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Analysis of coupled van der Pol oscillators and implementation to a myriapod robot

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Abstract: According to the study of nervous system ethology, it is thought that a walk movement of an animal is controlled by Central Pattern Generator (CPG). There are a lot of studies to try the control of the leg type robot based on CPG principle. However, most of the studies consider two or four leg type robots, and there are not many studies performed for robots more than six legs. One of its factor is probably difficulty of constituting stable and periodic CPG in the case of many number of the legs. Therefore, this paper proposes the novel CPG model which is based on coupled van del Pol oscillators. And this paper reports the analysis of proposed coupled van del Pol oscillators model and the implementation result to an actual myriapod robot.

Keywords: Coupled van der Pol Oscillators; Myriapod Robot; Central Pattern Generator; Legged Robot; Distributed Autonomous System.

1. INTRODUCTION

As mechanism to realize movement in the irregular ground, a lot of research of 2 and 4 legs walking robot have been performed till now.However, it is difficult to control the whole while controlling a system intensively because the system controlling a leg type walking robot is complicated. There are problems to become the poor system in tolerance for the trouble and flexibility when control the whole system in one place.

In late years it attracts its attention to apply the autonomy dispersion system which is kind of the disperse system system to the control of the leg type robot. The autonomy dispersion system consists of a subsystem, each subsystem does autonomy of that, and that trouble tolerance and flexibility are high (Ishii [2004]).

When apply an autonomy dispersion system to a leg type robot, the research of a control method based on the motion principle of the animal is performed , and even this research examines a method to use CPG(Central Pattern Generator) for motion pattern generation. Research of CPG are performed well, for example, there are the research that used a cat (Shink et al. [1976]). In addition, a research that showed what can generate the change of various walk patterns by CPG and a reserach that apply CPG to a robot and realized a walk. However, there are many things which intend for four or two legs robots and example which implemented is seen implemented to a robot of the number of the legs more than 6, for the research that applied CPG to the control of a myriapod. As the number of the legs increases, as for this, it seems that it is difficult to constitute periodic CPG by stability.

Therefore, by this paper, it is based on van der Pol oscillator well-known as a classic problem and proposes the CPG model that a plural number connected this. There is as similar research, but remains if that argue qualitatively in wave pattern of oscillators changing depending on a parameter. In this paper, report that analysis of coupled van der Pol oscillators and implementation to a myriapod.

2. MYRIAPOD ROBOT

A myriapod robot is shown in Fig.1. In addition, a specifications for one module of this robot is shown in Table 1. This myriapod robot consists of by connecting with modules with two legs right and left. It can be connected to 30 legs 15 modules at the maximum it, and the full length becomes about 1.8[m]. Each leg is equipped with two servomotors and becomes two degree of freedom of the top, bottom, front and back direction. In addition, it is equipped with the servomotor which can work in a horizontal direction between each modules, and the control of five degree of freedom is possible because of one module.

Each module is equipped with a CPU (a microcomputer). Each module interval is connected by a serial signal line, and a outside host PC or central control with it being connected to a high rank microcomputer and control of the autonomy dispersion by the communication between the module are possible. In addition, can develop the software

