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Robust Adaptive Control of Hard Disk Drives with Hysteresis Friction Nonlinearity in Mobile Applications ⋆

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Abstract: In this paper, robust adaptive NN control is investigated for small form factor hard disk drives which is mainly used in mobile applications. The hysteresis friction nonlinearity from the pivot bearing degrades the servo performance, as hard disk drive servo system operates in track-following mode. To deal with the effect of hysteresis friction nonlinearity, adaptive hysteresis friction compensation with RBFNN approximation is proposed. The effectiveness of the proposed adaptive control compared with the conventional proportional-integral-derivative (PID) control will be shown through comprehensive simulation results.

Keywords: Neural networks(NNs); hard disk drives; hysteresis; friction.

1. INTRODUCTION

Today's information storage industries are moving to the manufacturing of micro hard disk drives, which will be used in small hand-held devices such as portable computers, digital cameras, video cameras, MP3 players, car navigation and audio systems, even mobile phones in the near future. As the disk drives are smaller, the track placements become denser. A more precise head positioning accuracy is required due to the growth of areal density. The servo loop which is aimed to meet an error budget at sub-nanometer scales becomes the most significant effect in small form factor hard disk drives.

The existence of pivot bearing hysteresis friction and external disturbances in the voice-coil-motor (VCM) actuator results in large residual errors and high-frequency oscillations. As a consequence, the position error signal will be increased and the performance of hard disk drive servo systems will be decreased. In the worst case, the data may be lost due to damage of drives. These factors attract us to study and compensate for the nonlinear effects of pivot bearing hysteresis friction in the VCM actuator. Several models have been proposed to present the behavior of pivot nonlinearities in both time and frequency domain (Abramovitch *et al.*, 1994), (Wang *et al.*, 1994).

Cancelation of pivot hysteresis nonlinearities in HDDs have been presented by researchers using different kinds of control methods. For instance, acceleration feedforward control was investigated, (Ishikawa and Tomizuka, 1998) using friction observer with Kalman filter to estimate the difference between scaled VCM current and angular acceleration. In (Yan and Lin, 2003), a disturbance observer was also designed based on VCM current and the arm acceleration feedback to recover the gain loss at low frequency range. A nonlinear compensator, which behaves like a double integrator has been developed in (Gong *et al.*, 2002) to cancel the nonlinear effects.

Moreover, an integrated test apparatus for pivot bearing analysis was designed in (Liu and Liu, 1999), where a laser doppler vibrometer (LDV) was used to measure accurately the displacement and velocity of actuator for feedback control. In (Peng *et al.*, 2005), an enhanced composite nonlinear feedback (CNF) control technique has been proposed to improve tracking performance of the VCM actuator. As an attractive alternative solution, (Ge *et al.*, 2007) has designed a novel disturbance observer based on a series of integral filters to estimate unknown arbitrarily fast time-varying external disturbances with the exponential accuracy. In addition, observer-based adaptive friction compensation was adopted in (Wit and Ge, 1997), to achieve the asymptotic stability of the systems which have generalized by position/velocity static characteristics.

In contract to prior works, non-model based neural network (NN) controller was implemented in HDD-servo system (Herrmann *et al.*, 2005), in which notch filter was augmented for resonance cancelation while robust parameter estimation technique was employed to counteract parameter estimation errors. It has been proven that neural networks are the suitable candidature for friction modeling and adaptive control design for friction compensation (Ge *et al.*, 2001). To improve the control performance of piezo-positioning mechanism, an adaptive wavelet neural network (AWNN) control with hysteresis estimation has been studied (Lin *et al.*, 2006).

In this paper, adaptive neural network control is proposed based on the LuGre friction model (Wit *et al.*, 1995), which can capture all the static and dynamic characteristics of

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hystereis friction nonlinearity and provide the capability of accurate modeling both in sliding and pre-sliding regimes.

The organization of this paper is as follows. The problem formulation and preliminary knowledge including some notations, assumptions and RBFNN, which will be used in the later adaptive neural control design, are described in Section 2. The control design methodology using neural networks and the stability analysis through Lyapunov synthesis are developed in Section 3. In Section 4, the proposed method is evaluated through extensive simulation studies before conclusion is drawn in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Dynamics of Hard Disk Drive System

Consider the following dynamics of hard disk drive (HDD) system :

$$m\ddot{x} + h(x, \dot{x}) + d(t) = u \tag{1}$$

where m is the system inertial mass; x, $\dot{x}$ and $\ddot{x}$ are the position, velocity and acceleration of VCM actuator tip respectively; $d(t)$ is the external disturbance; u is the control input; $h(x, \dot{x})$ is the bearing frictional hysteresis of actuator pivot.

