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A Level Measurement Method based on Acoustic Resonance using Unscented Kalman Filter

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Abstract: An accurate and low-cost level measurement method based on acoustic resonance is presented. The method is useful in the cases that measurements are noisy which might be due to environment noise or due to low-cost instruments used in the measurement process. An iterative nonlinear filtering algorithm, called the "Unscented Kalman filter" (UKF) has been employed to obtain a good estimate of the noisy measurements. Simulation and experimental tests have been carried out indicating that the UKF greatly improves the accuracy of the measured level.

1. INTRODUCTION

Various classic and modern methods have been used for determining the level of liquids. These methods are useful in many industries such as chemical, petrochemical, water and food industries. Most level measurement methods are based on mechanical, conductivity, capacitive, wave and microwave reflection phenomena but the most presented methods are established on the wave reflection and operate in the sound and ultrasound regions (Angrisani L., 2006a,b; Burak Dalci K.; Isonaga S.; Meribout M.; Tanaka, S.). The use of ultrasound waves are generally limited due to the problem of parasitic reflections caused by foams, residues, surface ripple and deposits; however, there are some improvements which reduce this problem (Angrisani L. 2006a,b; Burak Dalci K.). Problem with the parasitic reflection in acoustic gauge can be reduced using long acoustic waves, since the wavelength of these waves is much longer than potential residue or deposit (Donlagic D.). But there is still a problem in the existing acoustic based techniques. This problem is the lack of accuracy which originates from the uncertainties in transmitter, receiver and temperature sensor. In addition, noise and shape distortions generally affect the transmitted signal and make the true level difficult to achieve. To overcome these limitations affecting the sound and ultrasound based techniques, iterative filtering algorithms such as "Kalman filter" (KF), "Extended Kalman filter" (EKF) and "Unscented Kalman filter" (UKF) can be employed (Angrisani L. 2006a,b; Tanaka S.). In general, the UKF can provide a better alternative for nonlinear filtering than the conventional EKF since it avoids errors associated with linearization (Julier S. J.). In this paper an accurate industrial level measurement method using UKF is proposed which can be used, especially in noisy environments or in the cases in which low-cost instruments such as low-cost microphone and speaker have been used. The proposed method can also reduce the effect of parasitic reflections caused by foam, residue, etc more than the existing methods.

2. THEORY AND BASIC SETUP

2.1 Basic setup

Fig. 1 shows the construction of the acoustic level gauge. It consists of an upright tube extending into a storage tank, a transmitter and a receiver of acoustic waves at the top of the tube (a loudspeaker and a microphone) to emit the sound waves into the tube and to receive the echo signals, a temperature sensor to measure the temperature of the air inside the tube during the measurement process and a controller. The controller transmits a sine wave to drive the speaker and the speaker emits the signal vertically toward the liquid surface. Then the microphone receives the echo or the reflected acoustic wave and sends the received signal to the controller (Donlagic D.).

When the speaker vibrates near the tube, there are certain frequencies at which the tube will amplify the sound from the speaker. These frequencies are called resonant frequencies, and occur because the dimensions of the tube are such that, at these frequencies, a maximum transfer of energy is accomplished between the speaker and the tube.

The resonant frequencies of the level gauge, can be found from the well-known equation

$$L = n\frac{\lambda_n}{2} + \frac{\lambda_n}{4} \quad , \; n = 0, 1, 2, \ldots \tag{1}$$

where λ_n is the wavelength of the n^{th} resonance and L is the distance from the top of the tube to the end which is the surface of liquid as shown in Fig. 1.

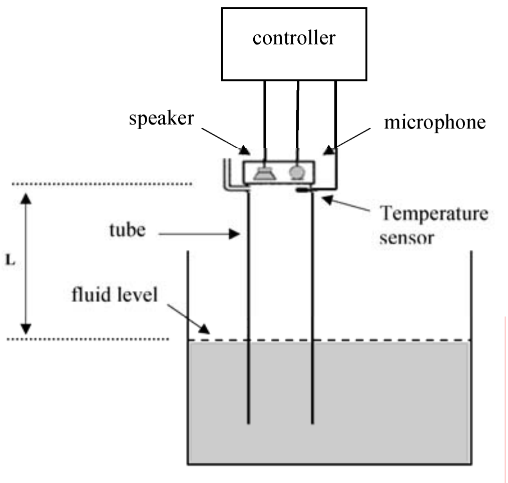

Fig. 1. The construction of the acoustic level gauge.

