Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Design of Observers for Takagi-Sugeno Systems with Immeasurable Premise Variables: an $\mathcal{L}_2$ Approach

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Abstract: A new observer design method is proposed for Takagi-Sugeno systems with immeasurable premise variables. Since the state estimation error can be written as a perturbed system, then the proposed method is based on the $\mathcal{L}_2$ techniques to minimize the effect of the perturbations on the state estimation error. The convergence conditions of the observer are established by using the second method of Lyapunov and a quadratic function. These conditions are expressed in terms of Linear Matrix Inequalities (LMI). Finally, the performances of the proposed observer are improved by eigenvalues clustering in LMI region.

Keywords: Multiple model approach; nonlinear observer; immeasurable premise variables; $\mathcal{L}_2$ optimization; eigenvalues assignment; linear matrix inequality.

1. INTRODUCTION

The problem of nonlinear state estimation is a very vast field of research, having many applications, among them one can cite the use of the observers to estimate the immeasurable states of a system or to replace sensors which are expensive and difficult to maintain; these observers are also used for the state feedback control or for system diagnosis.

The diagnosis methods of linear systems currently have a certain maturity, however assuming that the system to supervise can be correctly represented by a linear system is highly restrictive. Moreover, the direct extension of the methods developed in the linear case, to the nonlinear case is delicate. Nevertheless, interesting results have been obtained if the nonlinear systems are represented by a multiple model. This structure consists in a set of local linear models, each local model describing the behavior of the system in a particular region of the state-space.

In the context of the linear models, fault detection can be carried out by methods using state observers (Maquin and Ragot, 2000) and residual generation. In general, fault isolation methods use banks of observers where each observer is driven by a subset of the inputs. The preceding technique cannot be immediately extended to the multiple model because of the couplings introduced into the structure. Generally, the design of an observer for a multiple model begins with the design of local observers, then a weighted interpolation is performed to obtain the estimated state. This design allows the extension of the analysis and synthesis tools developed for the linear systems, to the nonlinear systems.

(Tanaka *et al.*, 1998) proposed a study concerning the stability and the synthesis of regulators and observers for multiple models. In (Chadli *et al.*, 2002), (Tanaka *et al.*, 1998) and (Guerra *et al.*, 2006) tools directly inspired of the study of the linear systems are adapted for the stability study and stabilization of nonlinear systems. The authors of (Patton *et al.*, 1998) proposed a multiple observer based on the use of Luenberger observers, which was then used for the diagnosis. In (Akhenak, 2004) and (Akhenak *et al.*, 2007) sliding mode observers developed for the linear systems, were transposed to the systems described by multiple model. The principal interest of this type of observers is the robustness with respect to modeling uncertainties. Moreover, the unknown input observers designed for linear systems, were transposed, in the same way, into the case of nonlinear systems and application to fault diagnosis is envisaged in (Marx *et al.*, 2007).

However, in all these works, the authors supposed that the weighting functions depend on measurable premise variables. In the field of diagnosis, this assumption forces to design observers with weighting functions depending on the input $u(t)$, for the detection of the sensors faults, and on the output $y(t)$, for the detection of actuator faults. Indeed, if the decision variables are the inputs, for example in a bank of observers, even if the ith observer is not controlled by the input u_i, this input appears indirectly in the weighting function and it cannot be eliminated. For this reason, it is interesting to consider the case of weighting functions depending on immeasurable premise variables, like the state of the system. This assumption makes it possible to represent a large class of nonlinear systems. Only few works are based on this approach, nevertheless, one can cite (Bergsten and Palm, 2000), (Palm and Driankov, 1999), (Bergsten *et al.*, 2001) and (Bergsten *et al.*, 2002), in which a Luenberger observer is proposed, by using Lipschitz weighting functions. The stability conditions of the observer are formulated in terms of linear matrix inequalities (LMI) (Boyd *et al.*, 1994). Unfortunately, the existence condition of the solution for the obtained set of LMI depends on the magnitude of the Lipschitz constants. In (Palm and Bergsten, 2000) and (Bergsten and Palm, 2000), the sliding mode observer compensates the unknown terms of the system.