The pivot hysteresis friction model $h(x, \dot{x})$ is given by:

$$h = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x}$$
$$\dot{z} = \dot{x} - \alpha(\dot{x})|\dot{x}|z$$

where h is the hysteresis friction force; z denotes the average deflection of the bristles, which is not measurable; σ_0, σ_1 and σ_2 are the hysteresis friction force parameters that can be physically explained as the stiffness of bristles, damping coefficient, and viscous coefficient; and the nonlinear function $\alpha(\dot{x})$ is a finite positive function which can be chosen to describe different hysteresis friction effects. $\alpha(\dot{x})$ is used to characterize the Stribeck effect which is given in (Wit *et al.*, 1995) as follows:

$$\alpha(\dot{x}) = \frac{\sigma_0}{f_c + (f_s - f_c)e^{-(\dot{x}/\dot{x}_s)^2}} \tag{2}$$

where f_c is the Coulomb friction level, f_s is the level of the stiction force and $\dot{x}_s$ is the constant Stribeck velocity.

Remark 1. According to (Ge *et al.*, 2000), there are no terms which explicitly account for position dependence of the hysteresis friction force in the above model. However, there may exist some applications where the function $\alpha(\cdot)$ in the LuGre model also depends on the actual position, or on a more complex combination of position and velocity. Therefore, we assume that $\alpha(x, \dot{x})$ is an upper and lower bounded positive smooth function of x and $\dot{x}$, and consider the LuGre model in the following form:

$$h = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x} \tag{3}$$
$$\dot{z} = \dot{x} - \alpha(x, \dot{x})|\dot{x}|z \tag{4}$$

To facilitate control design later in Section 3, we need the following assumptions.

Assumption 1. There exist positive constants α_{min} and α_{max} such that $0 < \alpha_{min} \le \alpha(x, \dot{x}) \le \alpha_{max}$, $\forall (x, \dot{x}) \in R^2$.

Assumption 2. States x and $\dot{x}$ are measurable for feedback controller design.

Assumption 3. The desired trajectories, x_d, and its first and second derivatives, $\dot{x}_d$ and $\ddot{x}_d$, are bounded and continuous signals.

Assumption 4. The external disturbance $d(t)$ satisfies the following conditon:

$$|d(t)| \le d^*$$

where d^* is an unknown positive constant.

The following lemma will be used in our design and analysis.

Lemma 1. (Wit *et al.*, 1995) Noting Assumption 1, if $|z(0)| \le 1/\alpha_{min}$ then $|z(t)| \le 1/\alpha_{min}$, $\forall t \ge 0$.

The control objective is to design a robust adaptive controller u for system (1) such that the output x follows the specified desired trajectory x_d.

2.2 Gaussian RBF Neural Network

In control engineering, radial basis function neural network (RBFNN) has been successfully used as a linearly parameterized function approximator to solve different problems due to its good capabilities (Ge *et al.*, 2002). In this paper, the following RBFNN is used to approximate the continuous function $h(z) : R^q \to R$,

$$h_{nn}(z, W) = W^T S(z) \tag{5}$$

where the input vector is $z \in \Omega \subset R^q$, weight vector is denoted as $W = [w_1, w_2, ..., w_l]^T \in R^l$ with the NNs node number $l > 1$; and $S(z) = [s_1(z), ..., s_l(z)]^T$ is defined as the basic function vector with $s_i(z)$ which has been expressed in the form of Gaussian functions as follow:

$$s_i(z) = \exp\left[\frac{-(z - \mu_i)^T (z - \mu_i)}{\eta^2}\right], \ i = 1, 2, ..., l \tag{6}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, ..., \mu_{iq}]^T$ is the center of the receptive field and η is the width of the Gaussian function. It has been proven that network (5) can approximate any continuous function over, $h(z)$, to any desired accuracy over a compact set $\Omega_z \subset R^q$ to arbitrary any accuracy as $h(z) = h_{nn}(z, W^*) + \varepsilon(z)$, $\forall z \in \Omega_z$ with ideal NN weights,W^* and NN approximation error $\varepsilon(z)$. The following assumption is made for W^* and $\varepsilon(z)$.

Assumption 5. There exist ideal constant weights W^* such that $|\varepsilon(z)| \le \varepsilon^*$ with constant $\varepsilon^* > 0$ for all $z \in \Omega_z$. Moreover, W^* is bounded by $\|W^*\| \le w_m$ on the compact set Ω_z.

The ideal weights W^* and V^* are "artificial" required for analytical purposes. According to the discussion (Ge *et al.*, 2002), W^* is defined as follows:

$$W^* = \arg\min_{(W)} \left[\sup_{z \in \Omega_z} |h_{nn}(z, W) - h(z)|\right] \tag{7}$$

As the ideal constant W^* is unknown and needs to be estimated in control design. Let $\hat{W}$ be the estimate of W^* and the weight estimation error be $\tilde{W} = \hat{W} - W^*$.

