Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Asymptotic Stability of Uncalibrated Eye-in-Hand Visual Servoing: An Affine Invariance Perspective ⋆

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Abstract: In this paper, asymptotic stability of uncalibrated eye-in-hand visual servoing is proved in an affine invariance perspective. After a brief retrospection on the uncalibrated eye-in-hand visual servoing, the affine invariance framework is introduced with discussion in depth. Then the visual servoing algorithm is reconstructed in an affine invariance framework, or more precisely as an affine contravariance algorithm, with its complete asymptotic stability proved, proposed as a convergence theorem. The affine invariance perspective enroots the series algorithm on a more solid and fruitful mathematical background and finally, the paper would discuss several potential research realms of the topic of visual servoing.

Keywords: Intelligent control; Robotics; Jacobian matrices; Servo systems; Affine.

1. INTRODUCTION

For numerous industrial applications of robotics and automation today, robot control with visual clues, or more technically, visual servoing, has demonstrated great power in large volumes of industries and has been regarded as one of the most promising research realms in Artificial Intelligence and Robotics (Hutchinson, 1996). Technically, current visual servoing methods could be categorized mainly into two types, Position Based Visual Servoing (PBVS) and Image Based Visual Servoing (IBVS) respectively (Kragic, 2001). According to (Kragic, 2001, Deng, 2003, Hutchinson, 1996), most of the visual servoing systems reported in previous literature, have to utilize the robot and especially the camera model, i.e. camera calibration is needed, if satisfying results and performance required (Deng, 2003). It's well known that the typical camera calibration process is in most cases elaborate, time-consuming yet not robust enough to system and environmental noises. In another word, a high accuracy camera calibration is obtained usually in constructed or at least semi-constructed environment.

Therefore, the uncalibrated visual servoing without the priori knowledge of robot and especially camera model has been paid fairly extensive attentions in recent years (Jang, 1991, Malis, 2002). (Hosoda, 1998) has presented uncalibrated visual servoing for static targets using fixed cameras. (Tanaka, 1999) has improved the control scheme to eye-in-hand stereo tracking of moving targets using static reference points to estimate the target motion, through the real-time estimation of Jacobian matrix. (Jagerand, 1997) has introduced a Broyden method in non-linear least square optimization and experiment using a trust region and (Piepmeier, 2004) as a review, has expanded them with convergence analysis. Especially, (Piepmeier, 2003) has proposed in detail, a dynamic visual servoing method to tracking a moving target, i.e. the velocity estimation is also done within the estimation of expanded Jacobian matrix and compared the partitioned Broyden method with the Recursive Gauss-Newton method in depth with a 6DOF robot simulation. (Piepmeier, 2004) has investigated an improved control law for moving target yet unfortunately, experimentally verified by only a 3DOF robot experiment and its convergence analysis seems incomplete which we would discussed later. Theoretically, the best result reported in previous literature is (Piepmeier, 2003) which is based on the non-linear optimization of affine invariance and adaptive algorithms, which has been utilized from (Hosoda, 1994, Hosoda, 1998) to (Piepmeier, 2003, Piepmeier, 2004) without significant change of theoretical basis. However, the Broyden method could be categorized as local Newton method of affine cotravariance.

Currently speaking, the research on uncalibrated eye-in-hand visual servoing faces potentially two major challenges. The first one is that nearly all the related algo-

⋆ This work was supported in part by The National Basic Research Program (973) of China (2002CB312205) and The National Natural Science Foundation of China (60772063).

rithms in visual servoing simulation, including those of fixed or moving camera, original or improved, seems to be derived mainly from Broyden method or technically in terms of numerical analysis, Gauss-Newton or Quasi-Newton method of affine contravarience, and result in highly demand of introducing other fruitful mathematical models to extend the research realms. The second is the lack of implementation on real world industrial applications, most of them are simulation using robotics toolbox of (Corke, 1996).