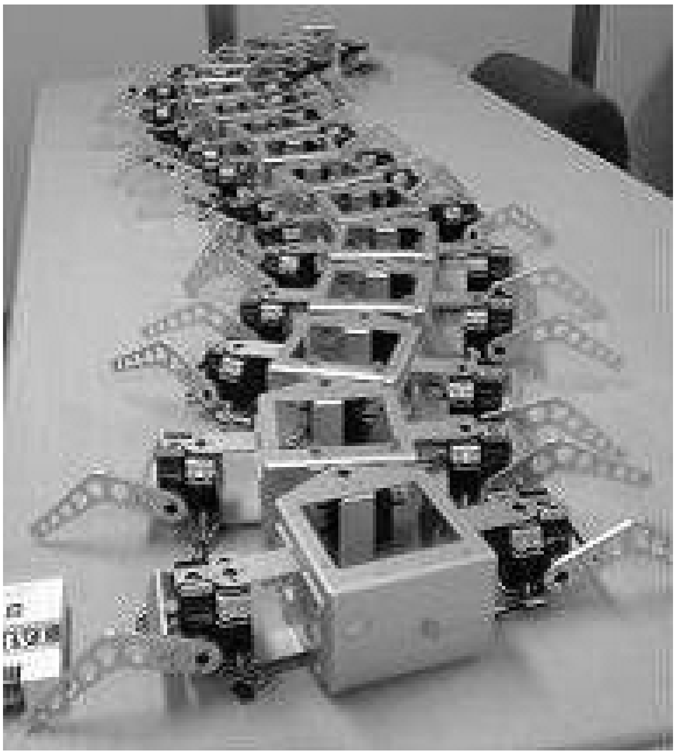

Fig. 1. Appearance of a myriapod robot.

Table 1. Specification of a module.

Number of the operation	Each left and right leg 2, connection 1 (Five degree of freedom)
Actuator	Servomotor for radio control (Futaba S9206)
Size	400(W)×120(D)×120(H)[mm]
Weight	1.2[kg]
Serial communication	RS485 or RS232C (original protocol)
CPU	SH7047 (SH-2 Core)
Memory	SRAM 256KB (battery backup) FlashROM 256KB (in CPU)
Power supply	cable broadcasting in DC12V from the outside
Battery for memory	CR123A (3V lithium battery for cameras) electric double layer condenser

of the CPU by C language, RAM and the Flash ROM can be written in.

The power supply becomes the method to supply to each module from the outside. A panel made by acrylic sticks to the top surface of the module and can be superior to the conservatism of the inside base because the putting on and taking off by the hand-operation is possible. In addition, expansibility is high because putting on and taking off between each modules is hand-operated.

3. VAN DER POL OSCILLATOR

3.1 Single van der Pol Oscillator

van der Pol oscillator (van der Pol [1926]) is well known as a classic problem, and it is expressed by the next differential equation.

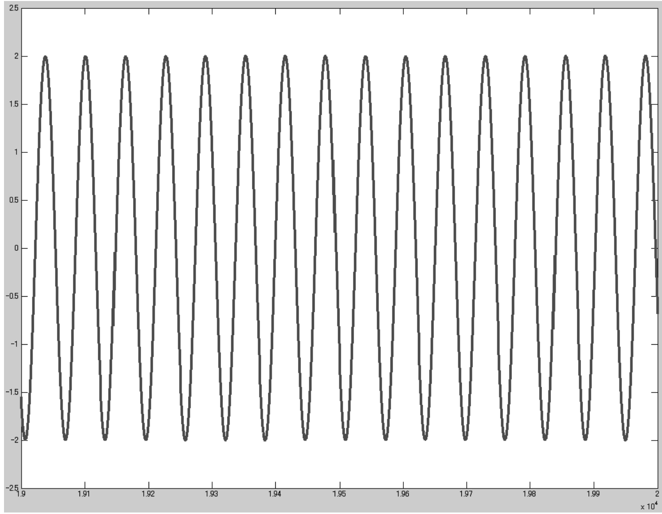

Fig. 2. Single van der Pol oscillator.

$$x'' - \varepsilon(1 - x^2)x' + x = 0, \quad t > 0 \tag{1}$$

If a solution of this equation is $\varepsilon > 0$, it is proved that become limit cycle by Pincaré-Bendixson theorem. Therefore, $x(t)$ generates the periodic output such as Fig.2 after time passed enough.