In this paper, observer error dynamics is written as a perturbed system. So, with the use of $\mathcal{L}_2$ design (which is an extension of the H_∞ design to nonlinear systems), the influence of the immeasurable terms on the state estimation error is minimized. According to this objective, we propose a new observer design for multiple model with immeasurable premise variables. The observer synthesis is carried out using the second method of Lyapunov with a quadratic function and $\mathcal{L}_2$ optimization. The paper is organized as follows: section 2 introduces some previous results about state estimation of multiple model with immeasurable premise variables. In section 3, the proposed observer is presented, convergence conditions of the proposed multiple observer are established. A design procedure to satisfy pole clustering constraints is also given. Simulation results are presented in section 4 and some conclusions and perspectives are given in section 5.

2. BACKGROUND RESULTS AND NOTATION

In this section, we summarize some results on observer design for Takagi-Sugeno systems of the form:

$$\dot{x}(t) = \sum_{i=1}^{N} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input of the system, $y(t) \in \mathbb{R}^p$ is the output of the system. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are real known constant matrices. The weighting functions μ_i depend on immeasurable premise variables (state of the system), and verify:

$$\begin{cases} \sum_{i=1}^{N} \mu_i(x(t)) = 1 \\ 0 \leqslant \mu_i(x(t)) \leqslant 1 \;\; \forall i \in \{1, ..., N\} \end{cases} \tag{3}$$

Few works can be found concerning this class of systems with the assumption of immeasurable premise variables. (Bergsten and Palm, 2000), propose a Luenberger-like observer, namely:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{N} \mu_i(\hat{x}(t))(A_i \hat{x}(t) + B_i u(t) + G_i(y(t) - \hat{y}(t))) \tag{4}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{5}$$

The observer error is given by:

$$e(t) = x(t) - \hat{x}(t) \tag{6}$$

and its dynamics is described by:

$$\dot{e} = \sum_{i=1}^{N} \mu_i(\hat{x})(A_i - G_i C)e + \Delta(x, \hat{x}, u) \tag{7}$$

with:

$$\Delta(x, \hat{x}, u) = \sum_{i=1}^{N} (\mu_i(x) - \mu_i(\hat{x}))(A_i x + B_i u) \tag{8}$$

where (8) satisfies the following condition:

$$\|\Delta(x, \hat{x}, u)\| \leqslant \alpha \|x - \hat{x}\| \tag{9}$$

Lemma 1. (Bergsten and Palm, 2000) The state estimation error between the multiple model (1) and the multiple observer (4) converges globally asymptotically toward zero, if there exists matrices $P = P^T > 0$ and $Q = Q^T > 0$ such that the following conditions hold for $i = 1, \ldots, N$:

$$A_i^T P + PA_i - C^T K_i^T - K_i C < -Q \tag{10}$$

$$\begin{bmatrix} -Q + \alpha^2 I & P \\ P & -I \end{bmatrix} < 0 \tag{11}$$

The observer gains are given by $G_i = P^{-1} K_i$.

Lemma 1 recalls the design of the Thau-Luenberger observer introduced in (Bergsten and Palm, 2000). Unfortunately, the considered perturbed term depend on the input $u(t)$ and the state $x(t)$, so for a large value of the bound of the input lead to a large value of the constant α, solving the set of LMI (10-11) may be unfeasible. Another method for state estimation of the system (1) is proposed in (Ichalal *et al.*, 2007). The contribution of that paper is to obtain less restrictive existence conditions for the observer. In this approach, the matrices A_i are decomposed into:

$$A_i = A_0 + \overline{A}_i \tag{12}$$

where A_0 is defined by:

$$A_0 = \frac{1}{N} \sum_{i=1}^{N} A_i \tag{13}$$

By substituting (12) in the equation of the multiple model (1) we obtain:

