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Influence of actuator size and location on robust stability of actively controlled flexible beams ⋆

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Abstract:
This paper addresses the influence of the actuator size on the closed loop stability of collocated and non-collocated transfer functions utilized in the structural control of flexible beams. Besides the well known robustness advantages of collocated transfer functions it is shown, that if the actuator is small compared to the flexible structure, a non-collocated actuator/sensor configuration provides larger stability margins. It is shown, that this effect arises from the uncertainty associated with the steady state gain of collocated transfer functions. The applicability to other types of boundary conditions and limitations due to length and spatial frequencies of the beam are also addressed.

Keywords: actuator/sensor placement, active damping, flexible beam, robustness.

1. INTRODUCTION

The placement of actuators and sensors along a flexible structure for active vibration damping has a direct effect on the achievable performance. A vast amount of positioning criteria can be found in literature. In many cases optimality is defined as the maximum of either the $\mathcal{H}_2$-, $\mathcal{H}_\infty$- or the Hankel system norm of the transfer function matrix from the actuators to the sensors. Some examples are given in Gawronski [1997], Leleu et al. [2001], Hać [1995], Moheimani and Ryall [1999], and Gawronski [1999]. All of these criteria will lead to almost the same optimal positions for actuators as well as sensors when the devices are of the same type and size (*collocation*).

A collocated actuator/sensor placement leads to minimum phase transfer functions (e.g. Preumont [1997]) which are of advantage in terms of closed loop stability. Furthermore, the minimum phase behavior of collocated transfer functions can be shown to be superior in terms of robustness for any controller design method when compared to non-collocation (MacMartin [1995]).

For any controller design a low order system model is needed. It is a well known fact that the truncation of higher order dynamics leads to a perturbation of the in-bandwidth zeros and the steady state transfer function gain (Clark [1997]). Additionally, for collocation a decreasing actuator size leads to a stronger influence of the higher order modes and therefore a stronger perturbation due to direct truncation (Benatzky and Kozek [2005]). The present contribution also extends and formalizes the results given in Moheimani and Ryall [1999]. Thus, for a small collocated actuator/sensor pair applied to highly flexible structures a higher number of modes is needed to accurately describe transfer function gains in the lower frequency region.

In the following it is shown for the flexible beam that if only a limited number of flexible modes is available for model correction, the non-collocated approach leads to a more robust control system due to a smaller low-frequency uncertainty. This is especially important in the presence of parameter uncertainties.

The remainder of this paper is organized as follows: First the simply supported beam is defined and the necessary transfer functions are given. Then the functionality of a transfer function correction as well as its effect on collocated and non-collocated transfer functions is discussed. In the next section a state vector feedback controller and an observer are designed, both by pole placement. Finally, utilizing the closed loop system and the small gain theorem, the robustness properties of the closed loop system are investigated to demonstrate the effects of non-collocation on robustness.

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2. THE SIMPLY SUPPORTED BEAM

2.1 Model equations

For the homogeneous Euler-Bernoulli beam depicted in Fig.1, E, J, F, ρ, h, b, and l are the modulus of elasticity, area moment of inertia, cross sectional area, density, height, width and length of the beam. Here, a piezoelectric actuator is modeled as a pair of moments acting on the beam as external excitation. Furthermore, x_{a1} and x_{a2} are the start and end position of the actuator. The beam's equation of motion is given by Timoshenko et al. [1974]

$$EJ\frac{\partial^4 w(x,t)}{\partial x^4} + \rho F\frac{\partial^2 w(x,t)}{\partial t^2} = \frac{\partial^2 M(x,t)}{\partial x^2} \qquad (1)$$

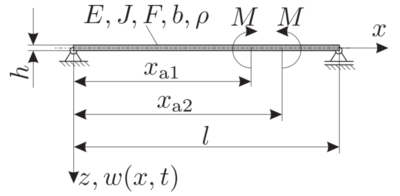

Fig. 1. Simply supported beam with patch actuator

with $M(x)$ being the bending moment of the beam. From the partial differential equation (1) the static curvature distribution is found by setting $\ddot{w}(x,t) = 0$ to be $w''''(x) = M_a''(x)/EJ$ or

$$w''(x) = M(x)/EJ = (M_a(x) + ax + b)/EJ. \qquad (2)$$

$M_a(x)$ is the actuator moment

$$M_a(x) = M[\sigma(x - x_{a1}) - \sigma(x - x_{a2})] \qquad (3)$$

and the integration constants a and b follow from the boundary conditions of the beam.