3.2 Coupled van der Pol Oscillators

It is thought that the van der Pol oscillator can function as CPG controlling the periodic walk exercise of the animal in a thing self-excitation vibration. However, it is necessary to connect the output of the osillator with the movement of each leg to apply to the control of the myriapod robot having plural legs. Therefore, it is necessary to constitute oscillator system having the number of legs and the output of the same number. By this paper, We assume that control the myriapod robot of the N leg and propose connection type van der Pol oscillator to have the output of an N unit showing in eq.(2).

$$x_i'' - \varepsilon(1 - x_i^2)x_i' + x_i = \begin{cases} k(x_i - x_{i-1}), & i \neq 1 \\ k(x_1 - x_N), & i = 1 \end{cases} \tag{2}$$

Here, k is a real number to become the design parameter. The output example in case of $N = 2$ seems to become Fig.3.

3.3 Existence characteristics of in-phase solution and out-of-phase solution

In eq.(2), assume it $x_1 = y, x_2 = z$ and think about the next system consisting of two continuous function of real variable $y = y(t), z = z(t)$.

$$\begin{cases} y'' - \varepsilon(1 - y^2)y' + y = k(y - z) \\ z'' - \varepsilon(1 - z^2)z' + z = k(z - y), \quad t > 0 \end{cases} \tag{3}$$

Here, define the next for the eq.(3) output.

Definition 1. When a steady solution after enough time passed becomes $y(t) = z(t)$ in expression eq.(3), this is

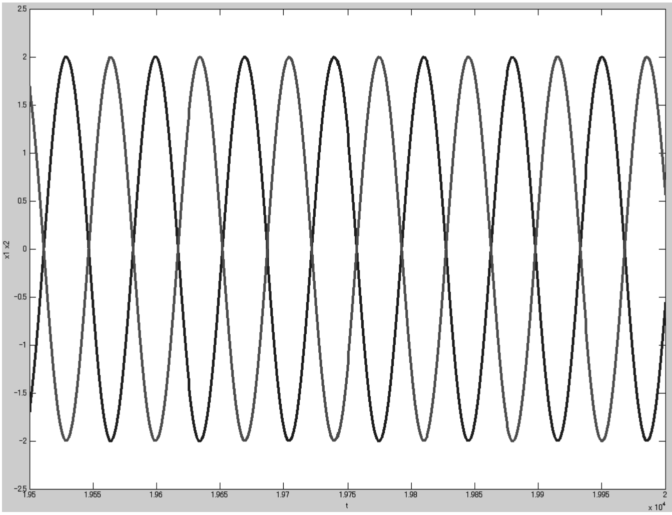

Fig. 3. Coupled van der Pol oscillator.

called in-phase solution. In addition, this is called out-of-phase solution when it becomed $y(t) = -z(t)$.

By the output of Fig.3, it is it with out-of-phase solution that phase of two output just reversed in a thing of a solution becoming $x_1(t) = -x_2(t)$ in a steady state being provided. It is thought that this is the output corresponding to two pairs of normal walks movement that the leg of right and left exercises in turn. When $x_1(t) = x_2(t)$ clearing up eq.(2) from a different initial value for this and the solution that it is provided, there is thought that is corresponding to two pairs of animal that the jump movement. About the eq.(3) steady solution, the next theorem holds good.

Theorem 2. Suppose it to be $\varepsilon \ll 1$ in expression eq.(3). There is stability in-phase solution or out-of-phase solution depending on an initial value after time passed enough then.

Proof. See the appendix A.

The coupled van der pol oscillator eq.(3) than this is dependent on an initial value and generates the output of in-phase or out-of-phase by a steady state. When fixed $\dot{x}_1(0)$ and $\dot{x}_2(0)$ among initial values and simulated it while changing initial value $x_1(0)$ and $x_2(0)(N = 2)$, there were the relations that seemed to be Fig.4 about phase relations of two output. In other words, in a phase plane consisting of it by initial value $x_1(0)$ and $x_2(0)$, as for initial value, it is it with in-phase solution of the first and the third quadrant, and, as for initial value, it is it with out-of-phase solution of the second and the fourth quadrant. From this, setting of an appropriate initial value becomes important to generate a desired walk movement.

