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Piecewise Linear Steady–State Target Optimization for Control Systems with MPC: a Case Study

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Abstract: This paper is concerned with predictive control combined with set–point optimization in the case of fast changing disturbances. The problem is encountered in many practical applications. Because of high computational complexity, nonlinear economic optimization cannot be repeated frequently. Therefore, in practice an additional steady–state target optimization repeated as often as the MPC (Model Predictive Control) algorithm is used. Typically, the steady–state target optimization is based on a linear steady–state process model. Unfortunately, in some cases, as the one studied in the paper, the target set–point optimization based on linear or linearized models fails. It is demonstrated in the paper that a solution to this problem can be the piecewise linear approximation of the nonlinear steady–state process model in the target optimization. The research is done for the control system of a MIMO chemical reactor. The presented results clearly show the effectiveness of the proposed approach.

1. INTRODUCTION

An important problem in the control systems with classical hierarchical approach is caused by disturbances of dynamics not much slower than the dynamics of the process. If the steady–state economic optimization is repeated infrequently, it usually leads to solutions with lower economic effectiveness (Brdys and Tatjewski, 2005; Findeisen *et al.*, 1980). The best solution, in theory, would be to increase the frequency of intervention of the economic optimization layer. However, in most cases it would be unrealistic because of the complexity of the nonlinear steady–state optimization problem solved at the economic optimization layer.

In modern control systems Model Predictive Control (MPC) is often applied in practice (Camacho and Bordons, 1999; Maciejowski, 2002; Qin and Badgwell, 2003; Rossiter, 2003). MPC algorithms dominate as the algorithms of the advanced control layer. One of their main advantages is the unique ability to take into account in a natural way the constraints imposed on both manipulated variables and output variables (included in the optimization problem solved at each algorithm iteration). Moreover, the MPC algorithms can be relatively easy designed for MIMO processes.

There are two possible solutions to the problem. The first one consists in supplementing the MPC controller with a simplified linear economic optimization task, called usually a steady–state target optimization problem, solved as frequently as the MPC controller executes (Qin and Badgwell, 2003; Kassmann *et al.*, 2003, Ławryńczuk *et al.*, 2006; Tatjewski, 2007; Tatjewski *et al.*, 2006). The second solution is to integrate the MPC algorithm with the economic steady–state optimization into one optimization problem. It is done in such a way that a computationally efficient algorithm requiring solving on–line a quadratic programming problem is obtained (linearization of the nonlinear steady–state process model is performed at each iteration) (Lawrynczuk *et al.*, 2006; 2007a; 2007b; Tvrzska *et al.*, 1998; Zanin *et al.*, 2000; 2002).

Unfortunately, in the case of some control plants, the linearization approach gives very disappointing results due to highly nonlinear nature of the controlled process. Such a case, of a highly nonlinear MIMO chemical reactor, is thoroughly studied in the paper. The solution applied is to use a piecewise linear approximation of the nonlinear steady–state process model in the target set–point optimization. It is demonstrated that thanks to such an approach very good control system operation can be obtained, almost as good as in the theoretically best possible case when a nonlinear steady–state economic optimization is repeated as often as the MPC controller executes.

The paper is organized as follows. In Sections 2 the classical multilayer control system structure is described. Section 3 details the additional target set–point optimization with linearization. Section 4 contains description of the proposed method using the piecewise linear approximation of the nonlinear steady–state process model. In Section 5 the example of a highly nonlinear control plant – a MIMO chemical reactor is presented. Moreover, simulation results obtained in the control system using the proposed approach, show the good performance offered by it comparing to the usually used control structures. Section 6 shortly concludes the paper.

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2. STANDARD CONTROL SYSTEM STRUCTURE

In Fig. 1 the structure of the standard hierarchical control system with the MPC advanced control layer is shown. A supervisory global plant–wide optimization layer aims at maximizing the economic yield obtained from many technological processes. The Local Steady–State Optimization (LSSO) layer takes into account a single plant. Each layer operates with different frequency of intervention.