Admittedly, the second requires more robust and flexible visual seroving algorithms yet the first one seems more challenging and the introduction of affine invariance framework is believed one of the very solutions.

In vast areas of engineering and applied sciences, Newton based method is regarded as one of the most widely used series. Theoretically, affine invariance framework could be regarded as a large series of Newton based methods connected by a common ground: affine invariance, which could be divided into four major categories(Deuflhard, 2004): affine covariance, affine contravariance, affine conjugacy and affine similarity which would be discussed in detail later in this paper. The elegant affine invariance framework consists of large mount of numerical analysis methodologies and algorithms derived from. Most importantly, theoretical analysis of affine invariance would benefit a lot in guiding the construction of corresponding algorithms.

This paper contributes the first time, in a complete asymptotic stability prove of the uncalibrated eye-in-hand visual seroving algorithm in an affine invariance perspective. The previous version of that is in (Piepmeier, 2004) and the stability proof in it is believed not complete(Hao, 2007a). More importantly, this paper introduces the first time, a grand affine invariance framework into the algorithm design and analysis of uncalibrated eye-in-hand visual servoing which is believed to be one of fruitful sources of new algorithm with complete theoretical analysis.

This paper is organized as follows. After a brief yet instructive investigation of the research realm in Section 1, the classical uncalibrated eye-in-hand visual seroving algorithm would be constructed in Section 2. Hereafter the affine invariance framework would be introduced briefly in Section 3 and then the complete asymptotic stability analysis of the algorithm is constructed in an affine invariance perspective in Section 4. Finally, conclusion and discussion of some future work are summarized in Section 5.

2. THE UNCALIBRATED EYE-IN-HAND VISUAL SERVOING

The task description of the uncalibrated eye-in-hand visual servoing could be described as follows (Hao, 2007a): given a robot $\mathcal{R}$ and an eye-in-hand camera $\mathcal{C}$ without calibration, the objective is to move the robot from the eye-in-hand image feature $y(\theta)$ under current joint value θ, to the desired image feature $y^*(\theta)$. No robot kinetics or dynamics model needed. Nor does the camera model.

The error function is defined as

$$f(\theta,t) = y(\theta) - y^*(t) \tag{1}$$

The squared error function is defined as

$$F(\theta,t) = \frac{1}{2} f^T(\theta,t) f(\theta,t) \tag{2}$$

And the squared error function could be ex-pressed in first order Taylor form as

$$\begin{aligned} &F(\theta+\Delta\theta, t+\Delta t) = \\ &\qquad F(\theta,t) + \frac{\partial F}{\partial \theta}\Delta\theta + \frac{\partial F}{\partial t}\Delta t + O(\Delta\theta, \Delta t) \end{aligned} \tag{3}$$

considering that the sampling period is assumed to be a constant, the minimum of $F(\theta,t)$ could be calculated as

$$\frac{\partial F(\theta+\Delta\theta, t+\Delta t)}{\partial \theta} = 0 \tag{4}$$

Omit the high order infinite population $O(\Delta\theta, \Delta t)$, approximately we have

$$\frac{\partial F}{\partial \theta} + \frac{\partial^2 F}{\partial \theta^2}\Delta\theta + \frac{\partial^2 F}{\partial \theta \partial t}\Delta t = 0 \tag{5}$$

then

$$\Delta\theta = -\left(\frac{\partial^2 F}{\partial \theta^2}\right)^{-1}\left(\frac{\partial F}{\partial \theta} + \frac{\partial^2 F}{\partial \theta \partial t}\Delta t\right) \tag{6}$$

Substituting (6) with (2) and (3) would result in

$$\Delta\theta = -\left(J^T J + S\right)^{-1} J^T \left(f + \frac{\partial f}{\partial t}\Delta t\right) \tag{7}$$

where

$$\begin{aligned} &J = \frac{\partial f}{\partial \theta}, S = \frac{\partial J^T}{\partial \theta} f, \frac{\partial F}{\partial \theta} = J^T f, \\ &\frac{\partial F^2}{\partial \theta \partial t} = J^T \frac{\partial f}{\partial t}, \frac{\partial^2 F}{\partial \theta^2} = J^T J + S \end{aligned} \tag{8}$$