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{N} \mu_i(x(t))(\overline{A}_i x(t) + B_i u(t)) \tag{14}$$

$$y(t) = Cx(t) \tag{15}$$

Based on this model, the following multiple observer is proposed:

$$\dot{\hat{x}}(t) = A_0 \hat{x}(t) + \sum_{i=1}^{N} \mu_i(\hat{x}(t))(\overline{A}_i \hat{x}(t) + B_i u(t) + G_i(y(t) - \hat{y}(t))) \tag{16}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{17}$$

Lemma 2. (Ichalal *et al.*, 2007) The state estimation error between the multiple model (1) and the multiple observer (16) converges globally asymptotically toward zero, if there exists matrices $P = P^T > 0$, $Q = Q^T > 0$ and positive scalars λ_1, λ_2 and γ such that the following conditions hold for $i = 1, \ldots, N$:

$$A_0^T P + PA_0 - K_i^T P - PK_i < -Q \quad (18)$$

$$\begin{bmatrix} -Q + \lambda_1 M_i^2 I & P\overline{A}_i & PB_i & N_i\gamma I \\ \overline{A}_i^T P & -\lambda_1 I & 0 & 0 \\ B_i^T P & 0 & -\lambda_2 & 0 \\ N_i\gamma I & 0 & 0 & -\lambda_2 I \end{bmatrix} < 0 \quad (19)$$

$$\gamma - \beta_1\lambda_2 > 0 \quad (20)$$

where β_1 is the bound on the input $u(t)$ and N_i are the Lipschitz constants of the weighting functions $\mu_i(x)$. The gains of the observer are computed by $G_i = P^{-1}K_i$.

The conditions expressed in the lemma 2 are less restrictive than that of lemma 1, i.e, the set of LMI (18-20) admits a solution even for great values of the constant α and of the bound on the input β_1. The drawback of this method is that, if the bound β_1 increases, then the band-width of the observer increases and thus the observer also reconstructs the measurement noise.

The contribution of this paper is to obtain a minimal influence of the unknown premise variables on the estimation quality, and moreover to satisfy pole clustering in prescribed regions of the complex plane. In order to quantify the influence of an input signal on the output of a system, the $\mathcal{L}_2$-norm of a system, based on the $\mathcal{L}_2$-norm of a signal, is introduced.

Definition ($\mathcal{L}_2$-norm) The $\mathcal{L}_2$-norm of a signal $z(t)$, denoted $\|z(t)\|_2$ is defined by

$$\|z(t)\|_2^2 = \int_0^\infty z^T(t)z(t)dt \quad (21)$$

It is supposed that all the signals studied in this paper are measurable functions (or square integrable) that is to say: of finite energy. The space of measurable functions is denoted $\mathbb{L}_2$.

Definition ($\mathcal{L}_2$-gain) Consider a system with input $u(t) \in \mathbb{L}_2$ and output $y(t) \in \mathbb{L}_2$. The $\mathcal{L}_2$-gain of the system is defined by:

$$\gamma = \sup_{u(t)\in\mathbb{L}_2} \frac{\|y(t)\|_2}{\|u(t)\|_2} \quad (22)$$

It is well known that the $\mathcal{L}_2$-norm is a extension to the nonlinear systems of the H_∞-norm of the linear systems (for a linear system $G(s)$, the $\mathcal{L}_2$-norm and the H_∞-norm defined by $\|G(s)\|_\infty = \sup_{\omega\in\mathbb{R}} \sigma_{max}(G(j\omega))$, where σ_{max} denotes the maximal singular value, are equal).

3. MAIN RESULT

Consider the structure of a multiple model presented in (14) and the following Luenberger observer is proposed:

$$\dot{\hat{x}}(t) = A_0\hat{x}(t) + \sum_{i=1}^{N} \mu_i(\hat{x}(t))(\overline{A}_i\hat{x}(t) + B_iu(t) + G_i(y(t) - \hat{y}(t))) \quad (23)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (24)$$

Our aim is to find the observer gains G_i which minimize the influence of the perturbation terms on the estimation error.