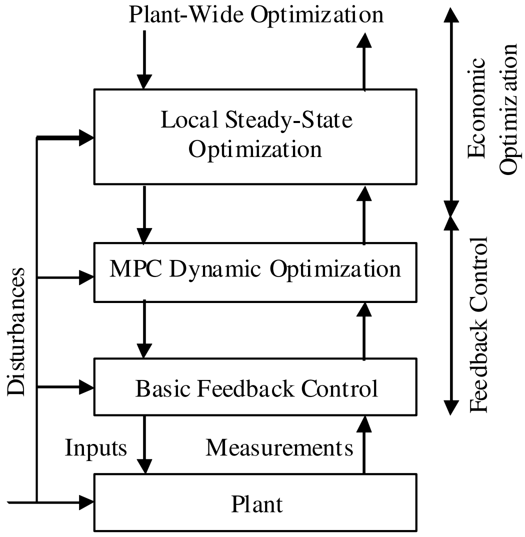

Fig. 1. Hierarchical control system structure with MPC advanced control layer (LSSO+MPC)

In the classical multilayer control system structure the economic LSSO layer usually solves the following optimization problem

$$\begin{aligned} &\min_{u^{ss}}\{J_E(k) = c_u^T u^{ss} - c_y^T y^{ss}\} \\ &\text{subject to}: \\ &\quad u_{\min} \le u^{ss} \le u_{\max} \\ &\quad y_{\min} \le y^{ss} \le y_{\max} \\ &\quad y^{ss} = F(u^{ss}, \tilde{w}) \end{aligned} \tag{1}$$

where $F : \Re^{n_u} \times \Re^{n_w} \to \Re^{n_y}$ denotes a comprehensive nonlinear steady–state process model relating outputs $y^{ss} \in \Re^{n_y}$ with controls $u^{ss} \in \Re^{n_u}$, n_u, n_w, n_y are the numbers of: manipulated variables, disturbances affecting the plant and controlled variables, respectively, $\tilde{w}$ is the current estimate or measurement of disturbances. The vectors $c_u \in \Re^{n_u}$, $c_y \in \Re^{n_y}$ are the prices resulting from economic considerations, $u_{\min}$, $u_{\max}$, $y_{\min}$, $y_{\max}$ are vectors of constraints imposed on inputs and outputs.

Let $\hat{u}^{ss}$ denote the optimal solution to the optimization problem (1). Using the nonlinear steady–state model the value $\hat{y}^{ss}$ corresponding to $\hat{u}^{ss}$ is calculated. The vector $\hat{y}^{ss}$ is then passed as the desired set–point to the MPC optimization problem. The MPC problem with a linear model and soft constraints written in a vector–matrix notation is (Maciejowski, 2002; Rossiter, 2003; Tatjewski, 2007)

$$\begin{aligned} &\min_{\Delta \boldsymbol{u}(k),\ \boldsymbol{\varepsilon}_{\min}\ \boldsymbol{\varepsilon}_{\max}} \{J_{MPC}(k) = \left\|\boldsymbol{y}^{ss} - \boldsymbol{G}\Delta\boldsymbol{u}(k) - \boldsymbol{y}^0(k)\right\|_{\boldsymbol{M}}^2 + \\ &\qquad + \left\|\Delta u(k)\right\|_{\boldsymbol{\Lambda}}^2 + \rho_{\min}\left\|\boldsymbol{\varepsilon}_{\min}\right\|^2 + \rho_{\max}\left\|\boldsymbol{\varepsilon}_{\max}\right\|^2\} \\ &\text{subject to}: \\ &\quad \boldsymbol{u}_{\min} \le \boldsymbol{J}\Delta\boldsymbol{u}(k) + \boldsymbol{u}^{k-1} \le \boldsymbol{u}_{\max} \\ &\quad -\Delta\boldsymbol{u}_{\max} \le \Delta\boldsymbol{u}(k) \le \Delta\boldsymbol{u}_{\max} \\ &\quad \boldsymbol{y}_{\min} - \boldsymbol{\varepsilon}_{\min} \le \boldsymbol{G}\Delta\boldsymbol{u}(k) + \boldsymbol{y}^0(k) \le \boldsymbol{y}_{\max} + \boldsymbol{\varepsilon}_{\max} \\ &\quad \boldsymbol{\varepsilon}_{\min} \ge 0,\ \boldsymbol{\varepsilon}_{\max} \ge 0 \end{aligned} \tag{2}$$