Throughout this paper, $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$, $\|\cdot\|$ denotes the 2-norm, $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the smallest and largest eigenvalues of a square matrix $(\cdot)$, respectively.

3. CONTROL DESIGN AND MAIN RESULTS

In this section, motivated by the work (Ge *et al.*, 2001) and (Ortega *et al.*, 1998), the dynamic hysteresis friction model (3) and (4) can be separated into two parts: (i) the term with unknown constant coefficient, and (ii) the term which is a function of the unmeasurable internal state $z(t)$ and is bounded by a function which is independent of $z(t)$. We can use RBFNN to approximate this unknown bounding function. Based on Lyapunov synthesis, adaptation algorithms for both the NN weights and the unknown system and hysteresis friction parameters are presented.

Define the tracking error e and the filtered tracking error r as follows:

$$\begin{aligned} e &= x - x_d \\ r &= \dot{e} + \lambda e \end{aligned} \tag{8}$$

where $\lambda > 0$. Define the reference velocity and acceleration signals as follows:

$$\dot{x}_r = \dot{x}_d - \lambda e \tag{9}$$

$$\ddot{x}_r = \ddot{x}_d - \lambda \dot{e} \tag{10}$$

Substituting (4) into (3), we obtain

$$\begin{aligned} h &= \sigma_1 \dot{x} + \sigma_2 \dot{x} + \sigma_0 z - \sigma_1 \alpha(x,\dot{x})|\dot{x}|z \\ &= \sigma \dot{x} + h_z(x,\dot{x},z) \end{aligned} \tag{11}$$

where $\sigma = \sigma_1 + \sigma_2$, and $h_z(x,\dot{x},z) = \sigma_0 z - \sigma_1 \alpha(x,\dot{x})|\dot{x}|z$, which depends on z.

From Lemma 1, we know that h_z is bounded by

$$\begin{aligned} |h_z(x,\dot{x},z)| &= |(\sigma_0 - \sigma_1 \alpha(x,\dot{x})|\dot{x}|)||z(t)| \\ &\le \frac{\sigma_0 + \sigma_1 \alpha(x,\dot{x})|\dot{x}|}{\alpha_{min}} = \bar{h}_z(x,\dot{x}) \end{aligned} \tag{12}$$

where $\bar{h}_z(x,\dot{x})$ is the bounding function of $h_z(x,\dot{x},z)$ and is independent of the unmeasurable internal state z. Then, we use RBFNN $\hat{\bar{h}}_z(x,\dot{x}) = W^T S(x,\dot{x})$to approximate the unknown $\bar{h}_z(x,\dot{x})$. Therefore, there exists the following function approximation

$$\bar{h}_z(x,\dot{x}) = W^{*T} S(x,\dot{x}) + \epsilon(x,\dot{x}) \tag{13}$$

where W^* is the optimal weight vector, and $\epsilon(x,\dot{x})$ is the NN approximation error, which satisfies $|\epsilon(x,\dot{x})| \le \epsilon^*, \quad \forall (x,\dot{x}) \in \Omega$, with a positive constant ϵ^*.

Then, we can have the following tracking error dynamics

$$\begin{aligned} m\dot{r} &= u - \sigma \dot{x} - h_z(x,\dot{x},z) - d(t) - m\ddot{x}_r \\ &\le u - \sigma \dot{x} + W^{*T} S(x,\dot{x}) + \phi - m\ddot{x}_r \end{aligned} \tag{14}$$

where $\phi = \epsilon^* + d^*$.

Consider the following control

$$u = -kr + \hat{\sigma}\dot{x} + \hat{m}\ddot{x}_r - \hat{W}^T S(x,\dot{x})\mathrm{sgn}(r) - \hat{\phi}\mathrm{sgn}(r) \tag{15}$$

where constant $k > 0$, $\hat{\sigma}$, $\hat{m}$ and $\hat{\phi}$ are the estimates of unknown θ, m and ϕ respectively.

