the TSM (2) as

$$\dot{x} + \beta'|x|^{\gamma'} sign(x) = 0, \quad \beta' = \beta^{-\frac{1}{\gamma}} > 0, \frac{1}{2} < \gamma' = \frac{1}{\gamma} < 1 \tag{4}$$

Remark 1. According to the definition of finite-time stability, the equilibrium point $x = 0$ of the continuous non-Lipschitz differential equation (1) is globally finite-time stable, i.e., for any given initial condition $x(0) = x_0$, the system state converges to $x = 0$ in finite time

$$T(x_0) = \frac{1}{\beta(1-\gamma)}|x_0|^{1-\gamma}, \quad T(x_0) = \frac{1}{\alpha(1-\gamma)}\ln\frac{\alpha|x_0|^{1-\gamma}+\beta}{\beta} \tag{5}$$

respectively and stay there forever, i.e., $x = 0$ for all $t > T(x_0)$.

Definition 4. The recursive structure based on the TSM and FTSM concept (Yu & Man, 2002) for higher order systems is expressed as

$$\begin{aligned} s_1 &= \dot{s}_0 + \beta_0 s_0^{q_0/p_0} \\ s_2 &= \dot{s}_1 + \beta_1 s_1^{q_1/p_1} \\ &\vdots \\ s_{n-1} &= \dot{s}_{n-2} + \beta_{n-2} s_{n-2}^{q_{n-2}/p_{n-2}} \end{aligned} \tag{6}$$

$$\begin{aligned} s_1 &= \dot{s}_0 + \alpha_0 s_0 + \beta_0 s_0^{q_0/p_0} \\ s_2 &= \dot{s}_1 + \alpha_1 s_1 + \beta_1 s_1^{q_1/p_1} \\ &\vdots \\ s_{n-1} &= \dot{s}_{n-2} + \alpha_{n-2} s_{n-2} + \beta_{n-2} s_{n-2}^{q_{n-2}/p_{n-2}} \end{aligned} \tag{7}$$

Here α_i, $\beta_i > 0$ and q_i, p_i are positive odd integers $(i = 1,2,...,n-2)$. One can easily see that if s_{n-1} reaches zero, $s_{n-2}, s_{n-3},..., s_0$ will reach zero subsequently according to the dynamical structure of the TSM and FTSM (1).

Remark 2. In the TSM controller design with TSM (6) or FTSM (7), we usually do derivative of s_{n-1}, then the negative fractional power will appear in the control law because of $0 < q_i / p_i < 1$, and then should do derivative of s_{n-2}, and so on till s_1, which leads the negative fractional power always appear in every step. Although the conditions are given to avoid the singularity, it only seems reasonable in mathematical point of view, because the exact mathematical relations are impossible to be satisfied exactly in practice and computer simulation, then the singularity with TSM controller must be considered.

3. NEW FORMS OF TSM AND FTSM

So far, there is no recursive form of the old version of TSM (3) reported. In this section, we will design a new recursive expression of TSM and FTSM. The finite-time convergence is retained and the usual singularity problem relating to conventional TSM is circumvented.

Consider the single-input nonlinear system represented by the model

$$\begin{aligned} \dot{x}_{1,} &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(x_1, x_2, \cdots, x_n) + g(x_1, x_2, \cdots, x_n)u \\ y &= x_1 \end{aligned} \tag{8}$$

Or equivalently

$$\begin{aligned} \dot{x}_i &= x_{i+1} \qquad i = 1,2,\cdots,n-1 \\ \dot{x}_n &= f(\boldsymbol{x}) + g(\boldsymbol{x})u \end{aligned} \tag{9}$$

where $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T = [x, \dot{x}, \cdots, x^{(n-1)}]^T \in R^n$ is the state vector, the scalars u and y the control input and system output, respectively, and $f, g : R^n \to R$ are nonlinear system functions.

3.1 New Form of TSM

Here we propose a new recursive form of TSM (a >1) as

$$\begin{aligned} s_0 &= x_1 \\ s_1 &= s_0 + \beta_0 \frac{a}{na-1}|\dot{s}_0|^{n-\frac{1}{a}} sign(\dot{s}_0) \\ s_2 &= s_1 + \beta_1 \frac{a}{(n-1)a-1}|\dot{s}_1|^{n-1-\frac{1}{a}} sign(\dot{s}_1) \\ &\vdots \\ s_{n-1} &= s_{n-2} + \beta_{n-2} \frac{a}{2a-1}|\dot{s}_{n-2}|^{2-\frac{1}{a}} sign(\dot{s}_{n-2}) \end{aligned} \tag{10}$$