2.2 Determined or under-determined beam

In this case the solution of (2) using a patch actuator always yields a bending moment $M(x) = M_a(x)$ since the actuator moment $M_a(x)$ and both constants a and b from (2) become zero. Therefore, the bending moment $M(x)$ is proportional to the curvature $w''(x)$. This is shown for the simply supported (hinged–hinged) beam in Fig.2 with a resulting deflection $w(x)$ of

$$w(x) = \frac{M}{2EJ}\left[x(1/l(x_{a2}^2 - x_{a1}^2) + 2l_{\text{p}}) + \sigma(x - x_{a1})(x - x_{a1})^2 - \sigma(x - x_{a2})(x - x_{a2})^2\right], \qquad (4)$$

where $\sigma(\cdot)$ is the unit step-function and $l_{\text{p}} = x_{a2} - x_{a1}$ is the actuator length. For the free–floating, hinged–free, and clamped–free beam the bending moment will be identical to (3), and hence the actuator action does not extend along the beam.

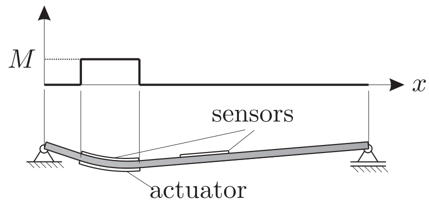

Fig. 2. Bending moment and deflection of the simply supported beam

2.3 Over-determined beam

For the over-determined beam the bending moment $M(x)$ strongly depends on the boundary conditions and the constants a and b are non-zero. For the clamped–clamped beam the integration constants in (2) are

$$a = 6Ml_{\text{p}}/l^2[1 - l_{\text{p}}/l - 2x_{a1}/l]$$
$$b = -4Ml_{\text{p}}/l[1 - 0.75l_{\text{p}}/l - 1.5x_{a1}/l], \qquad (5)$$

and for the clamped–hinged beam the result is

$$a = 3Ml_{\text{p}}/l^2[1 - 0.5l_{\text{p}}/l - x_{a1}/l]$$
$$b = -3Ml_{\text{p}}/l[1 - 0.5l_{\text{p}}/l - x_{a1}/l]. \qquad (6)$$

2.4 Modal representation and transfer function

The beam's deflection $w(x,t)$ can also be defined as

$$w(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t), \qquad (7)$$

where $\phi_i(x)$ is the i-th undamped natural mode shape and $q_i(t)$ is the according modal coordinate. Inserting (7) into (1), integrating over the beams length and utilizing the orthogonality properties of the mode shapes, leads to an infinite number of decoupled ordinary differential equations for the modal coordinates $q_i(t)$

$$\ddot{q}_i + 2\dot{q}_i\zeta_i\omega_i + \omega_i^2 q_i = \frac{M(t)}{\rho F}\psi_{i,u}. \qquad (8)$$

In (8) the modal damping coefficient ζ_i, the undamped natural frequency of the i-th mode ω_i, and the actuator influence coefficient $\psi_{i,u}$ are introduced, where the latter is given by:

$$\psi_{i,u} = [\phi_i'(x_{\text{a2}}) - \phi_i'(x_{\text{a1}})]. \qquad (9)$$

For a piezoelectric patch $M(t) = \bar{K}V_a(t)$ in (8) holds, where $V_a(t)$ is the actuator voltage and $\bar{K}$ is a constant factor from data of beam and patch.