where $\Delta\boldsymbol{u}(k) = \left[\Delta u(k \mid k)^T \quad \ldots \quad \Delta u(k+N_u-1 \mid k)^T\right]^T$ is the vector of future control increments, $\boldsymbol{G}$ is the dynamic matrix of the dimension $n_yN \times n_uN_u$, composed of step response coefficients, N and N_u denote prediction and control horizons, respectively, $\boldsymbol{y}^0(k) = \left[y^0(k+1 \mid k)^T \quad \ldots \quad y^0(k+N \mid k)^T\right]^T$ is the free response vector of the length n_yN, $\boldsymbol{M} \ge 0$ and $\boldsymbol{\Lambda} > 0$ are diagonal weighting matrices of dimension $n_yN \times n_yN$ and $n_uN_u \times n_uN_u$, $\boldsymbol{y}^{ss}$ is a Kronecker tensor product of the vector $e_y^{ss} = [1 \quad \ldots \quad 1]^T$ of length n_u and $\hat{y}^{ss}$

$$\boldsymbol{y}^{ss} = e_y^{ss} \otimes \hat{y}^{ss} = \left[\left(\hat{y}^{ss}\right)^T \quad \ldots \quad \left(\hat{y}^{ss}\right)^T\right]^T \tag{3}$$

$\rho_{\min}$, $\rho_{\max}$ are positive weights,

$$\begin{aligned} &\boldsymbol{u}_{\min} = \left[u_{\min}^T \quad \ldots \quad u_{\min}^T\right]^T, \quad \boldsymbol{u}_{\max} = \left[u_{\max}^T \quad \ldots \quad u_{\max}^T\right]^T \\ &\boldsymbol{u}^{k-1} = \left[u(k-1)^T \quad \ldots \quad u(k-1)^T\right]^T \\ &\Delta\boldsymbol{u}_{\max} = \left[\Delta u_{\max}^T \quad \ldots \quad \Delta u_{\max}^T\right]^T \end{aligned} \tag{4}$$

are vectors of the length n_uN_u and

$$\begin{aligned} &\boldsymbol{y}_{\min} = \left[y_{\min}^T \quad \ldots \quad y_{\min}^T\right]^T, \quad \boldsymbol{y}_{\max} = \left[y_{\max}^T \quad \ldots \quad y_{\max}^T\right]^T \\ &\boldsymbol{y}(k) = \left[y(k+1 \mid k)^T \quad \ldots \quad y(k+N \mid k)^T\right]^T \\ &\boldsymbol{\varepsilon}_{\min} = \left[\varepsilon_{\min} \quad \ldots \quad \varepsilon_{\min}\right]^T \\ &\boldsymbol{\varepsilon}_{\max} = \left[\varepsilon_{\max} \quad \ldots \quad \varepsilon_{\max}\right]^T \end{aligned} \tag{5}$$

are vectors of the length n_yN and

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{0}_{n_u \times n_u} & \boldsymbol{0}_{n_u \times n_u} & \cdots & \boldsymbol{0}_{n_u \times n_u} \\ \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{0}_{n_u \times n_u} & \cdots & \boldsymbol{0}_{n_u \times n_u} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{I}_{n_u \times n_u} & \cdots & \boldsymbol{I}_{n_u \times n_u} \end{bmatrix} \tag{6}$$

is a matrix of dimension $n_uN_u \times n_uN_u$.

The solution of the nonlinear optimization problem (1) is in most cases time consuming. Thus, it is usually repeated much less often than the control derivation by the MPC algorithm. If the dynamics of disturbances is comparable to the dynamics of the control plant, such an approach can result in economic process performance degradation.

3. MULTILAYER STRUCTURE WITH LINEAR STEADY–STATE TARGET OPTIMIZATION

Since at the LSSO layer a comprehensive nonlinear steady–state model of the process is typically used, the economic optimization problem (1) is usually difficult to solve and time consuming, with constraints which significantly decrease the set of possible solutions. When dynamics of disturbances is comparable with the dynamics of the process, the economic performance is likely to be below expectations (Blevins *et al.*, 2003; Kassmann *et al.*, 2000; Qin and Badgwell, 2003; Tatjewski, 2007).