Remark 3. According to the same derivative process with conventional TSM, after the first derivative of s_{n-1}, we can have

$$\dot{s}_{n-1} = \dot{s}_{n-2} + \beta_{n-2}|\dot{s}_{n-2}|^{1-\frac{1}{a}} \ddot{s}_{n-2} \tag{11}$$

In s_{n-1}, because of the exponential index is $1 < 2 - 1/a < 2$, after the derivative, it becomes $0 < 1 - 1/a < 1$ in $\dot{s}_{n-2}$, and another item is $\ddot{s}_{n-2}$. As s_{n-2} is expressed as

$$s_{n-2} = s_{n-3} + \beta_{n-3}\frac{a}{3a-1}\left|\dot{s}_{n-3}\right|^{3-\frac{1}{a}} sign(\dot{s}_{n-3}) \tag{12}$$

After the twice derivatives of s_{n-2}, we can have

$$\dot{s}_{n-2} = \dot{s}_{n-3} + \beta_{n-3}\left|\dot{s}_{n-3}\right|^{2-\frac{1}{a}}\ddot{s}_{n-3} \tag{13}$$

$$\begin{aligned}\ddot{s}_{n-2} &= \ddot{s}_{n-3} + \beta_{n-3}\frac{2a-1}{a}\left|\dot{s}_{n-3}\right|^{1-\frac{1}{a}}(\ddot{s}_{n-3})^2 \\ &\quad + \beta_{n-3}\left|\dot{s}_{n-3}\right|^{2-\frac{1}{a}}\dddot{s}_{n-3}\end{aligned} \tag{14}$$

Then the exponential index $3-1/a$ in s_{n-2} becomes $0<1-1/a<1$, and another higher derivative item $\dddot{s}_{n-3}$ appears. With the same procedure, the exponential index $4-1/a$ in s_{n-3} also becomes $0<1-1/a<1$ in $\dddot{s}_{n-3}$, and so on, until s_1. Because no negative fractional power appears in every procedure, so the singularity in TSM control is not an issue with the new TSM formulation.

Proposition 1. The first derivative of s_{n-1} in (10) is a kind of nonlinear function with the elements $\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(n)}$, i.e., $\dot{s}_{n-1} = f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(n)})$.

Proof. From (10), the first derivative of $s_{n-1}, \cdots, s_1$ can be expressed as

$$\begin{aligned}\dot{s}_{n-1} &= \dot{s}_{n-2} + \beta_{n-2}\left|\dot{s}_{n-2}\right|^{1-\frac{1}{a}}\ddot{s}_{n-2} \\ \dot{s}_{n-2} &= \dot{s}_{n-3} + \beta_{n-3}\left|\dot{s}_{n-3}\right|^{1-\frac{1}{a}}\ddot{s}_{n-3} \\ &\vdots \\ \dot{s}_1 &= \dot{s}_0 + \beta_0\left|\dot{s}_0\right|^{1-\frac{1}{a}}\ddot{s}_0\end{aligned} \tag{15}$$

Therefore, from (15) we can further have

$$\dot{s}_{n-1} = \dot{s}_0 + \sum_{k=0}^{n-2}\beta_k\left|\dot{s}_k\right|^{1-\frac{1}{a}}\ddot{s}_k \tag{16}$$

We will use the principle of induction to prove it as follows:

For s_1 in (13), i.e., $n=2$, it is obvious that

$$\dot{s}_1 = f(\dot{s}_0, \ddot{s}_0) \tag{17}$$

For s_2 in (13), i.e., $n=3$, we have

$$\begin{aligned}\dot{s}_2 &= \dot{s}_1 + \beta_1\left|\dot{s}_1\right|^{1-\frac{1}{a}}\ddot{s}_1 \\ &= \dot{s}_0 + \beta_0\left|\dot{s}_0\right|^{2-\frac{1}{a}}\ddot{s}_0 + \beta_1\left|\dot{s}_1\right|^{1-\frac{1}{a}}\ddot{s}_1 \\ &= f(\dot{s}_0, \ddot{s}_0, \dddot{s}_0)\end{aligned} \tag{18}$$

Now we assume that for $n=k$,

$$\begin{aligned}\dot{s}_{k-1} &= \dot{s}_0 + \sum_{j=0}^{k-2}\beta_j\left|\dot{s}_j\right|^{1-\frac{1}{a}}\ddot{s}_j \\ &= f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(k)})\end{aligned} \tag{19}$$

Then for $n=k+1$, we can further produce the following equation:

$$\begin{aligned}\dot{s}_k &= \dot{s}_0 + \sum_{j=0}^{k-1}\beta_j\left|\dot{s}_j\right|^{1-\frac{1}{a}}\ddot{s}_j \\ &= f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(k)}) + \beta_{k-1}\left|\dot{s}_{k-1}\right|^{1-\frac{1}{a}}\ddot{s}_{k-1} \\ &= f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(k+1)})\end{aligned} \tag{20}$$

With the principle of induction, we can directly get the following result:

For any *n*,

$$\dot{s}_{n-1} = f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(n)}) \tag{21}$$

is always satisfied.