The measurement $y(t)$ is defined to be proportional to the integral over the beam's curvature between x_{s1} and x_{s2} (start and end positions of a piezoelectric sensor patch)

$$y(t) = \Upsilon \int_{x_{\text{s1}}}^{x_{\text{s2}}} w''(x,t)dx = \Upsilon \sum_{i=1}^{\infty} \int_{x_{\text{s1}}}^{x_{\text{s2}}} \phi_i''(x) q_i(t) dx = \Upsilon \sum_{i=1}^{\infty} \psi_{i,y} q_i(t), \qquad (10)$$

where, similar to (9),

$$\psi_{i,y} = [\phi_i'(x_{\text{s2}}) - \phi_i'(x_{\text{s1}})] \qquad (11)$$

and Υ is a constant factor. The factors $\bar{K}$ and Υ are given in Halim and Moheimani [2001].

The transfer function from the moment pair $M(t)$ to the measurement $y(t)$ is obtained by Laplace-transforming and combining (8) and (10):

$$\frac{Y(s)}{M(s)} = \frac{\Upsilon}{\rho F} \sum_{i=1}^{\infty} \frac{\psi_{i,y}\psi_{i,u}}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}. \qquad (12)$$

In (12) the influence coefficients $\psi_{i,y}$ and $\psi_{i,u}$ depend on the boundary conditions of the beam.

3. CORRECTION OF REDUCED TRANSFER FUNCTIONS

The modal description (12) consists of an infinite number of modes

$$G(s) = \sum_{i=1}^{\infty} \frac{\alpha_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}. \qquad (13)$$

In (13) α_i is the influence coefficient of the i-th mode that describes the contribution of this mode to the overall solution. For a reduced order model the series expansion (13) is restricted to the first N modes

$$G_{\text{red}}(s) = \sum_{i=1}^{N} \frac{\alpha_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}. \qquad (14)$$

Due to direct truncation the in-bandwidth poles of (14) remain the same as in (13) while the in-bandwidth zeros are perturbed. Therefore, different correction methods have been proposed to correct the in-bandwidth zeros. The addition of a feed-through term K_c was first proposed in Bisplinghoff and Ashley [1962] and then applied to flexible structures by Clark [1997]. There the corrected model is given by

$$G_{\text{corr}}(s) = \sum_{i=1}^{N} \frac{\alpha_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} + K_c, \tag{15}$$

where K_c is

$$K_c = \sum_{i=N+1}^{\infty} \frac{\alpha_i}{\omega_i^2} \approx \sum_{i=N+1}^{L} \frac{\alpha_i}{\omega_i^2}, \quad L \gg N. \tag{16}$$

Utilizing the factor K_c, the corrected model G_{corr} will have a small steady-state gain error. According to Fig.2, the steady-state gain of the non-collocated sensor is zero in the case of an determined or under-determined beam, and for the over-determined beam it will be very small (see section 4.1).The correction term K_c for the *non-collocated* transfer function is given by

$$K_c = -G_{\text{red}}|_{s=0}. \tag{17}$$

This K_c is the exact solution for any order N of the reduced model leading to zero steady-state error.

In the case of collocated actuator and sensor the measurement signal according to (10)is given by $y(x,t){=}M\Upsilon l_p/EJ$. Therefore, the correction factor for *collocated* transfer functions of statically determined, under-determined, and approximately also of over-determined homogenous beams is defined as

$$K_c = \frac{\Upsilon l_p}{EJ} - G_{\text{red}}|_{s=0}. \tag{18}$$

A correction factor for collocated transfer functions according to (18) has to be calculated from the infinite series (16) by an approximation of order L. It is important to note that for collocated transfer functions all the influence factors α_i in (16) have the same sign (Martin [1978]), and the correction factor K_c has to include a high number of out-of-bandwidth modes (Clark [1997]).

Other methods for the calculation of K_c are given in Halim and Moheimani [2002] , Moheimani and Halim [2004] , and Benatzky and Kozek [2005]. Halim and Moheimani [2002] propose a feed-through term to compensate for out-of-bandwidth modes with respect to weighted point-wise and spatial $\mathcal{H}_2$-norm error minimization (using analytic results for the damped case). A similar approach is presented by Moheimani and Halim [2004], where an LMI optimization problem is formulated and the corresponding $\mathcal{H}_2$- and H-inf-error norms are minimized. Finally, in Benatzky and Kozek [2005] the use of additional frequency response modes is proposed in order to efficiently model the local actuator action. All of these methods are considerably more complex than the corrections (17) and (18), and no results on robustness with respect to model uncertainties are included.