Substituting (15) into (14) leads to

$$\begin{aligned} m\dot{r} \le & -kr + \tilde{\sigma}\dot{x} + \tilde{m}\ddot{x}_r - \hat{W}^T S(x,\dot{x})\mathrm{sgn}(r) \\ & + W^{*T} S(x,\dot{x}) - \hat{\phi}\mathrm{sgn}(r) + \phi \end{aligned} \tag{16}$$

Adding and subtracting $W^{*T} S(x,\dot{x})\mathrm{sgn}(r) + \phi\mathrm{sgn}(r)$ on the right hand side of (16), we have

$$\begin{aligned} m\dot{r} \le & -kr + \tilde{\sigma}\dot{x} + \tilde{m}\ddot{x}_r - \tilde{W}^T S(x,\dot{x})\mathrm{sgn}(r) \\ & - W^{*T} S(x,\dot{x})\mathrm{sgn}(r) + W^{*T} S(x,\dot{x}) \\ & - \tilde{\phi}\mathrm{sgn}(r) - \phi\mathrm{sgn}(r) + \phi \end{aligned} \tag{17}$$

Theorem 1. Consider the closed-loop system consisting of system (1) with dynamic hysteresis friction given by (3) and (4), and the control law (15). If the Assumptions 1-4 are satisfied and the parameters $\hat{\sigma}$, $\hat{m}$, $\hat{\phi}$ and NN weight $\hat{W}$ are updated by

$$\dot{\hat{\sigma}} = -k_\theta(\dot{x}r + \sigma_\theta \hat{\theta}) \tag{18}$$

$$\dot{\hat{m}} = -k_m(\ddot{x}_r r + \sigma_m \hat{m}) \tag{19}$$

$$\dot{\hat{\phi}} = k_\phi(|r| - \sigma_\phi \hat{\phi}) \tag{20}$$

$$\dot{\hat{W}} = \Gamma[S(x,\dot{x})|r| - \sigma_w \hat{W}] \tag{21}$$

where k_θ, k_m, k_ϕ, σ_θ, σ_m and σ_ϕ are positive design constant parameters, $\Gamma = \Gamma^T > 0$ is a dimensionally compatible constant matrix, then given any initial compact set defined by

$$\Omega_0 = \Big\{ x(0), x_d(0), \hat{\theta}(0), \hat{m}(0), \hat{\phi}(0), \hat{W}(0) \Big| x(0), \hat{\theta}(0), \hat{m}(0), \hat{\phi}(0), \hat{W}(0) \ \ are\ chosen\ finite, x_d(0) \in \Omega_d \Big\}$$

(i) All the closed loop signals will be remained in a compact set which is given by:

$$\Omega = \Big\{ x(t), \hat{\sigma}, \hat{m}, \hat{\phi}, \hat{W} \Big| |x| \le \max_{[0,t]} |x_d| + |e(0)| + \frac{1}{\lambda}\sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{m}}, |\hat{\sigma}| \le |\sigma| + \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\sigma}, |\hat{m}| \le |m| + \sqrt{(2V(0) + \frac{2c_2}{c_1})k_m}, |\hat{\phi}| \le |\phi| + \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\phi}, \|\hat{W}\| \le \|W^*\| + \sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{\lambda_{\min}(\Gamma^{-1})}} \Big\}$$

(ii) All the closed loop signals will eventually converge to the compact sets which is defined by:

$$\Omega_s = \Big\{ x(t), \hat{\sigma}, \hat{m}, \hat{\phi}, \hat{W} \Big| \lim_{t\to\infty} |r| = \sqrt{\frac{2c_2}{mc_1}}, \lim_{t\to\infty} |\tilde{\sigma}| = \sqrt{\frac{2c_2 k_\sigma}{c_1}}, \ \lim_{t\to\infty} |\tilde{m}| = \sqrt{\frac{2c_2 k_m}{c_1}}, \lim_{t\to\infty} |\tilde{\phi}| = \sqrt{\frac{2c_2 k_\phi}{c_1}}, \ \lim_{t\to\infty} \|\tilde{W}\| = \sqrt{\frac{2c_2}{\lambda_{\min}(\Gamma^{-1})c_1}} \Big\}$$

Proof: Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}mr^2 + \frac{1}{2k_\sigma}\tilde{\sigma}^2 + \frac{1}{2k_m}\tilde{m}^2 + \frac{1}{2k_\phi}\tilde{\phi}^2 + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W}$$