4. IMPLEMENTATION OF VAN DER POL OSCILLATOR

4.1 Output of oscillator and correspondence of the real movement

If van der Pol oscillator works normally, the output becomes the periodic vibration, but, for the output and associating of the exercise of the leg, there are various methods. By this report, implementation to the robot by associating the output and the joint angle of the leg.

Fig.5 shows image of the movement. A wave line of Fig.5 expresses output of van der Pol oscillator($x(t)$) operates joint angle of leg($\theta(t)$) to become as follows equation.

$$\theta(t) = \alpha x(t) \tag{4}$$

α is relation coefficient of the output amplitude of oscillator and movement range of legs. When the output of the trembler is maximum, working a leg most forward and working a leg most backward when the output is minimum.

Only in swing movement, it supports the output of oscillator this paper. In rift movement, take down a leg when backward from front in swing movement, raise a leg when front from backward.

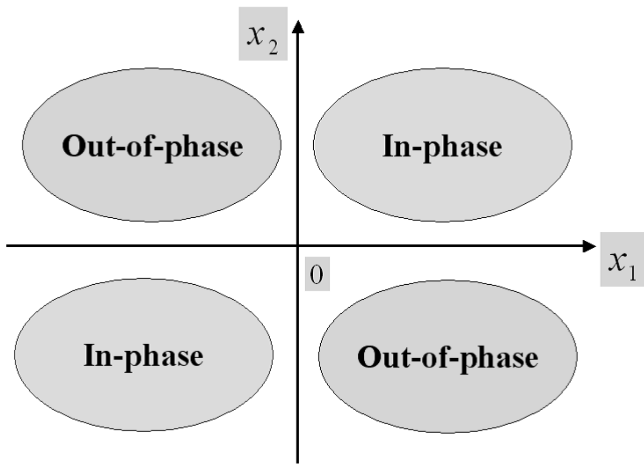

Fig. 4. Distribution of initial value.

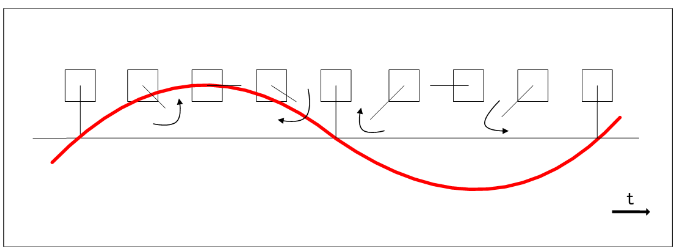

Fig. 5. Relationship between leg motion and output of van der Pol oscillator.

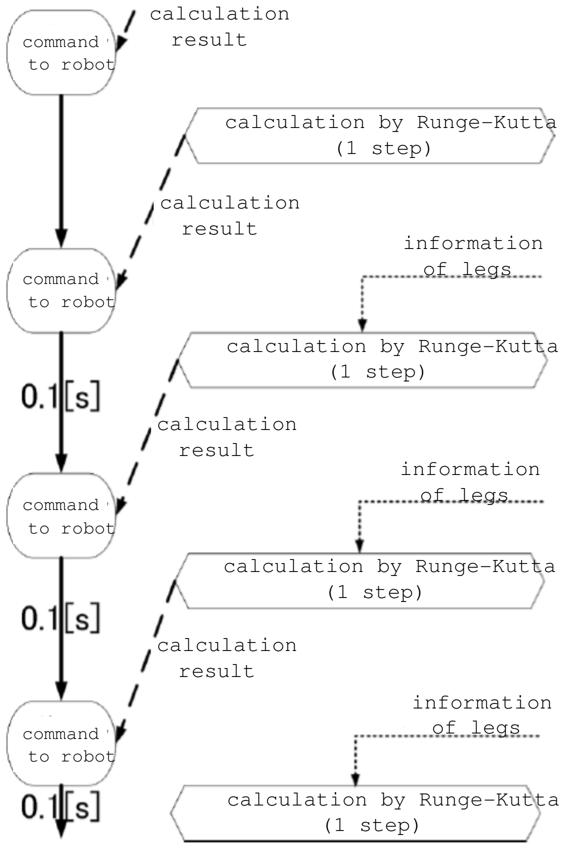

Fig. 6. Algorithm of implementation.