4. ACTUATOR SIZE AND LOCAL ACTION

The following definitions and formulations are derived for the static solutions of the flexible beam according to section 2. This is not a strong restriction since the spatial frequencies of the beam will be explicitly incorporated into the considerations.

4.1 Definition of local actuator action

The actuator action can be approximately regarded as local if the curvature along the beam is smaller than a predefined scalar factor μ of the curvature at the actuator location. The necessary condition can be formulated in terms of the bending moment as

Definition 1. A patch actuator placed at $x_{a1} \le x \le x_{a2}$ on a beam of length l is said to act locally if

$$|M(x)| \le \mu \min |M(\xi)|\,|_{x_{a1}\le\xi\le x_{a2}}\ , \ \ \forall x\epsilon[0;x_{a1}) \cup (x_{a2};l]$$

with $0 \le \mu < 1$ holds.

Under the assumption of static deformations of the determined or under-determined beam μ=0 holds. A suitable choice of μ for the over-determined beam is motivated by (5) and (6), which indicate that the extremal moment always occurs at either $x = 0$ or $x = l$. For the clamped–clamped beam the absolute value of the maximal bending moment M_{max} is in both cases

$$M_{max}(l_p, x_{a1}) = 4Ml_p/l[1 - 0.75l_p/l - 1.5x_{a1}/l], \tag{19}$$

and for the clamped–hinged beam the resulting moment is

$$M_{max}(l_p, x_{a1}) = 3Ml_p/l[1 - 0.5l_p/l - x_{a1}/l]. \tag{20}$$

4.2 Definition of a small actuator

A suitable choice of μ will depend on the required accuracy of the solution, however, by relating μ with $|M_{max}/M|$ and incorporating the spatial frequencies of the modes (Benatzky and Kozek [2005]) a necessary criterion for a small actuator size l_p/l results:

Definition 2. A patch actuator of length l_p is defined as small of order N with regard to a beam with length l if the quotient between maximum bending moment along the beam M_{max} and actuator moment M is smaller than some predefined constant $0 \le \mu < 1$ multiplied with the argument of the maximum $\lambda_{\max}(N)$ of the actuator influence function $F(N, l_p/l)$ of order N:

$$|M_{max}/M| \le \mu \arg\max_{l_p/l} \left(\frac{\psi_{N,u}(l_p/l)}{\psi_{N,u}(l_{p,\text{ref}}/l)} \right) = \mu\, \lambda_{\max}(N)$$

Note that the reference length $l_{p,\text{ref}}$ does not affect the criterion, since it is merely a scaling factor of the function $F(i, l_p/l)$ defined as

$$F(i, l_p/l) = \frac{\psi_{i,u}(l_p/l)}{\psi_{i,u}(l_{p,\text{ref}}/l)} = \nu(i)\, \psi_{i,u}(l_p/l). \tag{21}$$

In (21) $\nu(i)$ is a scaling factor only depending on the mode number i.

4.3 Definition of an upper bound for $\lambda(i)$

An upper bound of $\lambda(i)$ obviously depends on the spatial frequencies of the modes as given in (9) and (21) and discussed in Moheimani and Ryall [1999]. Therefore, a proper quantitative measure is given by the following definition:

Definition 3. An upper bound $\lambda_{\max}(i)$ for the scalar factor λ in Definitions2 and for the i-th mode is given by the argument of the maximum value of the actuator influence function $F(i, l_\mathrm{p}/l)$ with respect to l_p/l:

$$\lambda_{\max}(i) = \arg\max_{l_\mathrm{p}/l} \left(F(i, l_\mathrm{p}/l)\right) = \arg\max_{l_\mathrm{p}/l} \left(\psi_{i,u}(l_\mathrm{p}/l)\right).$$

For the hinged–hinged beam

$$F(i, l_\mathrm{p}/l) = \frac{\sin(\frac{i\pi l_\mathrm{p}}{2l})}{\sin(\frac{i\pi l_\mathrm{p,ref}}{2l})} \Rightarrow \lambda_{\max}(i) = \frac{1}{i} \qquad (22)$$

holds. In that case a small actuator is at least smaller by a factor μ than the argument of the maximum of the given surface for each mode i (depicted by a red line in Fig.3).