Its derivative along (17) is

$$\begin{aligned}\dot{V} &= mr\dot{r} + \frac{1}{k_\sigma}\tilde{\sigma}\dot{\tilde{\sigma}} + \frac{1}{k_m}\tilde{m}\dot{\tilde{m}} + \frac{1}{k_\phi}\tilde{\phi}\dot{\tilde{\phi}} + \tilde{W}^T\Gamma^{-1}\dot{\tilde{W}} \\ &\le -kr^2 + \tilde{\sigma}\dot{x}r + \tilde{m}\ddot{x}_r r - \tilde{W}^T S(x,\dot{x}) r\,\mathrm{sgn}(r) \\ &\quad -W^{*T}S(x,\dot{x})r\,\mathrm{sgn}(r) + W^{*T}S(x,\dot{x})r - \tilde{\phi}r\,\mathrm{sgn}(r) \\ &\quad -\phi r\,\mathrm{sgn}(r) + \phi r + \frac{1}{k_\sigma}\tilde{\sigma}\dot{\tilde{\sigma}} + \frac{1}{k_m}\tilde{m}\dot{\tilde{m}} + \frac{1}{k_\phi}\tilde{\phi}\dot{\tilde{\phi}} \\ &\quad +\tilde{W}^T\Gamma^{-1}\dot{\tilde{W}} \\ &\le -kr^2 + \tilde{\sigma}(\dot{x}r + \frac{1}{k_\theta}\dot{\tilde{\theta}}) + \tilde{m}(\ddot{x}_r r + \frac{1}{k_m}\dot{\tilde{m}}) \\ &\quad +\tilde{\phi}(-|r| + \frac{1}{k_\phi}\dot{\tilde{\phi}}) + \tilde{W}^T[-S(x,\dot{x})|r| + \Gamma^{-1}\dot{\tilde{W}}] \\ &\quad -W^{*T}S(x,\dot{x})|r| + W^{*T}S(x,\dot{x})r - \phi|r| + \phi r\end{aligned} \tag{22}$$

Since,

$$\dot{\tilde{\sigma}} = \dot{\hat{\sigma}}, \quad \dot{\tilde{m}} = \dot{\hat{m}}, \quad \dot{\tilde{\phi}} = \dot{\hat{\phi}} \tag{23}$$

and

$$\begin{aligned}-W^{*T}S(x,\dot{x})|r| + W^{*T}S(x,\dot{x})r \le 0, \\ -\phi|r| + \phi r \le 0\end{aligned} \tag{24}$$

Substituting (23), (24) and (18)- (21) into (22), we have

$$\dot{V} \le -kr^2 - \sigma_\sigma\tilde{\sigma}\hat{\sigma} - \sigma_m\tilde{m}\hat{m} - \sigma_\phi\tilde{\phi}\hat{\phi} - \sigma_w\tilde{W}\hat{W} \tag{25}$$

By completion of squares, the following inequalities hold

$$-\sigma_\sigma\tilde{\sigma}\hat{\sigma} \le -\frac{\sigma_\sigma}{2}\tilde{\sigma}^2 + \frac{\sigma_\sigma}{2}\sigma^2 \tag{26}$$

$$-\sigma_m\tilde{m}\hat{m} \le -\frac{\sigma_m}{2}\tilde{m}^2 + \frac{\sigma_m}{2}m^2 \tag{27}$$

$$-\sigma_\phi\tilde{\phi}\hat{\phi} \le -\frac{\sigma_\phi}{2}\tilde{\phi}^2 + \frac{\sigma_\phi}{2}\phi^2 \tag{28}$$

$$-\sigma_w\tilde{W}^T\hat{W} \le -\frac{\sigma_W}{2}\|\tilde{W}\|^2 + \frac{\sigma_W}{2}\|W^*\|^2 \tag{29}$$

Substituting (26)-(29) into (25), we have the following equations:

$$\begin{aligned}\dot{V} &\le -kr^2 - \frac{\sigma_\sigma}{2}\tilde{\sigma}^2 - \frac{\sigma_m}{2}\tilde{m}^2 - \frac{\sigma_\phi}{2}\tilde{\phi}^2 - \frac{\sigma_W}{2}\|\tilde{W}\|^2 \\ &\quad +\frac{\sigma_\sigma}{2}\sigma^2 + \frac{\sigma_m}{2}m^2 + \frac{\sigma_\phi}{2}\phi^2 + \frac{\sigma_W}{2}\|W^*\|^2 \\ &\le -c_1V + c_2\end{aligned} \tag{30}$$

where

$$c_1 = \min\left\{2k, \sigma_\sigma k_\sigma, \sigma_m k_m, \sigma_\phi k_\phi, \frac{\sigma_W}{\lambda_{\max}(\Gamma^{-1})}\right\} \tag{31}$$

$$c_2 = \frac{\sigma_\sigma}{2}\sigma^2 + \frac{\sigma_m}{2}m^2 + \frac{\sigma_\phi}{2}\phi^2 + \frac{\sigma_W}{2}\|W^*\|^2 \tag{32}$$