Since increasing the frequency of the LSSO layer is limited in practice because of its high computational burden, the MPC layer is often supplemented with an additional Steady–State Target Optimization (SSTO) layer, as it is shown in Fig. 2 (Blevins *et al.*, 2003; Kassmann *et al.*, 2000; Qin and Badgwell, 2003; Tatjewski, 2007). The SSTO closely co–operates with the MPC layer, the steady–state operating–point determined by the LSSO layer activated less often is recalculated as frequently as the MPC executes. Typically, in steady–state target optimization a linear model corresponding to the linear dynamic model used by the MPC algorithm is used

$$\min_{u^{ss}}\{J_E(k) = c_u^T \Delta u^{ss} - c_y^T \Delta y^{ss}\}$$

$$\begin{aligned} &\text{subject to:} \\ &u_{\min} \le u^{ss} \le u_{\max} \\ &y_{\min} \le y^{ss} \le y_{\max} \\ &\Delta y^{ss} = \boldsymbol{H} \Delta u^{ss} \\ &y^{ss} = F(u(k-1), \tilde{w}) + \Delta y^{ss} \\ &u^{ss} = u(k-1) + \Delta u^{ss} \end{aligned} \tag{7}$$

where $\boldsymbol{H}$ is the gains matrix corresponding to the linear model used in (2).

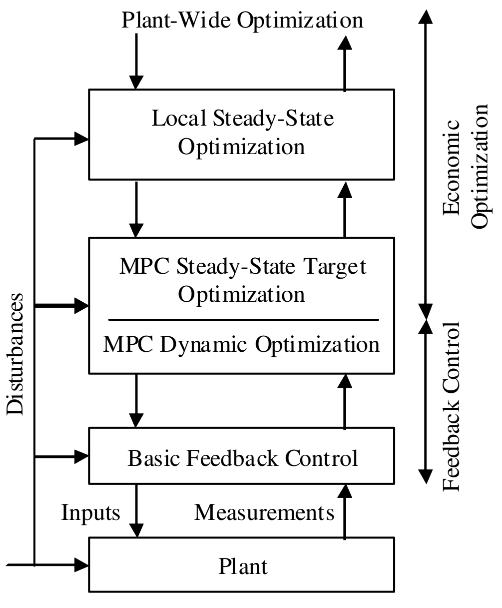

Fig. 2. Hierarchical control system structure with MPC advanced control layer and steady–state target calculation (LSSO+MPC+SSTO)

In order to increase the accuracy of the steady–state model used in the SSTO problem, the nonlinear steady–state model may be approximated on–line (Ławryńczuk *et al.*, 2007c; Ławryńczuk *et al.*, 2006; Tatjewski *et al.*, 2006). Instead of the constant model a linear updated on–line approximation

$$y^{ss} = F(u(k-1), \tilde{w}) + \boldsymbol{H}(k) \Delta u^{ss} \tag{8}$$

is employed, where the gain matrix $\boldsymbol{H}(k)$ of dimension $n_y \times n_u$ contains partial derivatives of the nonlinear function $y^{ss} = F(u(k-1), \tilde{w})$

$$\boldsymbol{H}(k) = \left[\frac{\partial F(u(k-1), \tilde{w})}{\partial u_1} \quad \cdots \quad \frac{\partial F(u(k-1), \tilde{w})}{\partial u_{n_u}} \right] \tag{9}$$

It is usually computed numerically using finite difference approach. Thus, the steady–state model used at the SSTO layer is consistent with the nonlinear model used at the LSSO layer rather than with the linear dynamic model used at the MPC layer as it is done in the standard SSTO approach with a constant linear model. The linearized steady–state model (8) is finally used to calculate the value $\hat{y}^{ss}$ corresponding to $\hat{u}^{ss}$ which is then passed to the MPC optimization problem.

4. MULTILAYER STRUCTURE WITH PIECEWISE LINEAR STEADY–STATE TARGET OPTIMIZATION

To approximate nonlinearity of the steady–state process model of constrained outputs which preserves a relatively simple and robust SSTO problem, a local piecewise linear approximation may be proposed. The approximation can be generally presented in a form

$$y^{ss} = \begin{cases} h_1 + \boldsymbol{H}^1(k)\Delta u^{ss} & \text{appr. within a subset } \Delta_1 \\ \vdots & \vdots \\ h_{n_s} + \boldsymbol{H}^{n_s}(k)\Delta u^{ss} & \text{appr. within a subset } \Delta_{n_s}. \end{cases} \tag{10}$$