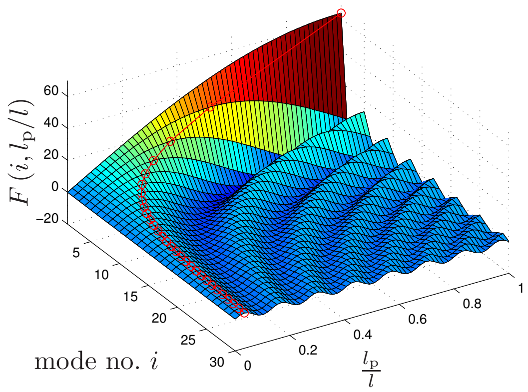

Fig. 3. Function $F(i, l_\mathrm{p}/l)$ for $l_\mathrm{p,ref}$=0.01m and l=1m for the simply supported beam

5. COLLOCATED AND NON-COLLOCATED TRANSFER FUNCTIONS

5.1 Steady-state correction

To demonstrate the influence of the actuator size on the transfer function (12), a simply supported beam with the data according to Table 1 is taken as example.

Table 1. Data of the simply supported beam

parameter	value	parameter	value
length l	1m	width b	$3{\cdot}10^{-2}$m
height h	$3{\cdot}10^{-3}$m	el. modulus E	$7{\cdot}10^{10}$N/m^2
density ρ	$2.7{\cdot}10^{3}$kg/m^3	mod. damp. ζ_j	0.01

A modal damping of 0.01 complies with many typical lightweight structures. The actuator is placed at $x_{\mathrm{a}1}$=0.22m using the criteria given in Gawronski [1997] - Gawronski [1999]. A patch length (actuator and sensor) of l_p=0.025m is considered. The sensor is either collocated with the actuator ($x_{\mathrm{s}1}$=0.22m) or placed at $x_{\mathrm{s}1}$=0.40m where a smaller local optimum of the above cited criteria exists. In Fig.4 and Fig.5 the collocated and non-collocated transfer functions for Υ=1 are given for a model consisting of N=500 modes. There the 500 mode model ("nom."), a truncated three mode model ("red."), a three mode model with a correction factor calculated from the steady-state contribution (16) of mode four to L = 50

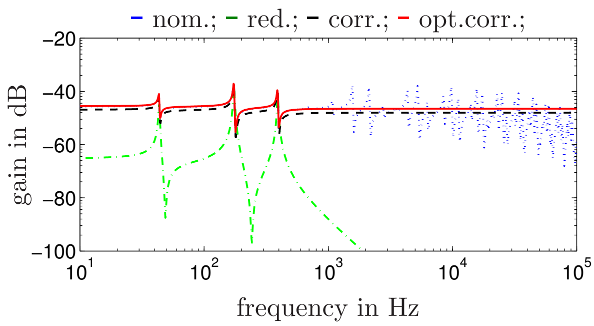

Fig. 4. Collocated transfer functions for l_p/l=0.025

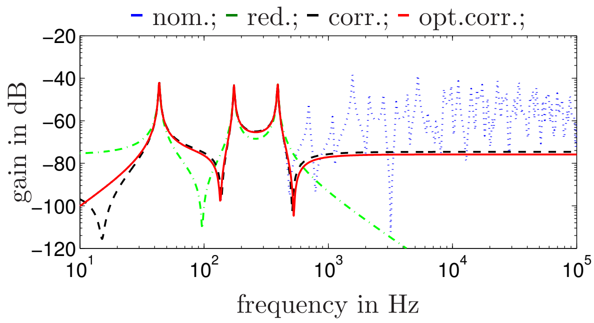

Fig. 5. Non-collocated transfer functions for l_p/l=0.025

("corr."), and the three mode model including the correcting factors K_c according to (17) or (18) ("opt.corr.") are shown, respectively. There is a visible difference between the model corrected with the contribution of mode $N+1 = 4$ to $L = 50$ and the model corrected with (18) for collocation, whereas this is not the case for non-collocation. In Fig.6 the steady-state gain of the full