Multiplying (30) by $e^{c_1 t}$ yields,

$$\frac{d}{dt}(V(t)e^{c_1 t}) \le c_2 e^{c_1 t} \tag{33}$$

Integrating (33) over $[0,t]$ leads to the following equation:

$$0 \le V(t) \le \left[V(0) - \frac{c_2}{c_1}\right]e^{-c_1 t} + \frac{c_2}{c_1} \tag{34}$$

where

$$\begin{aligned}V(0) = \frac{1}{2}mr^2(0) + \frac{1}{2k_\sigma}\tilde{\sigma}^2(0) + \frac{1}{2k_m}\tilde{m}^2(0) + \\ \frac{1}{2k_\phi}\tilde{\phi}^2(0) + \frac{1}{2}\tilde{W}^T(0)\Gamma^{-1}\tilde{W}(0)\end{aligned} \tag{35}$$

(i) Uniform Boundedness (UB)

From (34), we have

$$0 \le V(t) \le \left[V(0) - \frac{c_2}{c_1}\right]e^{-c_1 t} + \frac{c_2}{c_1} \le V(0) + \frac{c_2}{c_1} \tag{36}$$

From (22) and (36), we have

$$\begin{aligned}&|r| \le \sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{m}}, \\ &|\tilde{\sigma}| \le \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\sigma}, \quad |\tilde{m}| \le \sqrt{(2V(0) + \frac{2c_2}{c_1})k_m}, \\ &|\tilde{\phi}| \le \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\phi}, \|\tilde{W}\| \le \sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{\lambda_{\min}(\Gamma^{-1})}}\end{aligned} \tag{37}$$

Since $\tilde{\sigma} = \hat{\sigma} - \sigma$, $\tilde{m} = \hat{m} - m$, $\tilde{\phi} = \hat{\phi} - \phi$ and $\tilde{W} = \hat{W} - W^*$, we have

$$\begin{aligned}|\hat{\sigma}| - |\sigma| \le |\hat{\sigma} - \sigma| \le \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\sigma} \\ |\hat{m}| - |m| \le |\hat{m} - m| \le \sqrt{(2V(0) + \frac{2c_2}{c_1})k_m} \\ |\hat{\phi}| - |\phi| \le |\hat{\phi} - \phi| \le \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\phi} \\ \|\hat{W}\| - \|W^*\| \le \|\hat{W} - W^*\| \le \sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{\lambda_{\min}(\Gamma^{-1})}}\end{aligned} \tag{38}$$

i. e. ,

$$\begin{aligned}|\hat{\sigma}| \le |\sigma| + \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\sigma} \\ |\hat{m}| \le |m| + \sqrt{(2V(0) + \frac{2c_2}{c_1})k_m} \\ |\hat{\phi}| \le |\phi| + \sqrt{(2V(0) + \frac{2c_2}{c_1})k_\phi} \\ \|\hat{W}\| \le \|W^*\| + \sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{\lambda_{\min}(\Gamma^{-1})}}\end{aligned} \tag{39}$$

From the definition of r in (8), we have

$$\dot{e} = -\lambda e + r \tag{40}$$

Solving this equation results in

$$e = e^{-\lambda t}e(0) + \int_0^t e^{-\lambda(t-\tau)}|r|d\tau \tag{41}$$

Combining with (37), the following equation is obtained:

$$|e| \le |e(0)| + \frac{1}{\lambda}\sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{m}} \tag{42}$$

From (8), we have

$$|x| \le \max_{[0,t]}|x_d| + |e(0)| + \frac{1}{\lambda}\sqrt{\frac{2V(0) + \frac{2c_2}{c_1}}{m}} \tag{43}$$

(ii) Uniformly Ultimate Boundedness (UUB)

From (22) and (34), we have

$$
\begin{aligned}
|r| &\leq \sqrt{\frac{2[V(0)-\frac{c_2}{c_1}]e^{-c_1 t}+\frac{2c_2}{c_1}}{m}}, \\
|\tilde{\sigma}| &\leq \sqrt{\Big[2(V(0)-\frac{c_2}{c_1})e^{-c_1 t}+\frac{2c_2}{c_1}\Big]k_\sigma}, \\
|\tilde{m}| &\leq \sqrt{\Big[2(V(0)-\frac{c_2}{c_1})e^{-c_1 t}+\frac{2c_2}{c_1}\Big]k_m}, \\
|\tilde{\phi}| &\leq \sqrt{\Big[2(V(0)-\frac{c_2}{c_1})e^{-c_1 t}+\frac{2c_2}{c_1}\Big]k_\phi}, \\
\|\tilde{W}\| &\leq \sqrt{\frac{2[V(0)-\frac{c_2}{c_1}]e^{-c_1 t}+\frac{2c_2}{c_1}}{\lambda_{\min}(\Gamma^{-1})}}
\end{aligned} \tag{44}
$$