4.2 Algorithm of implementation

In this time, implementation of one module of the robot.($N = 2$) Algorithm is shown in below.

(1) Solve a differential equation fot take the trembler output
(2) Convert the output value into a movement angle and move a leg
(3) get the angle information of legs

The robot walks by repetition of $1 \to 2 \to 3 \to 1 \cdots$ every 0.1[s]. Fig.6 shows these a series of flows.

It is step 1 and solve of eq.(2) every 0.1[s] with Runge-Kutta method. By a joint angle provided in eq.(4), working swing movement. During the movement, feed back the joint angle information from a leg by step 3. And solving of van der Pol oscillators output by next step 1 again. Finish a calculation, and a waiting, and movement is completed, working the next movement by step 2.

In rift movement, take down a leg when (x' varies from $-$ to $+$), raise a leg when (x' varies from $+$ to $-$).

4.3 Implementation result

With the algorithm that above, show in below the result that implemented coupled van der Pol oscillators to a myriapod robot(1 module). Each fixed number and an initial value is shown in Table2,Table3. Table3 seted two pattern of initial values to coupled van der Pol oscillator become in-phase solution or out-of-phase solution(section 3.3). Fig.7 and Fig.8 shows a simulation result in case of a each pattern(movement start, steady state). The movement of the leg is inconsistent at the time of the movement start, becomes in-phase and out-of-phase at the time of steady state. We was able to realize movement corresponding to output of coupled van der Pol oscillator.

Table 2. A parameter value of oscillator.

parameter	value
ε	0.01
k	0.1
period	0.1[s]

Table 3. Initial value of oscillator.

	$x_1(0)$	$x_1'(0)$	$x_2(0)$	$x_2'(0)$
in-phase	1.0	0.0	0.1	0.0
out-of-phase	1.0	0.0	-0.1	0.0