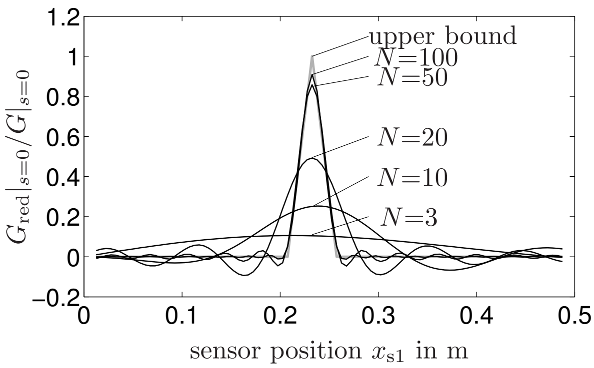

Fig. 6. Static gain over sensor position $x_{\mathrm{s}1}$ for different numbers of modes N in the truncated model G_red

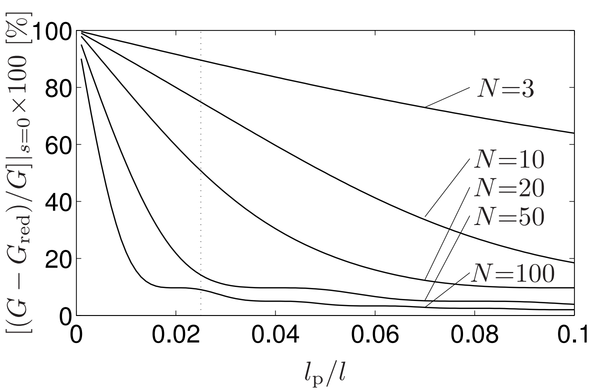

Fig. 7. Truncation error for models of order N

system G (upper bound) is compared to various truncated systems G_{red} when the sensor patch is moved along the beam. The upper bound again indicates, that the steady-state gain of G_{red} is zero as long as sensor and actuator patch do not overlap and increases linearly with the length of overlap. Moreover, for non-collocated transfer functions the steady-state gain error decreases with the separation of the patches. The correction error for fully overlapping patches is displayed in Fig.7 for increasing actuator length. It can be seen, that for an actuator size of l_{p}/l=0.025 even for a model including 100 modes an error of approximately 10% remains at steady-state.

6. STABILITY ANALYSIS AND ROBUSTNESS

In order to compare the closed loop stability for collocated and non-collocated transfer functions a control system based on state vector feedback methods is designed. Both the controller and observer are designed by pole placement for a reduced system of order three, and the feedback gain $\boldsymbol{L}$ is derived by placing the poles at

$$p_{i,\mathrm{des}} = -\omega_i \zeta_{i,\mathrm{des}} \pm \omega_i \sqrt{1-\zeta_{i,\mathrm{des}}}, \tag{23}$$

i.e. only the damping is increased by the controller. The damping coefficients of the first three modes are chosen to be $\boldsymbol{\zeta}_{\mathrm{des}}$=[0.3, 0.2, 0.01]. This choice is motivated by practical applications, where significant damping is applied only to the structural modes of interest. For the design of the observer feedback matrix $\boldsymbol{H}$ the desired damping is doubled. The structure of the controller $K(s)$ is shown