where h_i are constant vectors, Δ_i are subsets of the domain under consideration in which the gain matrices $\boldsymbol{H}^i(k)$, $i=1,\ldots,n_s$ of dimension $n_y \times n_u$ contain local approximations of the nonlinear function $y^{ss} = F(u^{ss}, \tilde{w})$, n_s – number of regions with local linear approximations. Piecewise linear approximation of a nonlinear function is a well known concept in mathematical programming, especially in separable programming, leading to mixed LP (Linear Programming) problems, or LP problems if the applied LP procedure allows for declaration of SOS (Special Ordered Sets) for additional variables describing the piecewise linear approximations (Williams, 1995). The problem can also be solved (with certain suboptimality) by checking values of the objective function at selected grid points defining the piecewise linear approximation mesh. It can be done because solutions to LP problems are always located at vertices of the constraint set. Such a method was used during the experiments.

In details, the applied approach is simply to check values of the objective function and constrained outputs only at input–feasible grid points of the piecewise linear approximation, chosen locally around a current operating point, in this way selecting an economically best output–feasible one. If finer solution is needed, the search can be repeated with finer grid mesh around the found point (i.e., using finer local piecewise linear approximation), but it was found not necessary in our study and may not be reasonable in practice due to unavoidable uncertainty.

5. SIMULATION RESULTS

The process under consideration is an isothermal, constant–volume, continuous, stirred–tank reactor (Soroush *et al.*, 2005) shown in Fig. 3. The reactor has two feed streams (A and B). The stream A does not contain the reactant B, and the stream B does not include the reactant A. Mathematical model of the reactor in continuous–time domain is

$$\begin{aligned} \dot{C}_A &= -KC_AC_B + (C_{Ai} - C_A)\frac{q_A}{V} - C_A\frac{q_B}{V} \\ \dot{C}_B &= -KC_AC_B - C_B\frac{q_A}{V} + (C_{Bi} - C_B)\frac{q_B}{V} \end{aligned} \tag{11}$$

where C_A and C_B are concentrations of reactants A and B in the outlet stream, respectively; C_{Ai} is concentration of the reactant A in the feed stream A and C_{Bi} is concentration of the reactant B in the feed stream B; q_A and q_B are the volumetric flow rates of the feed streams, respectively; K is the reaction rate constant; and V is the volume of the reactor. Parameters of the model are: K=2.778×10^{-3} m^3 kmol^{-1} s^{-1}, nominal concentrations of reactants in the feed streams are C_{Ai}=1 kmol m^{-3}, C_{Bi}=1.2 kmol m^{-3}. It is assumed that a fast controller stabilizes the volume of the reactor.

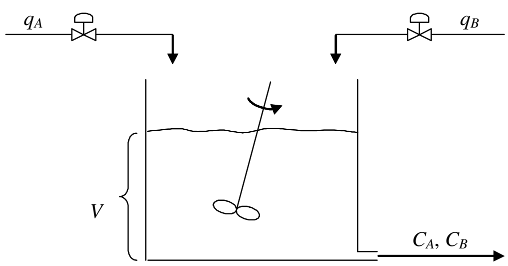

Fig. 3. Schematic diagram of the chemical reactor

The process has two input (manipulated) variables: $v_1=q_A/V$, $v_2=q_B/V$ and two output (controlled) variables: C_A and C_B. Manipulated variables are constrained as follows

$$0 \le v_1 \le v_1^{\max} \quad 0 \le v_2 \le v_2^{\max} \tag{12}$$

Where $v_1^{\max} = 1.9\times10^{-3}\ s^{-1}$, $v_2^{\max} = 6\times10^{-4}\ s^{-1}$. Nonlinear steady–state characteristics of the process are depicted in Fig. 4.

Since concentrations of the reactants A and B in the feed streams (C_{Ai} and C_{Bi}) can change, they are treated as disturbances. In this study it is assumed that C_{Bi} is equal to its nominal value. It is assumed that changes in the disturbance signal C_{Ai} are described by the equation

$$C_{Ai}(k) = C_{Ai0} - \alpha(\sin(\beta k) - \sin(\beta_0)) \tag{13}$$

where C_{Ai0} is the nominal value, α=0.45, β=0.015, β_0=0.045.

Maximum production rate is required, the following performance function in the economic optimization is used

$$J_E = -c_1 v_1^{ss} - c_2 v_2^{ss} \tag{14}$$

where c_1=c_2=1 are prices. Because of technological reasons, the lower bound on the composition of the first substance is imposed

$$C_A \ge 0.5\times10^{-3}\ \text{kmol m}^{-3} \tag{15}$$

Although the economic performance function (14) is linear, the constraint (15) is nonlinear. As a result, one obtains the nonlinear economic optimization problem.