The value of λ is calculated through the following equation

$$\lambda = \frac{c}{f} = \frac{331.4 + 0.6T}{f} \tag{2}$$

where f and c are the frequency and the speed of sound, respectively and T is the temperature in Celsius.

It should be noted that the sound speed is highly dependent on the temperature and on the composition of air inside the tube, and also on its pressure. So the temperature of the air should be measured continuously during the level measurement process to detect its variations with respect to time.

2.2 The algorithm of Level measurement

The resonator is excited from a frequency f_L to a higher frequency f_H. The speaker transmits a sine wave with a wavelength of $\lambda_L = c/f_L$ and waits for acoustic amplitude to settle. The controller samples the signals received via the microphone and stores the wavelengths and the amplitudes of the received signals in memory. This process is repeated over the entire frequency range with a frequency step of Δf. Fig. 2 shows the envelope of a typical acquired signal received through the microphone for a 3-meter long tube in the frequency range of 1525 *Hz* to 1675 *Hz* with the frequency step of 1 *Hz*. When the frequency range is totally scanned, the controller searches for local maxima and extracts the resonant wavelengths λ_n to λ_{n+N} where $N+1$ is the number of resonant wavelengths in the entire frequency range. It is seen in Fig. 2 that there are three resonant frequencies in this range which are $f_n = 1548$ *Hz*, $f_{n+1} = 1607$ *Hz* and $f_{n+2} = 1667$ *Hz*.

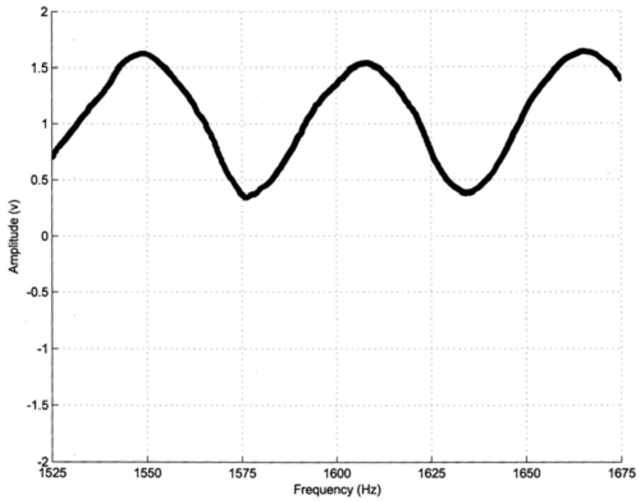

Fig. 2. The signal received through the microphone for a 3-meter long tube in the frequency range of 1525 *Hz* to 1675 *Hz*.

2.3 Problem Formulation

The objective of this subsection is to present a relation between the length L and the resonant wavelengths λ_n to λ_{n+k}.

If λ_n, λ_{n+1} and λ_{n+2} are the resonant wavelengths for n, $n+1$ and $n+2$ in (1), then the system equations can be written as

$$L = n\frac{\lambda_n}{2} + \frac{\lambda_n}{4} \tag{3}$$

$$L = (n+1)\frac{\lambda_{n+1}}{2} + \frac{\lambda_{n+1}}{4} \tag{4}$$

$$L = (n+2)\frac{\lambda_{n+2}}{2} + \frac{\lambda_{n+2}}{4} \tag{5}$$

Substituting (3) into (4) and solving for the length L, we obtain

$$L = \frac{\lambda_n \lambda_{n+1}}{2(\lambda_n - \lambda_{n+1})}. \tag{6}$$

Hence, the value of L can be calculated using two resonant wavelengths λ_n and λ_{n+1}. In addition, the system dynamics can be written using (3), (4) and (5) as follows

$$\lambda_{n+2} = \frac{\lambda_n \lambda_{n+1}}{2\lambda_n - \lambda_{n+1}}. \tag{7}$$

It is seen that (7) is a second-order nonlinear difference equation. Instead of working directly with this second order difference equation for state estimation, it is convenient to convert it into a first order nonlinear equation (Meditch J. S.) as follows:

$$\begin{bmatrix} \lambda_{n+1} \\ \lambda_{n+2} \end{bmatrix} = \begin{bmatrix} \lambda_{n+1} \\ \dfrac{\lambda_n \lambda_{n+1}}{2\lambda_n - \lambda_{n+1}} \end{bmatrix} . \tag{8}$$

The price we have paid to convert (7) into (8) is that of introducing a new vector which has twice as many elements as the original state vector.