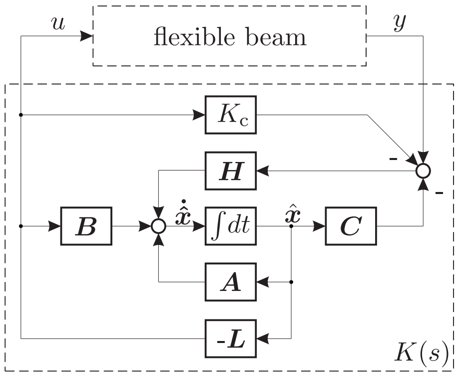

Fig. 8. Flexible beam control system

in Fig.8. Evaluating the controller transfer function $K(s)$ results in

$$K(s) = -\boldsymbol{L}\left[s\boldsymbol{I} - \boldsymbol{A} + (\boldsymbol{B} - \boldsymbol{H}K_{\mathrm{c}})\boldsymbol{L} + \boldsymbol{H}\boldsymbol{C}\right]^{-1}\boldsymbol{H}. \tag{24}$$

Equation (24) states, that K_{c} directly affects the controller poles. For collocation, the feed-through term K_{c} dominates the transfer function behavior. The pole locations of the closed-loop in the complex s-plane are plotted in Fig.9 for a collocated actuator/sensor pair of size l_{p}/l=0.025. There, □ are the closed-loop poles predicted by the controller design for a three mode model corrected with the contribution of mode four to 100. Since the real system has a higher transfer function gain, the actual closed-loop poles are shifted towards the imaginary axis (□→◇), and one of the controller poles becomes even unstable.

Finally, to more closely investigate the stability behavior the small gain theorem (Skogestad and Postlethwaite [1996]) is utilized to determine stability bounds for collocated and non-collocated transfer functions. For this

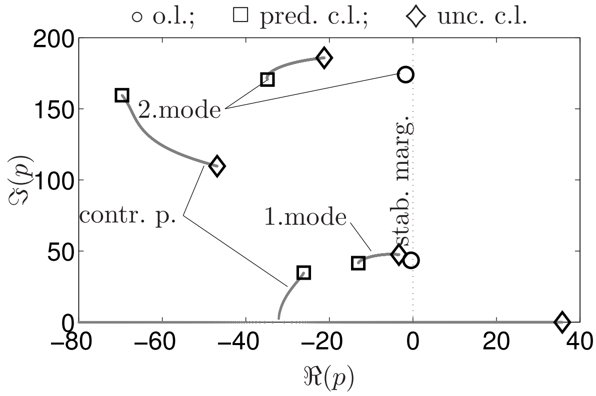

Fig. 9. Pole plot: stability loss due to uncertain correction factor K_{c}

reason, the correction factor K_{c} according to (16) is built from the compliance terms of mode four up to mode 100 for the small (l_{p}/l=0.025) and a large actuator (l_{p}/l=0.1). For the collocated case, the difference between K_{c} and its upper bound (18) is 11.3% and 3.3% for the small and the large actuator, respectively. These percentile errors are modeled as multiplicative input uncertainty for stability analysis. For the non-collocated case the same amount of multiplicative input uncertainty is utilized.

A system with multiplicative input uncertainty G_{d} is given by $G_{\mathrm{d}}=G_{\mathrm{corr}}(1+\Delta_{\mathrm{m}}W_{\mathrm{m}})$, where G_{corr} is the nominal plant (three modes corrected with the compliance contribution of modes four to 100), Δ_{m} is the uncertainty ($|\Delta_{\mathrm{m}}| \le 1$, $\Delta_{\mathrm{m}} \in \mathbb{C}$), and W_{m} is a weighting factor describing the size of the uncertainty over frequency. The loop of G_{d} is closed via the controller $K(s)$ (Fig.10). Then, the transfer

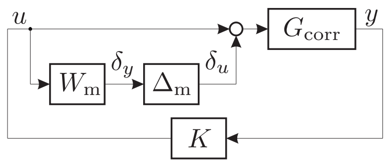

Fig. 10. Compliance uncertainty model

function M from the output δ_u of the uncertainty Δ_{m} to its input δ_y for positive feedback is given by

$$M = \frac{\delta_y}{\delta_u} = W_{\mathrm{m}} \frac{G_{\mathrm{corr}}K}{1 - G_{\mathrm{corr}}K}. \tag{25}$$