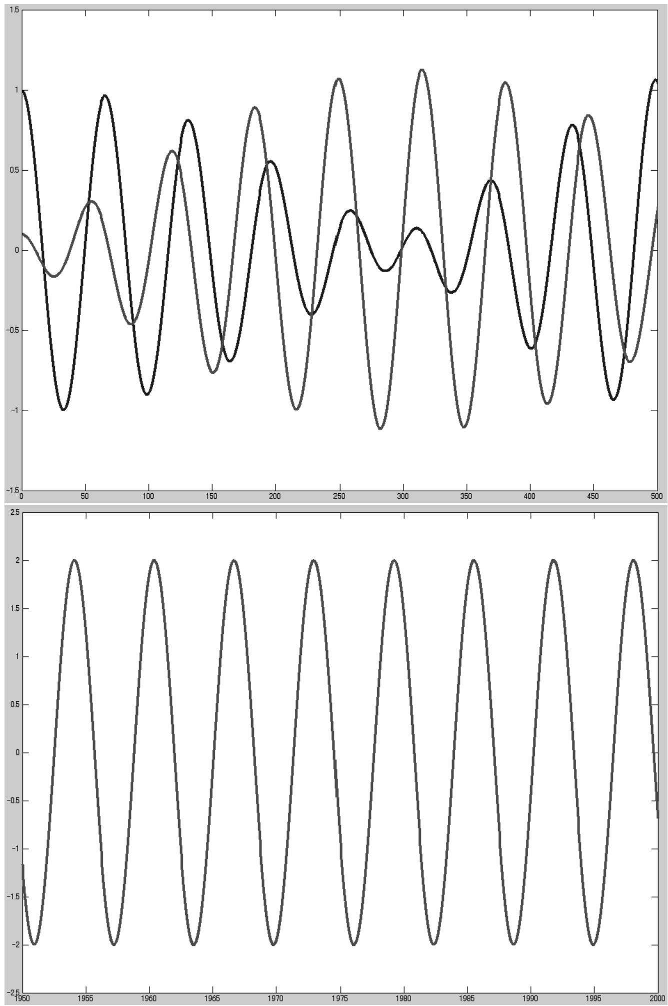

Fig. 7. In-phase.

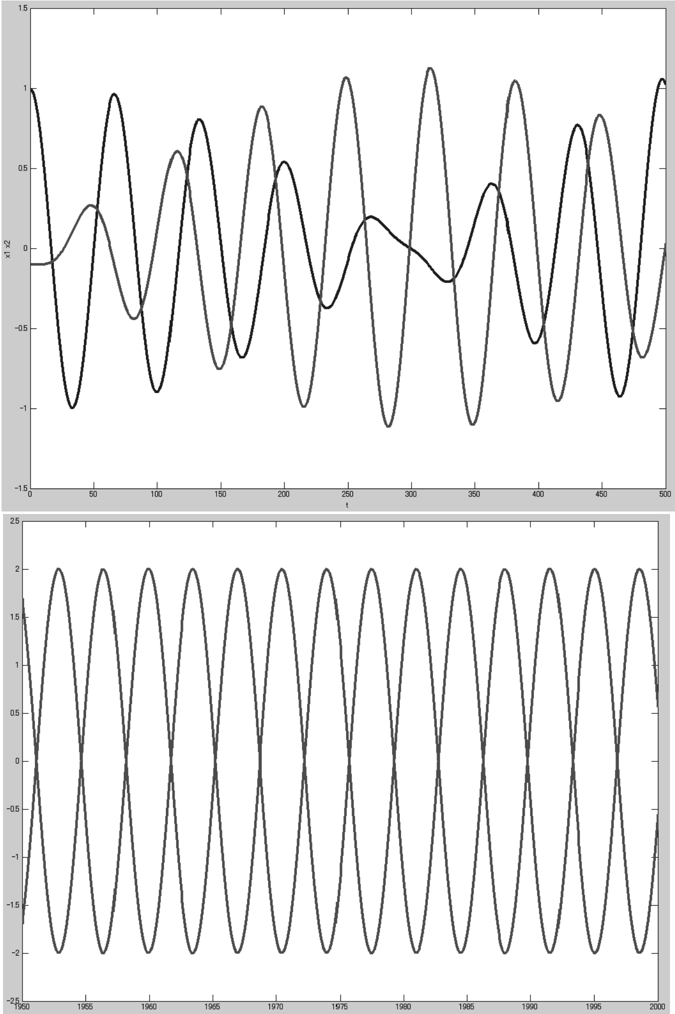

Fig. 8. Out-of-phase.

5. CONCLUSION

In this paper, We proposed coupled van der Pol oscillators and proved existence characteristics of in-phase solution and out-of-phase solution($N = 2$). We implemented it for one module of a myriapod robot and realized movement corresponding to oscillators output. Analysis about the initializing of the oscillators proposed and implementation to a myriapod robot of plural modules will be problems in future.