3.2 The Mechanism of Finite-time Convergence of TSM

We can design terminal sliding mode control law to make $\dot{s}_{n-1} = -Ksign(s_{n-1})$, then the sliding surface $s_{n-1}=0$ can be reached in finite time. After $s_{n-1}=0$, from (10), we have

$$s_{n-2} + \frac{\beta_{n-2}a}{2a-1}\left|\dot{s}_{n-2}\right|^{2-\frac{1}{a}} sign(\dot{s}_{n-2}) = 0 \tag{22}$$

This means that

$$\begin{aligned}\dot{s}_{n-2} &= -\left|\frac{2a-1}{\beta_{n-2}a}s_{n-2}\right|^{\frac{a}{2a-1}} sign\left(\frac{2a-1}{\beta_{n-2}a}s_{n-2}\right) \\ &= -\left(\frac{2a-1}{\beta_{n-2}a}\right)^{\frac{a}{2a-1}}\left|s_{n-2}\right|^{\frac{a}{2a-1}} sign(s_{n-2})\end{aligned} \tag{23}$$

where we should note that $0 < a/(2a-1) < 1$. According to the TSM (2), TSM $s_{n-2}=0$ will be reached in finite time. Furthermore, $s_{n-2}=0$ means that

$$s_{n-3} + \frac{\beta_{n-3}a}{3a-1}\left|\dot{s}_{n-3}\right|^{3-\frac{1}{a}} sign(\dot{s}_{n-3}) = 0 \tag{24}$$

With the same procedure as (22) and (23), we can know that $s_{n-3}=0$ will also be reached in finite time. Recursively we can get the conclusion that $s_0 = x_1 = 0$ will be finally reached in finite time. Because no negative fractional power appears in the whole procedure, therefore this new TSM does not incur the singularity problem while maintaining the major advantages of the conventional TSM control.

3.3 New Form of FTSM

In the similar way, we can design the new form of FTSM as

$$
\begin{aligned}
s_0 &= x_1 \\
s_1 &= s_0 + \beta_0 \frac{a}{na-1} |\dot{s}_0 + \alpha_0 s_0|^{n-\frac{1}{a}} sign(\dot{s}_0 + \alpha_0 s_0) \\
s_2 &= s_1 + \beta_1 \frac{a}{(n-1)a-1} |\dot{s}_1 + \alpha_1 s_1|^{n-1-\frac{1}{a}} \\
&\quad sign(\dot{s}_1 + \alpha_1 s_1) \\
&\vdots \\
s_{n-1} &= s_{n-2} + \beta_{n-2} \frac{a}{2a-1} |\dot{s}_{n-2} + \alpha_{n-2} s_{n-2}|^{2-\frac{1}{a}} \\
&\quad sign(\dot{s}_{n-2} + \alpha_{n-2} s_{n-2})
\end{aligned}
\tag{25}
$$

Remark 4.The first derivative of s_{n-1} in (25) is

$$
\begin{aligned}
\dot{s}_{n-1} &= \dot{s}_{n-2} + \\
&\beta_{n-2} |\dot{s}_{n-2} + \alpha_{n-2} s_{n-2}|^{1-\frac{1}{a}} (\ddot{s}_{n-2} + \alpha_{n-2} \dot{s}_{n-2})
\end{aligned}
\tag{26}
$$

As s_{n-2} in (25) is expressed as

$$
\begin{aligned}
s_{n-2} &= s_{n-3} + \beta_{n-3} \frac{a}{3a-1} |\dot{s}_{n-3} + \alpha_{n-3} s_{n-3}|^{3-\frac{1}{a}} \\
&\quad sign(\dot{s}_{n-3} + \alpha_{n-3} s_{n-3})
\end{aligned}
\tag{27}
$$