The small gain theorem states, that the closed loop system is stable under all perturbations Δ_{m} if and only if

$$\max_{s=j\omega} \bar{\sigma}(M(s)) = \max_{s=j\omega}(|M|) < 1. \tag{26}$$

To evaluate (26), $\bar{\sigma}(M(j\omega))$ is plotted for the small and the large actuator in Fig.11 and Fig.12 for the collocated and the non-collocated case, respectively. For collocated sensor and actuator (Fig.11) the maximum of $\bar{\sigma}(M(s))$ is 3.65 for the small actuator and 0.24 for the large actuator, thus clearly indicating the instability of the closed loop with the small actuator/sensor pair. To achieve robust stability for the small actuator pair the uncertainty has to be reduced to 27.4% of its current value, whereas for the large actuator/sensor pair only a 4.18 times or larger uncertainty would lead to instability. Finally, from Fig.12 it is clear, that in the case of non-collocation for the same uncertainty stability is guaranteed. As a matter of fact, for the small actuator/sensor pair the uncertainty could

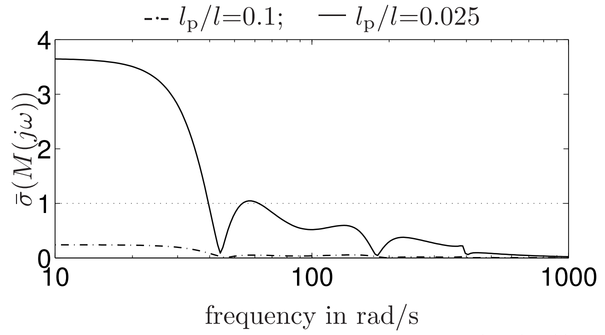

Fig. 11. *Collocation:* closed loop stability $\bar{\sigma}(M(j\omega))$

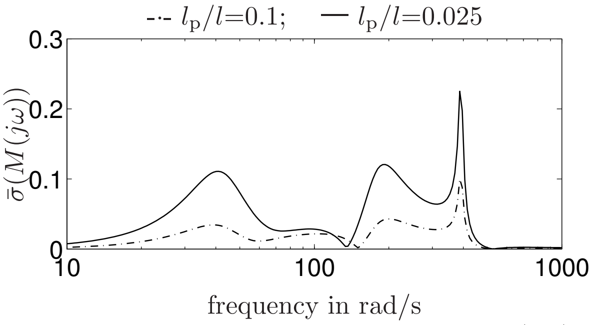

Fig. 12. *Non-collocation:* closed loop stability $\bar{\sigma}(M(j\omega))$

be increased by a factor of 4.44 and for the large pair by 9.98 before instability occurs.

For small actuator/sensor pairs the in-bandwidth part of the transfer function is more strongly influenced by the out-of-bandwidth modes for collocation than for non-collocation. Therefore, regarding robustness issues associated with the accuracy of the correction terms, the non-collocated placement of actuators and sensors is better suited than the utilization of collocated pairs. The importance of this proposition increases when the actuator size decreases compared to the size of the structure to be controlled.

7. CONCLUSION

In this paper the influence of the actuator size on the closed loop stability of collocated and non-collocated transfer function models for flexible beams has been investigated. The notion of a small actuator was defined, and a criterion for the applicability of the proposed methods on different beam supports was formulated. It was found, that for collocation the correction of the steady state transfer function gain is of vital importance to achieve closed loop stability. For statically determined and under-determined flexible structures the correction factors for non-collocation can be computed. For statically over-determined structures an approximation with a quantitative criterion was derived, and a stability analysis for parametric reduction-induced uncertainties was carried out. Therefore, for small actuators and highly flexible structures, the non-collocated placement of actuators and sensors is of advantage for flexible structure control, especially in the presence of uncertain parameters.

Future work will be focused on appropriate modifications to optimal positioning criteria as well as the problem of MIMO-control systems. Although that case is already covered in principle by the methods presented here, some open questions related to closely neighboring actuators or sensors still remain.

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