Note the definition $J = \partial f / \partial \theta$ is known as composite Jacobian matrix. Moreover, since S is system dependent and actually difficult to estimate, yet it could be regarded as an infinite population, because when the robot position is near the desired, the error function $f(\theta,t)$ could be regarded as zero. Therefore, we could rewrite (7) as

$$\Delta\theta = -\left(\widetilde{J}_k^T \widetilde{J}_k\right)^{-1} \widetilde{J}_k^T \left(f + \frac{\partial f}{\partial t}\Delta t\right) \tag{9}$$

where $\widetilde{J}_k$ stands for the estimation of the composite Jacobian matrix at the Kth iteration. And (8) is the joint value update formula.

Besides, the affine model of error function is defined as

$$m_k(\theta,t) = f(\theta_k,t_k) + \widetilde{J}_k(\theta-\theta_k) + \frac{\partial f_k}{\partial t}(t-t_k) \tag{10}$$

which could be regarded as also the first order expansion of $m_k(\theta,t)$ at (θ_k,t_k). And the corresponding target function is

$$\begin{aligned} \min \mathbf{E}(k) &= \sum_{i=0}^{k-1} \lambda^{k-i-1} \|\Delta m_{ki}\|^2 \\ \Delta m_{ki} &= m_k(\theta_i,t_i) - m_i(\theta_i,t_i) \\ &= [f_k - f_i - \frac{\partial f_k}{\partial t}(t_k - t_i)] - J_k h_{ki} \\ &= [f_k - f_i] - J_k h_{ki} \end{aligned} \tag{11}$$

where we denote

$$\begin{aligned} h_{ki} &= (\theta_k - \theta_i) \\ h_k &= t_{k+1} - t_k \end{aligned} \tag{12}$$

Now if corresponding substitutions made in the Recursive Least Squares algorithm, as stated in (Macchi, 1995), the Visual Servoing using Recursive Least Squares (VS-RLS) algorithm could be expressed as (Piepmeier, 2003)(Hao, 2007a)

Initialization:

Given $f : \mathbb{R}^n \to \mathbb{R}^m; \theta_0, \theta_1 \in \mathbb{R}^{m\times n}$; $(\widetilde{f}_t)_0 \in \mathbb{R}^{m\times 1}; P_0 \in \mathbb{R}^{n+1\times n+1}; \lambda \in (0, 1]$;

Iteration body:

$$\begin{aligned} \Delta f &= f_k - f_{k-1} \\ h_\theta &= \theta_k - \theta_{k-1} \\ h_t &= t_k - t_{k-1} \\ h &= \begin{bmatrix} \theta_k - \theta_{k-1} \\ t_k - t_{k-1} \end{bmatrix} \\ J_k &= [\widetilde{J}_k \ (\widetilde{f}_t)_{k-1}] \\ \Delta J_k &= (\Delta f - J_{k-1}h)(\lambda + h^T P_{k-1} h)^{-1} h^T P_{k-1} \\ J_k &= J_{k-1} + \Delta J_k \\ P_k &= \frac{1}{\lambda}\left[P_{k-1} - \frac{P_{k-1}hh^T P_{k-1}}{\lambda + h^T P_{k-1} h}\right] \\ \theta_{k+1} &= \theta_k - (\widetilde{J}_k^T \widetilde{J}_k)^{-1} (\widetilde{J}_k^T f_k + \widetilde{J}_k^T (\widetilde{f}_t)_k h) \end{aligned} \tag{13}$$