Let us analyze the property of $|r|$ first. If $V(0) = c_2/c_1$, then $|r| \leq \sqrt{\frac{2c_2}{mc_1}}$, $\forall t \geq 0$. If $V(0) \neq c_2/c_1$, from (44), we can conclude that given any $\mu_r > \frac{2c_2}{mc_1}$, there exists T_r , such that for any $t > T_r$, we have $|r| \leq \mu_r$. Specially, given any μ_r

$$
\begin{aligned}
\mu_r &= \sqrt{\frac{2[V(0)-\frac{c_2}{c_1}]e^{-c_1 T_r}+\frac{2c_2}{c_1}}{m}}, V(0) \neq \frac{c_2}{c_1} \\
T_r &= -\frac{1}{c_1}\ln\Big(\frac{\mu_e^2 m-\frac{2c_2}{c_1}}{2[V(0)-\frac{c_2}{c_1}]}\Big) \\
\lim_{t\to\infty}|r| &= \sqrt{\frac{2c_2}{mc_1}}
\end{aligned} \tag{45}
$$

Similar analysis can be made about $|\tilde{\sigma}|$, $|\tilde{m}|$, $|\tilde{\phi}|$, $\|\tilde{W}\|$, and $\lim_{t\to\infty}|\tilde{\sigma}| = \sqrt{\frac{2c_2k_\theta}{c_1}}$, $\lim_{t\to\infty}|\tilde{m}| = \sqrt{\frac{2c_2k_m}{c_1}}$, $\lim_{t\to\infty}|\tilde{\phi}| = \sqrt{\frac{2c_2k_\phi}{c_1}}$, $\lim_{t\to\infty}\|\tilde{W}\| = \sqrt{\frac{2c_2}{\lambda_{\min}(\Gamma^{-1})c_1}}$. ■

4. SIMULATION STUDIES

To demonstrate the effectiveness of our proposed control design, we perform simulations on the plant model which is described in (1),(3) and (4) under the following choices of the parameter values: $m = 1.0$, $\sigma_0 = 10^5$, $\sigma_1 = \sqrt{10^5}$, $\sigma_2 = 0.4$, $\dot{x}_s = 0.001$, $f_c = 1$, $f_s = 1.5$, $d(t) = 0.1\sin(20t)$. The proposed control is mainly investigated for the purpose of trajectory tracking in which the output x is required to follow the reference trajectory $x_d = \sin(5t)$.

For the neural network $\hat{W}^T S(x, \dot{x})$, the centers for $S(x, \dot{x})$ has been considered to be evenly spaced in a regular lattice in R^2. Employing five nodes for each input dimension, it ends up with $5^2 = 25$ nodes. The design parameters in control (15) and adaption laws (18)-(21) are chosen as: $\lambda = 1.0, k = 10.0, k_\theta = 1.0, k_\phi = 1.0, k_m = 1.0, \sigma_\theta = 0.1, \sigma_\phi = 0.1, \sigma_m = 0.1$, $\Gamma = diag\{1.0\}$, $\sigma_w = 1.0$. The initial conditions are taken as: $x(0) = \dot{x}(0) = 0.0$, $\hat{W}(0) = 0.0$, $\hat{\theta}(0) = \hat{\phi}(0) = \hat{m}(0) = 0.0$.

The simulation results are detailed in Figs 1-5. From Fig. 1, it can be seen that the tracking performance has been improved compared with conventional PID control. The bounededness of control signals u are shown in Fig. 2, while norm of NN weights $\|W\|$, and the estimated parameters $\hat{\theta}$, $\hat{\phi}$ and $\hat{m}$, are shown in Fig. 3 and Fig. 4, respectively.

To illustrate the effect of parameter λ on the tracking performance, the comparison studies of different λ values are shown in Fig. 5. It can be observed that the larger the value of λ, the smaller the tracking errors can be achieved. However, if the λ is chosen too small, the control gain becomes very large, which is not expected in the application. Therefore, there is a compromise between the tracking performance and the control efforts. Here, we recommend that λ is chosen between 0.1 and 15.

5. CONCLUSION

In this paper, robust adaptive NN control for mobile hard disk drive system has been presented. All the signals in the closed-loop have been ensured semi-globally uniformly ultimately bounded and the output can track a desired trajectory to a neighborhood of zero. Simulation results show that the proposed robust adaptive NN control is effective for compensation of hysteresis friction nonlinearities and the external disturbances.