The observer error dynamics is given by:

$$\dot{e} = \sum_{i=1}^{N} \left(\mu_i(\hat{x})\Phi_i e + \overline{A}_i\delta_i + \Delta_i B_i u\right) \quad (25)$$

where:

$$\begin{cases} \delta_i(t) = \mu_i(x(t))x(t) - \mu_i(\hat{x}(t))\hat{x}(t) \\ \Delta_i(t) = \mu_i(x(t)) - \mu_i(\hat{x}(t)) \\ \Phi_i = A_0 - G_iC \end{cases} \quad (26)$$

This error dynamics can be written as:

$$\dot{e}(t) = \sum_{i=1}^{N} (\mu_i(\hat{x}(t))\Phi_i e(t) + H_i\omega(t)) \quad (27)$$

where:

$$H_i = \begin{bmatrix} \overline{A}_i & B_i \end{bmatrix}$$

and:

$$\omega(t) = \begin{bmatrix} \delta_i(t) \\ \Delta_i(t)u(t) \end{bmatrix}$$

In the remaining, the proposed observer is said to be optimal, if the $\mathcal{L}_2$-gain from $\omega(t)$ to $e(t)$ is minimal.

3.1 Observer design

Theorem 3. The optimal observer (23)-(24) for the system (14) is obtained by minimizing $\tilde{\gamma} > 0$ under the constraints

$$P = P^T > 0$$

$$\begin{bmatrix} \dfrac{S_i}{N} & PH_j \\ H_j^T P & -\dfrac{\tilde{\gamma}}{N} \end{bmatrix} < 0, \quad \forall i, j = 1, ..., N$$

where

$$S_i = A_0^T P + PA_0 - K_iC - C^T K_i^T + I$$

The observer gains are given by $G_i = P^{-1}K_i$ and the $\mathcal{L}_2$-gain from $\omega(t)$ to $e(t)$ is $\gamma = \sqrt{\tilde{\gamma}}$.

Proof. To prove the convergence of the estimation error toward zero, let us consider the following quadratic Lyapunov function:

$$V(t) = e(t)^T Pe(t), P = P^T > 0 \quad (28)$$

The observer converges and the $\mathcal{L}_2$-gain from $\omega(t)$ to e(t) is bounded by γ if the following holds:

$$\dot{V}(t) + e(t)^T e(t) - \gamma^2\omega(t)^T\omega(t) < 0 \quad (29)$$

Then, using (27), it follows:

$$\dot{V}(t) = \sum_{i=1}^{N} (\omega^T H_i^T Pe + e^T PH_i\omega + \mu_i(\hat{x})(e^T\Phi_i^T Pe + e^T P\Phi_i e)) \quad (30)$$

Inequality (29) can then be written in the following way:

$$\sum_{i=1}^{N}(\omega^T H_i^T Pe + e^T PH_i\omega + \mu_i(\hat{x})(e^T\Phi_i^T Pe + e^T P\Phi_i e)) \\ +e^T e - \gamma^2\omega^T\omega < 0 \quad (31)$$

That can be expressed under the following form:

$$\sum_{i=1}^{N}\mu_i(\hat{x})\begin{bmatrix} e \\ \omega \end{bmatrix}^T \begin{bmatrix} P\Phi_i + \Phi_i^T P + I & 0 \\ 0 & -\gamma^2 \end{bmatrix}\begin{bmatrix} e \\ \omega \end{bmatrix} \\ +\sum_{j=1}^{N}\begin{bmatrix} e \\ \omega \end{bmatrix}^T \begin{bmatrix} 0 & PH_j \\ H_j^T P & 0 \end{bmatrix}\begin{bmatrix} e \\ \omega \end{bmatrix} < 0 \quad (32)$$