3. THE AFFINE INVARIANCE FRAMEWORK

Let's start with classical and well-known Newton method

$$F'(x^k)\Delta x^k = -F(x^k), \ x^{k+1} = x^k + \Delta x^k, \ k = 0, 1, ... \tag{14}$$

and classical Lipschitz condition

$$\|F'(x) - F'(\overline{x})\|_{X\to Y} \le \gamma \|x - \overline{x}\|_X \tag{15}$$

when nonsigular diagonal scaling matrix D_L and D_R selected for left and right scaling, i.e.

$$(D_L F'(x^k) D_R)(D_R^{-1} \Delta x^k) = -D_L F(x^k) \tag{16}$$

Now the affine transformation could be defined as

$$G(y) = AF(By) = 0, \ x = By \tag{17}$$

for any $A, B \in \mathbb{R}^{n\times n}$. Therefore the transformed Newton methods could be defined as

$$G'(y^k)\Delta y^k = -G(x^k), \ y^{k+1} = y^k + \Delta y^k, \ k = 0, 1, ... \tag{18}$$

Theoretically, affine invariance framework consists of four modules: affine covariance, affine contravariance, affine conjugacy and affine similarity respectively. And more importantly, as stated in (Deuflhard, 2004), different affine invariant Lipschitz conditions, lead to different characterizations of local convergence demains in terms of error oriented norms, residual norms, or energy norms, which in turn, give rise to corresponding variants of Newton algorithms.

Now we can specify the four categories in detail. If $B = I$ in (17), it could be defined as affine covariance

$$G(x) = AF(x) = 0 \tag{19}$$

with corresponding Lipschitz condition

$$\|G'(x) - G'(\overline{x})\| \le \gamma(A)\|x - \overline{x}\| \tag{20}$$

If image space F fixed, i.e. $A = I$, the affine contravariance is defined as

$$G(y) = F(by) = 0, \ x = By \tag{21}$$

with corresponding Lipschitz condition

$$\|G'(x) - G'(\overline{x})\| \le \gamma(A)\|x - \overline{x}\| \tag{22}$$

For a optimization problem $f(x) = \min, \ f : D \subset \mathbb{R}^n \to \mathbb{R}$ where f is convex in neighborhood D, it equals to

$$F(x) = \text{grad} f(x) = f'(x)^T = 0, \ x \in D \tag{23}$$

Then affine conjugacy is defined as

$$g(y) = f(By) = \min, \ x = By \tag{24}$$

with corresponding Lipschitz condition

$$\|F'(x)^{-1/2}(F'(\overline{x}) - F'(x))\| \le \omega \|F'(x)^{1/2}(\overline{x} - x)\|^2 \tag{25}$$

In an equilibrium point of a dynamical system $\dot{x} = F(x)$ with an arbitrary transformation $A\dot{x} = AF(x) = 0$, affine similarity is defined as

$$G(y) = AF(A^{-1}y) = 0, \ y = Ax \tag{26}$$

with corresponding Lipschitz condition

$$\|(F'(\overline{x}) - F'(x))u\| \le \omega \|\overline{x} - x\| \|u\| \tag{27}$$

Note that since the underlying algorithm of uncalibrated eye-in-hand visual servoing is categorized as affine contravariance, then it would be discussed in detail.

4. ASYMPTOTIC STABILITY OF UNCALIBRATED EYE-IN-HAND VISUAL SERVOING: AN AFFINE INVARIANCE PERSPECTIVE

Historically, the first classical convergence theorems for Newton series methods are Newton-Kantorovich theorem and Newton-Mysovskikh theorem respectively. Newton-Kantorovich theorem introduces Kantorovich quantity $h_0 = \|\Delta x_0\|_X \beta_0 \gamma < \frac{1}{2}$ and a convergence ball round x_0 with radius $\rho_0 \sim 1/\beta_0\gamma$. Similarly, Newton-Mysovskikh theorem introduces Mysovskikh quantity, slightly different from the previous one, $h_0 = \|\Delta x_0\|_X \beta\gamma < 2$ and a convergence ball round x_0 with radius $\rho \sim 1/\beta\gamma$. However, such a quantity is certainly difficult to compute in realistic nonlinear systems, if not hopless (Deuflhard, 2004). Therefore, it's the very introduction of affine invariance framework that helps eliminate the gap between theoretical convergence analysis and realistic algorithm design and implementation.