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Appendix A. PROOF OF THEOREM 2

Here, proved only existence characteristics. The perfect proof refers to Nohara et al. [2007]. Introduce $t_1 = t,\ t_2 = \varepsilon t_1$ and use multiple scale analysis (Nayfeh [1973]). Time differential calculus used t_1, t_2.

$$\frac{d}{dt} \longrightarrow \frac{dt_1}{dt}\frac{\partial}{\partial t_1} + \frac{dt_2}{dt}\frac{\partial}{\partial t_2} = \frac{\partial}{\partial t_1} + \varepsilon\frac{\partial}{\partial t_2} \tag{A.1}$$

$\frac{\partial}{\partial t_1}$ is ∂_{t_1}, $\frac{\partial}{\partial t_2}$ is ∂_{t_2} for simplify it. Then eq.(3) is as follows.

$$\begin{cases} (\partial_{t_1}^2 + 2\varepsilon\partial_{t_1}\partial_{t_2} + \varepsilon^2\partial_{t_2}^2)y \\ \quad -\varepsilon(1-y^2)(\partial_{t_1} + \varepsilon\partial_{t_2})y + y = k(y-z) \\ (\partial_{t_1}^2 + 2\varepsilon\partial_{t_1}\partial_{t_2} + \varepsilon^2\partial_{t_2}^2)z \\ \quad -\varepsilon(1-z^2)(\partial_{t_1} + \varepsilon\partial_{t_2})z + z = k(z-y) \end{cases} \tag{A.2}$$

And expand solution $y(t),\ z(t)$

$$\begin{cases} y(t) \sim y_0(t_1,t_2) + \varepsilon y_1(t_1,t_2) + \varepsilon^2 y_2(t_1,t_2) + \cdots \\ z(t) \sim z_0(t_1,t_2) + \varepsilon z_1(t_1,t_2) + \varepsilon^2 z_2(t_1,t_2) + \cdots \end{cases} \tag{A.3}$$

Following as an equation of $O(1)$ about ε when substitute this for eq.(A.2).

$$\begin{cases} (\partial_{t_1}^2 + 1)y_0 = k(y_0 - z_0) \\ (\partial_{t_1}^2 + 1)z_0 = k(z_0 - y_0) \end{cases} \tag{A.4}$$

And soluve this.

$$\begin{cases} y_0 = A_1(t_2)\cos(\eta_1) + A_2(t_2)\cos(\eta_2) \\ z_0 = A_1(t_2)\cos(\eta_1) - A_2(t_2)\cos(\eta_2) \end{cases} \tag{A.5}$$

It is $\eta = t_1 + \phi_1(t_2),\ \eta_2 = \sqrt{1-2k}t_1 + \phi_2(t_2)$ here. We can suppose it to be $A_1 \geq 0,\ A_2 \geq 0$(Without losing generality). Equation $O(\varepsilon)$ likewise.

$$\begin{cases}(\partial_{t_1}^2+1)y_1 = (1-y_0^2)\partial_{t_1}y_0 - 2\partial_{t_1}\partial_{t_2}y_0 + k(y_1-z_1)\\(\partial_{t_1}^2+1)z_1 = (1-z_0^2)\partial_{t_1}z_0 - 2\partial_{t_1}\partial_{t_2}z_0 + k(z_1-y_1)\end{cases} \tag{A.6}$$

Eq.(A.6) is as follows by a simple calculation.

$$\begin{aligned}&(\partial_{t_1}^2+1)y_1 - k(y_1-z_1) =\\&\quad\left(\frac{1}{2}A_1A_2^2 - A_1 + \frac{1}{4}A_1^3 + 2A_1^{'}\right)\sin\eta_1\\&\quad+\sqrt{1-2k}\left(\frac{1}{2}A_1^2A_2 - A_2 + \frac{1}{4}A_2^3 + 2A_2^{'}\right)\sin\eta_2\\&\quad+(2A_1\phi_1^{'})\cos\eta_1\sqrt{1-2k}(2A_2\phi_2^{'})\cos\eta_2 + \frac{1}{4}A_1^3\sin 3\eta_1\\&\quad+\frac{1}{4}\sqrt{1-2k}A_2^3\sin 3\eta_2\frac{1}{2}A_1A_2^2\sin\eta_1\cos 2\eta_2\\&\quad+A_1^2A_2\sin 2\eta_1\cos\eta_2\frac{1}{2}\sqrt{1-2k}A_1^2A_2\sin\eta_2\cos 2\eta_1\\&\quad+\sqrt{1-2k}A_1A_2^2\cos\eta_1\sin 2\eta_2\end{aligned} \tag{A.7}$$