Using the properties (3) of the weighing functions, it follows:

$$\sum_{i=1}^{N}\mu_i(\hat{x})\begin{bmatrix} e \\ \omega \end{bmatrix}^T \begin{bmatrix} P\Phi_i + \Phi_i^T P + I & 0 \\ 0 & -\gamma^2 \end{bmatrix}\begin{bmatrix} e \\ \omega \end{bmatrix} \\ +\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\hat{x})\begin{bmatrix} e \\ \omega \end{bmatrix}^T \begin{bmatrix} 0 & PH_j \\ H_j^T P & 0 \end{bmatrix}\begin{bmatrix} e \\ \omega \end{bmatrix} < 0 \quad (33)$$

or, in a compact form:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\hat{x})\begin{bmatrix} e \\ \omega \end{bmatrix}^T \mathcal{M}_{ij}\begin{bmatrix} e \\ \omega \end{bmatrix} < 0 \quad (34)$$

where:

$$\mathcal{M}_{ij} = \begin{bmatrix} \frac{1}{N}(P\Phi_i + \Phi_i^T P + I) & PH_j \\ H_j^T P & -\frac{\gamma^2}{N} \end{bmatrix} \quad (35)$$

Thus, the negativity of (34) is assured if:

$$\mathcal{M}_{ij} < 0, \ \forall i, j = 1, ..., N \quad (36)$$

Inequalities (36) are not linear because of the products PG_i and γ^2. This problem can be solved by using the change of variables $K_i = PG_i$ and $\tilde{\gamma} = \gamma^2$. After the minimization of $\tilde{\gamma}$ under the constraint (36), the observer gains are computed by $G_i = P^{-1}K_i$ and the $\mathcal{L}_2$-gain from $\omega(t)$ to $e(t)$ is computed by $\gamma = \sqrt{\tilde{\gamma}}$. □

3.2 Eigenvalue assignment

From the results obtained in simulation one notes that, if the value of γ decreases, the eigenvalues of the matrices $(A_0 - G_iC)$ increase in absolute value which is not a desirable effect. However, eigenvalue assignment makes it possible to solve this problem. It is proposed to assign the eigenvalues of the multiple observer in particular regions. In this section, we propose an extension of the previous method of synthesis by placing the eigenvalues of the observer in LMI region S (Fig. 1) defined by:

$$S(\alpha, \beta) = \{ \ z \in C | \ \mathrm{Re}(z) < -\alpha, \ |z| \ < \beta\} \quad (37)$$

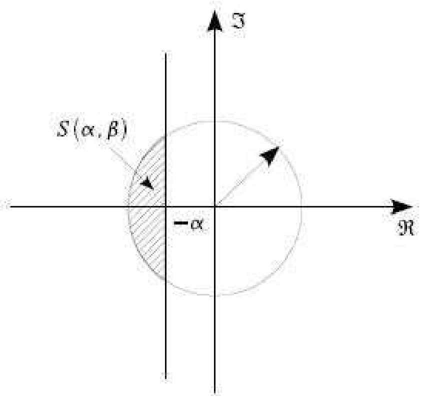

Fig. 1. LMI region

Theorem 4. The optimal observer (23)-(24) for the multiple model (14), satisfying the pole clustering in $S(\alpha, \beta)$ (37), is obtained by minimizing $\tilde{\gamma} > 0$ under the following constraints:

$$P = P^T > 0$$

$$\begin{bmatrix} \beta P & P(A_0 - G_iC) \\ (A_0 - G_iC)^T P & \beta P \end{bmatrix} > 0 \quad (38)$$

$$A_0^T P + PA_0 - C^T K_i^T - K_iC + 2\alpha P < 0 \quad (39)$$

$$\begin{bmatrix} \frac{S_i}{N} & PH_j \\ H_j^T P & -\frac{\tilde{\gamma}}{N} \end{bmatrix} < 0, \ \forall i, j = 1, ..., N \quad (40)$$

where:

$$S_i = PA_0 + A_0^T P - K_iC - C^T K_i + I$$

The observer gains are given by $G_i = P^{-1}K_i$, and the $\mathcal{L}_2$-gain is $\gamma = \sqrt{\tilde{\gamma}}$.