Firstly, the contraction factor, or convergence monitor would be introduced as

$$\Theta_k = \frac{\|\Delta\theta^{k+1}\|}{\|\Delta\theta^k\|} \tag{28}$$

whenever $\Theta_k \ge 1$ for simplicity, the algorithm monitored is classified as 'not convergent'.

Since the Broyden based method could be classified as Jacobian rank-1 update, then the Jacobian rank-1 update operator is defined as

$$E_k(J) = I - J_k J^{-1} \tag{29}$$

Lemma 1. For $0 < \Theta < 1$, $0 \leq \eta_0 < \Theta$ and

$$b \leq \frac{\Theta - \eta_0}{1 + \eta_0 + \frac{4}{3}(1-\Theta)^{-1}} \tag{30}$$

if

$$\eta = \eta_0 + \frac{t}{(1-b)(1-\Theta)}$$

we could have

$$\eta + (1+\eta)t < \Theta \tag{31}$$

The proof of Lemma 1 could be accessed at (Deuflhard, 2004).

Theorem 1. For $F \in C^1(D)$, $F : D \subset \mathbb{R}^n \to \mathbb{R}^m$, D convex, if θ^* indicates the unique solution of robot joint value with $F'(\theta^*)$ nonsingular. For a specified $\omega < \infty$, $\theta \in D$, the affine contravariance Lipschitz condition

$$\begin{aligned}\|(F'(\theta_k) - F'(\theta^*))(\theta - \theta_k)\| \leq \\ \omega\|F'(\theta^*)(\theta_k - \theta^*)\|\|F'(\theta^*)(\theta - \theta_k)\|\end{aligned} \tag{32}$$

holds. For specified $\Theta \in (0,1)$, if $\overline{\eta}_0 = \|E_0\| < \overline{\Theta}$, and the initial robot joint value condition

$$b = \omega\|F'(\theta^*)(\theta_0 - \theta^*)\| \leq \frac{\overline{\Theta} - \overline{\eta_0}}{1 + \overline{\eta_0} + 4/3(1-\overline{\Theta})^{-1}} \tag{33}$$

holds, the robot joint value series $\{\theta_k\}$ would converge to θ^* in terms of error as

$$\|F_{k+1}\| \leq \overline{\Theta}\|F_k\| \tag{34}$$

or in the terms of image feature vector as

$$\|y_{k+1} - y^*\| \leq \overline{\Theta}\|y_k - y^*\| \tag{35}$$

Proof. The Jacobian rank-1 update of F is

$$\begin{aligned} F_{k+1} &= F_k + \int_{q=0}^{1} F'(\theta_k + q\Delta\theta_k)\Delta\theta_k dq \\ &= \int_{q=0}^{1} (F'(\theta^k + q\Delta\theta_k) - F'(\theta^*))\Delta\theta_k dq \\ &\quad +(F'(\theta^*) - J_k)\Delta\theta_k \end{aligned} \tag{36}$$

Consider the Lipschitz condition (32), (36) yields

$$\begin{aligned} \|F_{k+1}\| &\leq \int_{q=0}^{1} \|(F'(\theta_k + q\Delta\theta_k) - F'(\theta^*))\Delta\theta_k\|dq \\ &\quad +\|(F'(\theta^*)J_k{}^{-1} - I)F_k\| \\ &\leq \int_{q=0}^{1} \omega\|F'(\theta^*)(\theta_k + q\Delta\theta_k - \theta^*)\| \\ &\quad \cdot\|F'(\theta^*)\Delta\theta_k\|dq + \|E_kF_k\| \\ &\leq \int_{q=0}^{1} \omega(\|F'(\theta^*)(1-q)(\theta_k - \theta^*)\| \\ &\quad +\|F'(\theta^*)q(\theta_{k+1} - \theta^*)\|) \\ &\quad \|F'(\theta^*)\Delta\theta_k\|dq + \overline{\eta_k}\|F_k\| \\ &= \frac{1}{2}(b_k + b_{k+1})\|F'(\theta^*)\Delta\theta_k\| + \overline{\eta_k}\|F_k\| \end{aligned} \tag{37}$$