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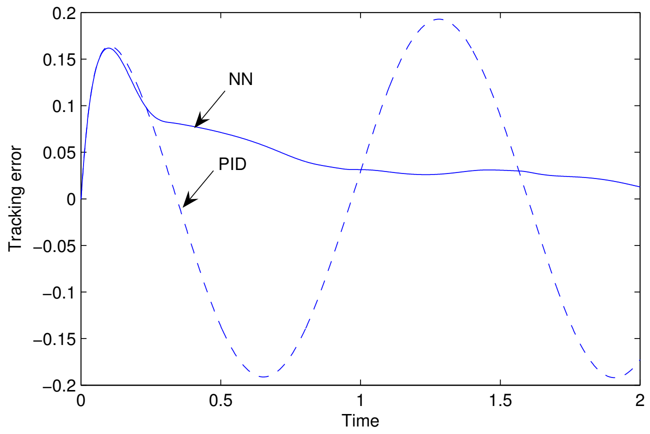

Fig. 1. Output tracking performance comparison

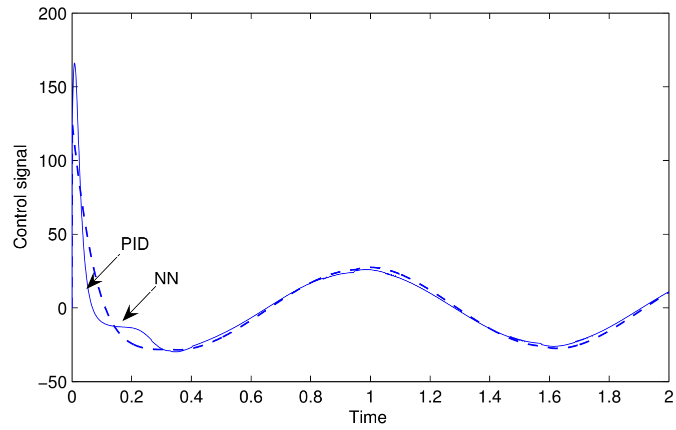

Fig. 2. Control inputs comparison

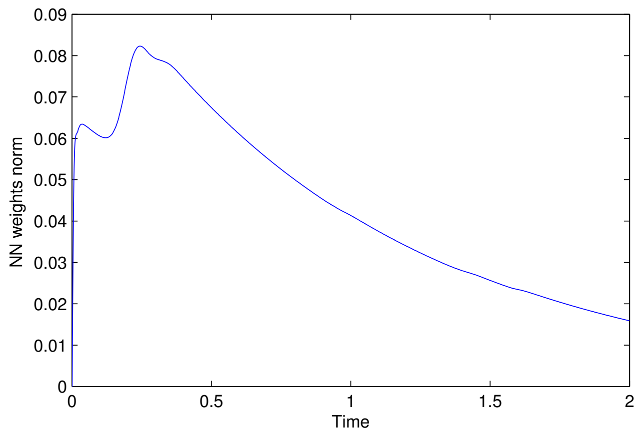

Fig. 3. NN weights norm $\|W\|$

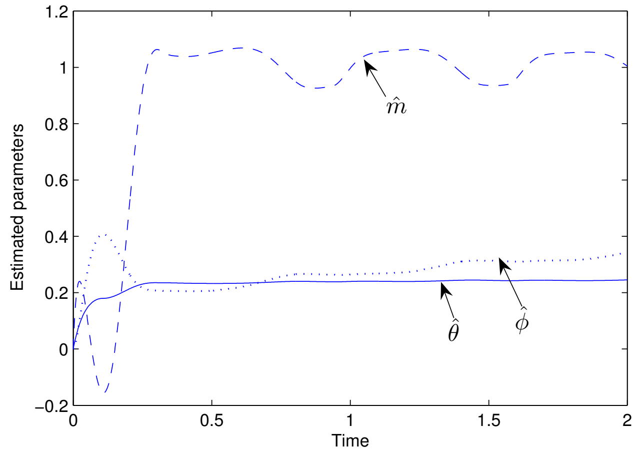

Fig. 4. Estimated parameters trajectories

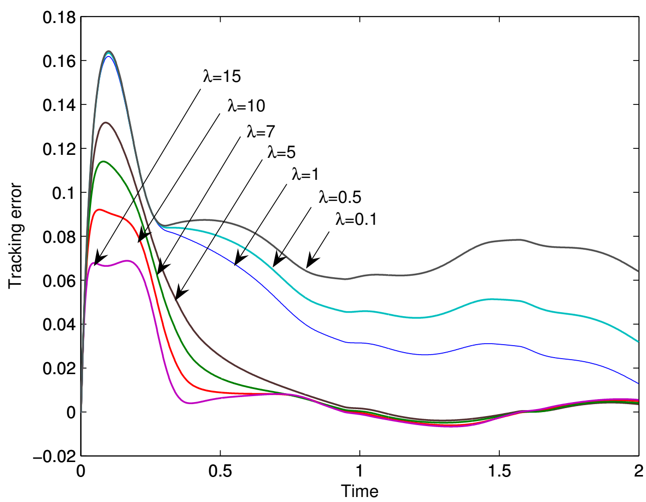

Fig. 5. Tracking errors with different λ values