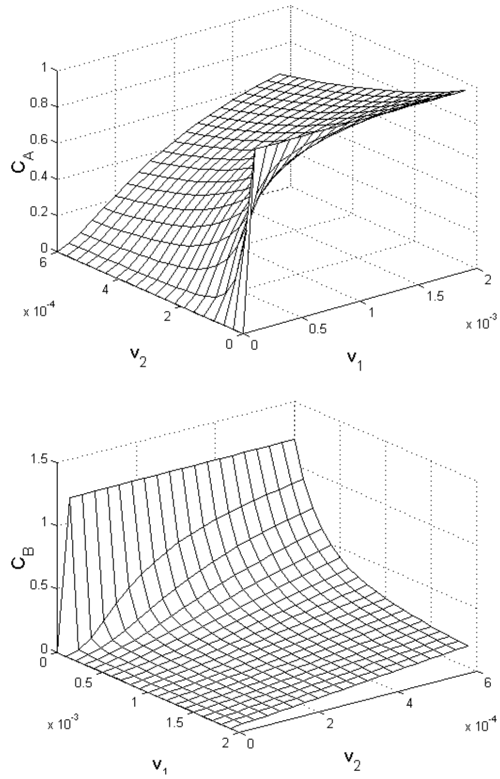

Fig. 4. Steady–state characteristics of the chemical reactor

GPC type MPC algorithm is developed. Parameters of this algorithm are: N=10, N_u=3, $\boldsymbol{M}$=[1 1] and $\boldsymbol{\Lambda}$=[1000 1000], the sampling period is 10 seconds.

At first the classical multilayer control structure with nonlinear economic optimization activated at each sampling instant was tested. Then it is used as the reference for the other structures. Next, the structure with steady–state target optimization based on steady–state model linearized on–line is used. Simulation results of these two structures are depicted in Fig. 5. It is clearly seen that starting from the sampling instant k=36 the linearization–based approach produces totally wrong operating points – the constraint (15) put on concentration C_A is violated due to inaccuracies of linearized process model. Such behavior of the system is caused by the shape of the steady–state characteristics and the objective function. As the result, the linear steady–state target optimization is unable to find the optimal point.

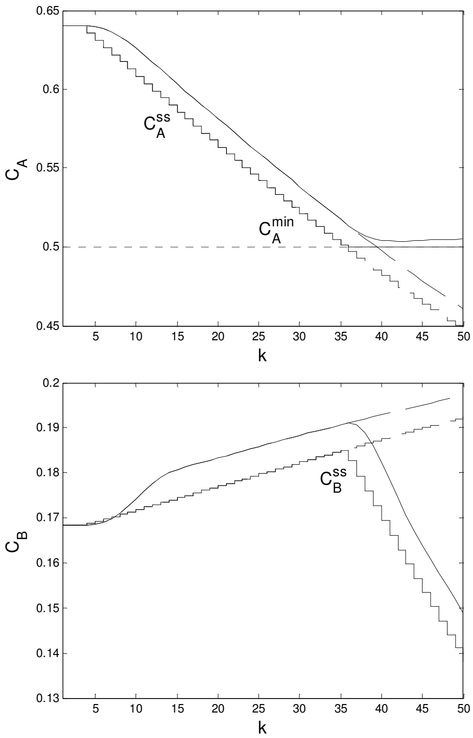

Fig. 5. Simulation results of the multilayer control structure with: nonlinear economic optimization activated at each sampling instant (solid line) and with steady–state target optimization based on steady–state model linearized on–line (dashed line); C_A^{ss}, C_B^{ss} – optimal steady–state set–points