Defining $\overline{b_k} = \frac{1}{2}(b_k + b_{k+1})$, then

$$\begin{aligned} \|F_{k+1} &\leq \overline{b_k}\|(E_k - I)F_k\|\| + \overline{\eta_k}\|F_k\| \\ &\leq (\overline{b_k}(1 + \overline{\eta_k}) + \overline{\eta_k})\|F_k\| \end{aligned} \tag{38}$$

Now we only have to prove

$$\overline{b_k}(1 + \overline{\eta_k}) + \overline{\eta_k} \leq \|\overline{\Theta}\| \tag{39}$$

The update formula for g_k is

$$\begin{aligned} g_{k+1} &= g_k - F'(\theta^*)J_k^{-1}F_k = F'(\theta^*)(\theta_k - \theta^*) \\ &\quad -F_k + E_kF_k \\ &= \int_{s=0}^{1} (F'(\theta^*) - F'(\theta^* + q(\theta_k - \theta^*))) \\ &\quad \cdot(\theta_k - \theta^*)dq + E_kF_k \end{aligned} \tag{40}$$

and

$$\begin{aligned} \|g_{k+1}\| &\leq \int_{s=0}^{1} q\omega\|F'(\theta^*)(\theta_k - \theta^*)\|\|F'(\theta^*) \\ &\quad \cdot(\theta_k - \theta^*)\|dq + \overline{\eta_k}\|F_k\| \\ &\leq \frac{\omega}{2}\|g_k\|^2 + \overline{\eta_k}(\|g_k - F_k\| + \|g_k\|) \end{aligned} \tag{41}$$

Similarly,

$$b_{k+1} \leq \frac{1}{2}b_k^2 + \overline{\eta_k}(\frac{1}{2}b_k^2 + b_k) = \left(\overline{\eta_k} + \frac{1 + \overline{\eta_k}}{2}\right) b_k \tag{42}$$

After that, the approximation properties of the Jacobian updates could be discussed by introducing the orthogonal projection

$$Q_k = \frac{\Delta F_{k+1}\Delta F_{k+1}^T}{\|\Delta F_{k+1}\|^2} \tag{43}$$

onto the secant direction ΔF_{k+1}, and the deterioration matrix could be noted as

$$E_{k+1} = E_kQ_k^{\perp} + E_{k+1}Q_k \tag{44}$$

From the 'good' Broyden update proof (Deuflhard, 2004)

$$\begin{aligned} \overline{\eta}_{k+1} &= \|E_{k+1}\| \leq \|E_kQ_k^{\perp}\| + \|E_{k+1}Q_k\| \\ &\leq \|E_k\| + \frac{\|E_{k+1}\Delta F_{k+1}\|}{\|\Delta F_{k+1}\|} \end{aligned} \tag{45}$$

The second right hand term could be expressed, utilizing the secant condition, as

$$\begin{aligned} E_{k+1}\Delta F_{k+1} &= \Delta F_{k+1} - F'(\theta^*)J_{k+1}^{-1}\Delta F_{k+1} \\ &= \Delta F_{k+1} - F'(\theta^*)\Delta\theta_k \\ &= \int_{q=0}^{1} (F'(\theta_k + q\Delta\theta_k) - F'(\theta^*))\Delta\theta_k dq \end{aligned} \tag{46}$$