$$\begin{aligned}&(\partial_{t_1}^2+1)z_1 - k(z_1-y_1) =\\&\quad\left(\frac{1}{2}A_1A_2^2 - A_1 + \frac{1}{4}A_1^3 + 2A_1^{'}\right)\sin\eta_1\\&\quad+\sqrt{1-2k}\left(-\frac{1}{2}A_1^2A_2 + A_2 - \frac{1}{4}A_2^3 + 2A_2^{'}\right)\sin\eta_2\\&\quad+(2A_1\phi_1^{'})\cos\eta_1\sqrt{1-2k}(2A_2\phi_2^{'})\cos\eta_2 + \frac{1}{4}A_1^3\sin 3\eta_1\\&\quad-\frac{1}{4}\sqrt{1-2k}A_2^3\sin 3\eta_2\frac{1}{2}A_1A_2^2\sin\eta_1\cos 2\eta_2\\&\quad-A_1^2A_2\sin 2\eta_1\cos\eta_2\frac{1}{2}\sqrt{1-2k}A_1^2A_2\sin\eta_2\cos 2\eta_1\\&\quad+\sqrt{1-2k}A_1A_2^2\cos\eta_1\sin 2\eta_2\end{aligned} \tag{A.8}$$

solvability conditions by these equation.

$$\begin{cases}2A_1^{'} = A_1\left(1-\frac{1}{4}A_1^2-\frac{1}{2}A_2^2\right)\\2A_2^{'} = A_2\left(1-\frac{1}{4}A_2^2-\frac{1}{2}A_1^2\right)\\2A_1\phi_1^{'}=0\\2A_2\phi_2^{'}=0\end{cases} \tag{A.9}$$

$$\begin{cases}2A_1^{'} = A_1\left(1-\frac{1}{4}A_1^2-\frac{1}{2}A_2^2\right)\\2A_2^{'} = -A_2\left(1-\frac{1}{4}A_2^2-\frac{1}{2}A_1^2\right)\\2A_1\phi_1^{'}=0\\2A_2\phi_2^{'}=0\end{cases} \tag{A.10}$$

Eq.(A.9) and eq.(A.10) are AND condition.

$$A_2(t_2) = const.(=c_2), \quad \phi_2(t_2) = const.(=c_3) \tag{A.11}$$

A_1 is as follows with $A_2(t_2) = c_2$

$$A_1(t_2) = \frac{\sqrt{2}\sqrt{c_2^2-2}}{\sqrt{2c_1(c_2^2-2)e^{\frac{c_2^2-2}{2}t_2}-1}}, \quad c_1 = const. \tag{A.12}$$

Finally

$$\phi_1(t_2) = const.(=c_4) \tag{A.13}$$

Than the above.

(1) In case of $A_2 = c_2 \neq 0$, A_1 is expressed in eq.(A.12).

$$\begin{cases}A_1 \to \sqrt{2}\sqrt{2-c_2^2}\ \text{ as } t\to\infty, & \text{for } 0 \le c_2 < \sqrt{2}\\A_1 \to 0\ \text{ as } t\to\infty, & \text{for } \sqrt{2} \le c_2 \le 2\end{cases}$$

It is out-of-phase at the time of $\sqrt{2} \le c_2 \le 2$. (get the condition of $c_2 < 2$ from the second equation of solvability condition)

(2) In case of $A_2 = 0$, following from first and third equation of eq.(A.9).

$$A_1 = \frac{\sqrt{2}\sqrt{-2}}{\sqrt{2c_1(-2)e^{-t_2}-1}} \to 2\ \text{ as } t\to\infty$$
$$\phi_1 = const.$$

There is in-phase.