Then the first and second derivatives of s_{n-2} are

$$
\begin{aligned}
\dot{s}_{n-2} &= \dot{s}_{n-3} + \\
&\beta_{n-3} |\dot{s}_{n-3} + \alpha_{n-3} s_{n-3}|^{2-\frac{1}{a}} (\ddot{s}_{n-3} + \alpha_{n-3} \dot{s}_{n-3})
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\ddot{s}_{n-2} &= \ddot{s}_{n-3} + \\
&\beta_{n-3} \frac{2a-1}{a} |\dot{s}_{n-3} + \alpha_{n-3} s_{n-3}|^{1-\frac{1}{a}} \\
&(\ddot{s}_{n-3} + \alpha_{n-3} \dot{s}_{n-3})^2 \\
&+ \beta_{n-3} |\dot{s}_{n-3} + \alpha_{n-3} s_{n-3}|^{2-\frac{1}{a}} (\dddot{s}_{n-3} + \alpha_{n-3} \ddot{s}_{n-3})
\end{aligned}
\tag{29}
$$

Then the higher derivative item $\dddot{s}_{n-3}$ appears. With the same trivial procedure as Remark 3, we also can find no negative fractional power appears in every procedure, so there is no the singularity problem in the new FTSM control formulation at all.

Similarly we can propose a proposition 2 as follows:

Proposition 2. The first derivative of s_{n-1} in (25) is a kind of nonlinear function with the elements $\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(n)}$, i.e., $\dot{s}_{n-1} = f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(n)})$.

Proof. From (25), the first derivative of $s_{n-1}, \cdots, s_1$ can be expressed as

$$
\begin{aligned}
\dot{s}_{n-1} &= \dot{s}_{n-2} + \beta_{n-2} |\dot{s}_{n-2} + \alpha_{n-2} s_{n-2}|^{1-\frac{1}{a}} \\
&\quad (\ddot{s}_{n-2} + \alpha_{n-2} \dot{s}_{n-2}) \\
\dot{s}_{n-2} &= \dot{s}_{n-3} + \beta_{n-3} |\dot{s}_{n-3} + \alpha_{n-3} s_{n-3}|^{1-\frac{1}{a}} \\
&\quad (\ddot{s}_{n-3} + \alpha_{n-3} \dot{s}_{n-3}) \\
&\vdots \\
\dot{s}_1 &= \dot{s}_0 + \beta_0 \| \dot{s}_0 + \alpha_0 s_0 \|^{1-\frac{1}{a}} (\ddot{s}_0 + \alpha_0 \dot{s}_0)
\end{aligned}
\tag{30}
$$

Therefore, from (30) we can further have

$$
\dot{s}_{n-1} = \dot{s}_0 + \sum_{k=0}^{n-2} \beta_k |\dot{s}_k + \alpha_k s_k|^{1-\frac{1}{a}} (\ddot{s}_k + \alpha_k \dot{s}_k) \tag{31}
$$

We will also use the principle of induction to prove it as follows:

For s_1 in (25), it is obvious that

$$
\begin{aligned}
\dot{s}_1 &= \dot{s}_0 + \beta_0 \| \dot{s}_0 + \alpha_0 s_0 \|^{1-\frac{1}{a}} (\ddot{s}_0 + \alpha_0 \dot{s}_0) \\
&= f(\dot{s}_0, \ddot{s}_0)
\end{aligned}
\tag{32}
$$

For s_2 in (25), we have

$$
\begin{aligned}
\dot{s}_2 &= \dot{s}_1 + \beta_1 |\dot{s}_1 + \alpha_1 s_1|^{1-\frac{1}{a}} (\ddot{s}_1 + + \alpha_1 \dot{s}_1) \\
&= f(\dot{s}_0, \ddot{s}_0, \dddot{s}_0)
\end{aligned}
\tag{33}
$$

Now we assume that for $n = k$,

$$
\begin{aligned}
\dot{s}_{k-1} &= \dot{s}_0 + \sum_{j=0}^{k-2} \beta_j |\dot{s}_j + \alpha_j s_j|^{1-\frac{1}{a}} (\ddot{s}_j + \alpha_j \dot{s}_j) \\
&= f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(k)})
\end{aligned}
\tag{34}
$$

Then for $n = k+1$, we can further produce the following equation:

$$
\begin{aligned}
\dot{s}_k &= \dot{s}_0 + \sum_{j=0}^{k-1} \beta_j |\dot{s}_j + \alpha_j s_j|^{1-\frac{1}{a}} (\ddot{s}_j + \alpha_j \dot{s}_j) \\
&= f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(k)}) + \\
&\quad \beta_{k-1} |\dot{s}_{k-1} + \alpha_{k-1} s_{k-1}|^{1-\frac{1}{a}} (\ddot{s}_{k-1} + \alpha_{k-1} \dot{s}_{k-1}) \\
&= f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(k+1)})
\end{aligned}
\tag{35}
$$

With the principle of induction, we can directly get the following result:

For any *n*,

$$
\dot{s}_{n-1} = f(\dot{s}_0, \ddot{s}_0, \cdots, s_0^{(n)}) \tag{36}
$$

is always satisfied.