Now the norm of (46) could be noted as

$$\begin{aligned} \|E_{k+1}\Delta F_{k+1}\| &\leq \bar{b}_k\|F'(\theta^*)\Delta\theta_k\| \\ &= \bar{b}_k\|E_{k+1}\Delta F_{k+1} - \Delta F_{k+1}\| \\ &\leq \bar{b}_k(\|E_{k+1}\Delta F_{k+1}\| + \|\Delta F_{k+1}\|) \\ &\leq \frac{\bar{b}_k}{1 - \bar{b}_k}\|\Delta F_{k+1}\| \end{aligned} \tag{47}$$

Substitute (45) into (47) yields fairly rough estimation

$$\overline{\eta}_{k+1} \leq \overline{\eta} + \frac{\bar{b}_k}{1 - \bar{b}_k} \tag{48}$$

Since

$$\overline{\eta}_k \leq \overline{\eta}_0 + \frac{\sum_{i=0}^{k-1}\overline{\Theta}_i t_0}{1 - b_0} \leq \overline{\eta} \tag{49}$$

with

$$\overline{\eta} = \overline{\eta}_0 + \frac{b_0}{(1-b_0)(1-\overline{\Theta})} \tag{50}$$

and

$$b_k \leq \overline{\Theta}_k b_0 \tag{51}$$

and combing (51) and Lemma 1, immediately we have

$$b_{k+1} \leq (\overline{\eta} + (1+\overline{\eta})b_0)b_k \leq \overline{\Theta} b_k \leq \overline{\Theta}_{k+1} b_0 \tag{52}$$

and therefore

$$\begin{aligned} \overline{\eta}_{k+1} &\leq \overline{\eta}_k + \frac{b_k}{1-b_0} \leq \overline{\eta}_0 + \frac{\sum_{i=0}^{k-1} \overline{\Theta}_i t_0}{1-b_0} + \frac{\overline{\Theta}_{k+1} b_0}{1-b_0} \\ &\leq \overline{\eta}_0 + \frac{\sum_{i=0}^{k} \overline{\Theta}_i b_0}{1-b_0} \leq \overline{\eta} \end{aligned} \tag{53}$$

When the 'bounded deterioration property'

$$\overline{\eta}_k \leq \overline{\eta} \tag{54}$$

holds, and the error contraction

$$b_{k+1} \leq b_k \tag{55}$$

for any k. Therefore in conclusion, combining (38) and again Lemma 1 would result in

$$\|F_{k+1}\| \leq \overline{\Theta} \|F_k\| \tag{56}$$

i.e.

$$\|y_{k+1} - y^*\| \leq \overline{\Theta} \|y_k - y^*\| \tag{57}$$

5. CONCLUSION

Generally speaking, the major contribution of this paper is to prove the asymptotic stability of the classical uncalibrated eye-in-hand visual seroving algorithm in an affine invariance perspective. The major work refers to the cutting-edge work specified in (Deuflhard, 2004) and is applied to solve completely the asymptotic stability prove in an affine contravariance framework. Yet maybe more importantly, the introduction of affine invariance framework might do far more than that. In such a grand framework with solid theoretical analysis ground, different affine invariance modules are believed to contribute a great deal to the fruitful development of the classical uncalibrated eye-in-hand visual seroving algorithm, not only derived from Broyden method as so far we've done. What's more interesting of the affine invariance framework is that its theoretical analysis could guide indeed the construction and implementation of realistic algorithm, which is also regarded as a promising research realm in the future. For example, affine covariance, or more famously known as 'good' Broyden method, could be considered to construct a new uncalibrated eye-in-hand visual seroving algorithm. So does the affine conjugacy and affine similarity ones.

Finally, introducing the affine invariance framework might extend the current underlying architecture of the classical uncalibrated eye-in-hand visual seroving algorithm dramatically with fairly complete and thorough theoretical analysis at hand.

ACKNOWLEDGEMENTS

The first author of this paper sincerely appreciates to Prof. Lipkin as well as Mr. Basong Chen for their kindness on the academic discussion. He also wants to thank and Ms. Kainan Zhao for her help beneficial to make this paper available.

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