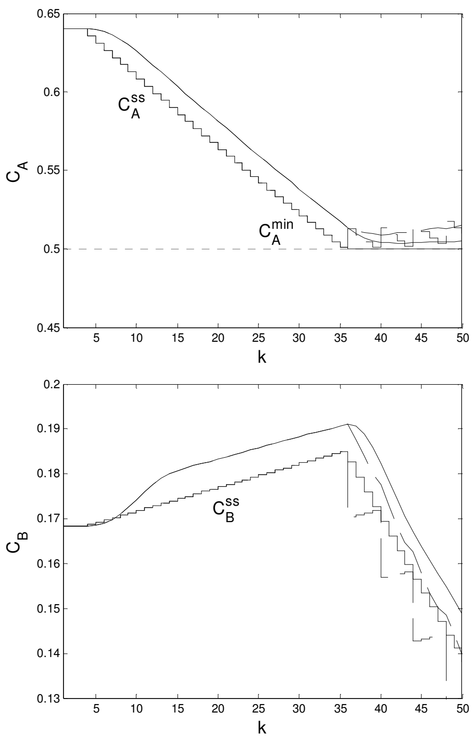

Fig. 6. Simulation results of the multilayer control structure with: nonlinear economic optimization activated at each sampling instant (solid line) and with *low accuracy* piecewise linear steady–state target optimization (dashed line)

In order to improve the operation of the control system, piecewise linear steady–state target optimization is used. Fig. 6 depicts responses obtained when low accuracy piecewise linear approximation is used, i.e. the number of grid points is 9. They are evenly located in the neighborhood of the optimal operating point obtained at the previous sampling instant. The neighborhood is as small as 10% of $v_1^{\max}$ and 16% of $v_2^{\max}$. Unlike the steady–state target optimization based on a linearized model, this approach gives good results because the constraint (15) is satisfied.

To further improve operation of the system the number of grid points is increased to 25 with the same area covered. Fig. 7 presents responses obtained. Naturally, the more accurate the approximation, the closer the obtained responses to the reference ones obtained in the multilayer control structure with nonlinear economic optimization activated at each sampling instant.

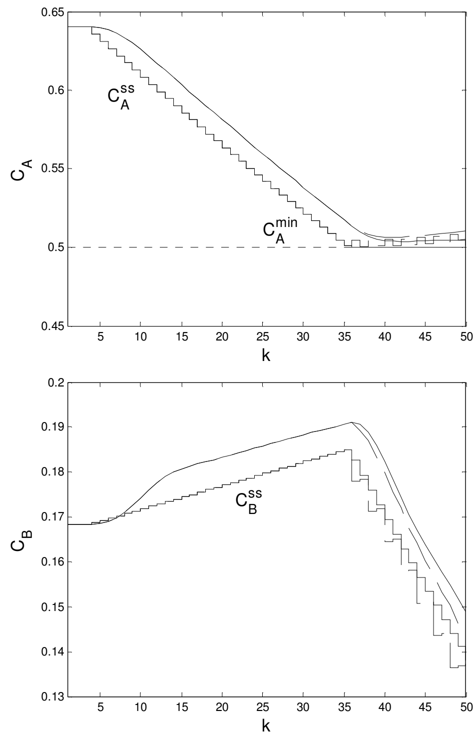

Fig. 7. Simulation results of the multilayer control structure with: nonlinear economic optimization activated at each sampling instant (solid line) and with *high accuracy* piecewise linear steady–state target optimization (dashed line)

6. CONCLUSIONS

Cooperation of Model Predictive Control (MPC) algorithms with nonlinear steady–state economic optimization is important when dynamics of disturbances is comparable with dynamics of the process. A reliable, comprehensive steady–state model of the process is usually complex and nonlinear, leading to a difficult economic optimization problem. Thus, it is very rarely possible in practice to repeat the nonlinear LSSO at each sampling instant of the MPC controller.

Therefore, in practice usually an additional MPC target set–point optimization is performed. The solutions proposed in the literature were based on linear models. However in the case of highly nonlinear plants (like the one considered in the paper) such an approach generates erroneous set–points.

The solution to the problem can be the piecewise linear target set–point optimization. The discussed approach was illustrated on a chemical reactor example. Because of the shape of the steady–state characteristics and the objective function the SSTO based on a linear model was unable to find the optimal operating point. Finally, the piecewise linear SSTO problem was formulated which could be solved by a linear programming routine, but a proposed simplified procedure was applied which calculates values of the economic objective function and constrained outputs for a chosen set of input–feasible grid points only, at each sampling instant. The used approach is computationally effective and the calculated steady–states can be close to the optimal ones, i.e. to those calculated by the LSSO using a comprehensive nonlinear model of the plant. Moreover, the accuracy of the solution can be easily improved by making the grid of points to be searched denser. Thus, the demanded compromise between quality of control and computational demand can be obtained.

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