3.4 The Mechanism of Finite-time Convergence of FTSM

In the same way, we can design FTSM control law to make $\dot{s}_{n-1} = -Ksign(s_{n-1})$, then the sliding surface $s_{n-1} = 0$ can be reached in finite time. After $s_{n-1} = 0$, from (25), we have

$$\begin{aligned} & s_{n-2} + \beta_{n-2}\frac{a}{2a-1}\left|\dot{s}_{n-2} + \alpha_{n-2}s_{n-2}\right|^{2-\frac{1}{a}} \\ & sign(\dot{s}_{n-2} + \alpha_{n-2}s_{n-2}) = 0 \end{aligned} \tag{37}$$

After rearrange the equation (37), we can achieve the original FTSM as

$$\dot{s}_{n-2} = -\alpha_{n-2}s_{n-2} - \left|\frac{2a-1}{\beta_{n-2}a}\right|^{\frac{a}{2a-1}}\left|s_{n-2}\right|^{\frac{a}{2a-1}} sign(s_{n-2}) \tag{38}$$

where we should note that $0 < a/(2a-1) < 1$. Therefore, according to the finite-time convergence property of FTSM, FTSM $s_{n-2} = 0$ will be reached in finite time. Furthermore, $s_{n-2} = 0$ means that

$$\begin{aligned} & s_{n-3} + \beta_{n-3}\frac{a}{3a-1}\left|\dot{s}_{n-3} + \alpha_{n-3}s_{n-3}\right|^{3-\frac{1}{a}} \\ & \qquad sign(\dot{s}_{n-3} + \alpha_{n-3}s_{n-3}) = 0 \end{aligned} \tag{39}$$

With the same procedure as (37) and (38), we can know that $s_{n-3} = 0$ will also be reached in finite time. Recursively we can get the conclusion that $s_0 = x_1 = 0$ will be finally reached in finite time. Because no negative fractional power appears in the whole procedure, therefore this new FTSM does not incur the singularity problem while maintaining the major advantages of the conventional FTSM control.

4. CONCLUSIONS

In this note, we present the novel recursive formulations of TSM and FTSM. The finite-time convergence can be retained recursively. Furthermore, in the recursive process, the stubborn negative fractional powers going with conventional TSM and FTSM completely disappear. Therefore the singular problem obsessing TSM control can be removed, at the same time, nothing about the merits of original TSM is damaged.

ACKNOWLEDGEMENTS

This work was supported in part by the National Natural Science Fund of China (NSFC) 60674061 and 60775060, and in part by Natural Science Fund of Liaoning Province of China 20072145.

REFERENCES

Bhat, S. P. and Bernstein, D.S. (1998). Continuous finite-time stabilization of the translational and rotational double integrators. *IEEE Transactions on Automatic Control*, **43(5)**, 678-682.

Feng, Y., Yu, X. and Man, Z. (2002). Non-singular terminal sliding mode control of rigid manipulators. *Automatica*, **38(12)**, 2159-2167.

Haimo, V. (1986). Finite time controllers. *SIAM Journal of Control and Optimization*, **24(4)**, 760-770.

Hong, Y. and Jiang, Z.P. (2006). Finite-Time Stabilization of Nonlinear Systems with Parametric and Dynamic Uncertainties. *IEEE Transactions on Automatic Control*, **51(12)**, 1950-1956.

Hong, Y., Huang, J., and Xu,Y. (2001). On an output finite-time stabilization problem. *IEEE Transactions on Automatic Control*, **46(2)**, 305–309.

Ryan, E. P. (1991). Finite-time stabilization of uncertain nonlinear planar systems. *Dynamics and Control*. **1(1)**, 83-94.

Yu, S., Yu, X. and Man, Z. (2004). A fuzzy neural network approximator with fast terminal sliding mode and its applications. *Fuzzy Sets and Systems*, **148(3)**, 469–486.

Yu, S., Yu, X., Shirinzadeh, B. and Man, Z. (2005). Continuous finite-time control for robotic manipulators with terminal sliding mode, *Automatica*, **41(11)**, 1957-1964.

Yu, X. and Man, Z. (2002). Fast terminal sliding-mode control design for nonlinear dynamical systems. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, **49(2)**, 261-264.