Proof. Using the concept of $\mathcal{D}$-stability presented in (Chilali and Gahinet, 1996) and (Bong-Jae and Sangchul, 2006), the constraints allowing to assign the eigenvalues of the matrix $(A_0 - G_iC)$ in S (Fig.1) can be expressed in terms of LMIs as:

$$\begin{bmatrix} \beta P & P(A_0 - G_iC) \\ (A_0 - G_iC)^T P & \beta P \end{bmatrix} > 0 \quad (41)$$

$$(A_0 - G_iC)^T P + P(A_0 - G_iC) + 2\alpha P < 0 \quad (42)$$

The constraint (40) has been demonstrated in the previous section. □

4. SIMULATION RESULTS

We consider the following example to show the advantages of using the proposed $\mathcal{L}_2$ observer. The system is defined by (1) with:

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -6 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 1 \\ 0.25 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The weighting functions are

$$\begin{cases} \mu_1(x) = \dfrac{1 - \tanh(x_1)}{2} \\ \mu_2(x) = 1 - \mu_1(x) = \dfrac{1 + \tanh(x_1)}{2} \end{cases} \quad (43)$$

A stable observer with $\mathcal{L}_2$ attenuation of the perturbation

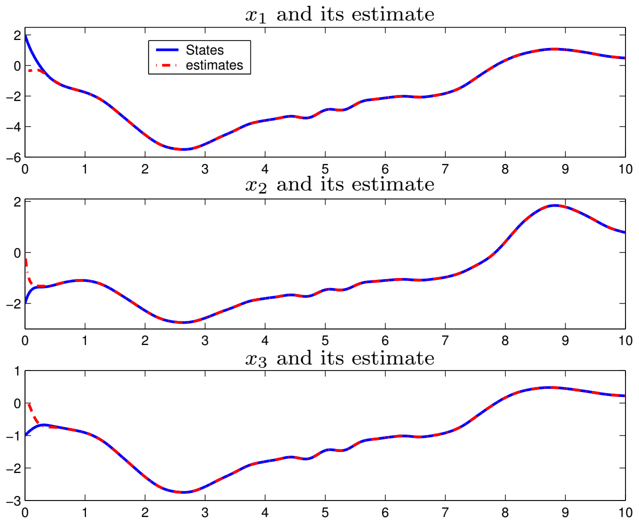

Fig. 2. State estimation

terms for the above system can be designed using Theorem 4. For this example, the minimal value of the attenuation of the perturbation terms is $\gamma = 0.46$. The eigenvalues are clustered in the region $S(\alpha, \beta)$ defined by $\beta = 15$ and $\alpha = 5$. Conditions in Theorem 4 are satisfied with:

$$P = \begin{bmatrix} 0.10 & 0.04 & 0.12 \\ 0.04 & 0.18 & 0.15 \\ 0.12 & 0.15 & 0.40 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 9.04 & 5.08 \\ 10.24 & -7.58 \\ -5.60 & 1.63 \end{bmatrix}, G_2 = \begin{bmatrix} 8.41 & 5.68 \\ 10.87 & -8.06 \\ -5.30 & 0.73 \end{bmatrix}$$

Given the initial conditions $x(0) = [2 \quad -2 \quad -1]^T$ and $\hat{x}(0) = [0 \quad 0 \quad 0]^T$, the simulation results are illustrated in (Fig.2).

The advantages of this method compared to those using lemma 1 and 2 are, on the one hand, the elimination of the Lipschitz assumption (9) of the weighting functions, needed by the previous method, and on the other hand, the method described in this paper does not require the knowledge of the input bound of the system like in lemma 2. As a result this method can be employed with a wider class of nonlinear systems.

5. CONCLUSION

In this paper, a new method is proposed to design an observer for the Takagi-Sugeno systems with immeasurable premise variables. The structure of the observer is inspired by the linear Luenberger observer. Estimation error is written like a perturbed system and conditions for convergence of the observer are studied by using a quadratic Lyapunov candidate function and $\mathcal{L}_2$ design to attenuate the effect of this perturbation on the state estimation error. These conditions are expressed in LMI terms. This method makes it possible to synthesize an observer for Takagi-Sugeno systems without Lipschitz weighing functions, and the knowledge of the input bound of the system is not required to find the gains of the observer. Until in previous works on this subject.

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