Since the noise outside the tube affects continually the amplitude and the phase of the stationary waves in the tube (especially in noisy environments such as factories) and on the other hand, the measurements of λ are not accurate and there are some uncertainties in the measurements through speaker, microphone, system noise, parasitic reflections, etc (especially for low-cost instruments such as low-cost microphone and speaker), a stochastic filter should be used to estimate the measured data. Here the UKF, which is a proper choice for state estimation in nonlinear systems such as (8), is applied.

2.4 The Unscented Kalman filter:

Kalman filter is an optimal, minimum mean square error estimator for linear systems. When system dynamics are intrinsically nonlinear, extended Kalman filter (EKF) has customarily been used. In general, the EKF performs a truncated first-order Taylor linearization on the system equations about the current state, to which the linear filter equations are applied. The EKF has been used extensively; however, it suffers from possible divergence problems because the linearization does not always capture the correct dynamics of the underlying system. As a result, several new filtering methods have recently been introduced on the basis of the Kalman filter. The UKF is a powerful nonlinear estimation technique and has been shown to be a superior alternative to the EKF in a variety of applications including state estimation (Julier S. J.). Unlike the EKF, the UKF does not explicitly approximate the nonlinear process and measurement models; it uses the true nonlinear models and approximates the distribution of the state random variable. The basic idea underlying the UKF is that "it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation" (Julier S. J.). Instead of linearizing using Jacobian matrices, a set of input domain points, which are referred to as sigma points, along with a proper collection of weights are deterministically chosen in such a way that a defined number of their central moments match those of input random variables (Van der Merwe R.).

Consider a discrete time nonlinear dynamic system with additive noise in process and measurement

$$X_{k+1} = F(X_k, U_k) + \omega_k \tag{9}$$

$$Y_k = H(X_k, U_k) + V_k \tag{10}$$

where ω_k ($\ell \times 1$) and V_k ($m \times 1$) are the process and measurement noise vectors with covariances R_ω and R_V; U_k is the input vector; Y_k is an $m \times 1$ measurement vector and X_k is an $\ell \times 1$ random vector with mean $\overline{X}_k = E[X_k]$ and covariance P_{X_k} where E is the expectation operator.

The algorithm of the UKF for this system can be performed as follows (Van der Merwe R.)

Let $\hat{X}_0 = E[X_0] = \overline{X}_0$,

$$P_{X_0} = E[(X_0 - \hat{X}_0)(X_0 - \hat{X}_0)^T] \tag{11}$$

then for $n = 1, \ldots, k$:

the random vector X_{k-1} is approximated by $2\ell + 1$ sigma points and their weights W_i based on the given values of $\hat{X}_{k-1}$ and covariance $P_{X_{k-1}}$:

$$(\chi_{k-1})_0 = \hat{X}_{k-1}$$

$$(\chi_{k-1})_i = \hat{X}_{k-1} + \left(\sqrt{(\ell + \kappa) P_{X_{k-1}}}\right)_i, \quad i=1,\ldots,\ \ell$$

$$(\chi_{k-1})_i = \hat{X}_{k-1} - \left(\sqrt{(\ell + \kappa) P_{X_{k-1}}}\right)_i, \quad i=\ell+1,\ldots,\ 2\ell$$

$$W_0 = \frac{\kappa}{\ell + \kappa}$$

$$W_i = \frac{1}{2(\ell + \kappa)}, \qquad i=1,\ldots,\ \ell$$

$$W_i = \frac{1}{2(\ell + \kappa)}, \qquad i=\ell+1,\ldots,\ 2\ell \tag{12}$$

where $\left(\sqrt{P}\right)_i$ is the i^{th} column of the matrix square root of P and W_i is the weight associated with the i^{th} sigma point. The scalar κ is a scaling parameter that is usually set to $3 - \ell$. In general, other choices of κ would lead to better or worst results, depending on specific characteristics of the problem.

(12) can be written in a simplified vector form as below

$$\chi_{k-1} = [\hat{X}_{k-1} \quad \hat{X}_{k-1} + \sqrt{(\ell + \kappa) P_{X_{k-1}}} \quad \hat{X}_{k-1} - \sqrt{(\ell + \kappa) P_{X_{k-1}}}\,] \tag{13}$$

where $\hat{X}_{k-1}$ is an $\ell \times 1$ vector and $\hat{X}_{k-1} \pm \sqrt{(\ell + \kappa) P_{X_{k-1}}}$ is an $\ell \times \ell$ matrix. Consequently the dimensions of sigma point matrix χ_{k-1} is $\ell \times (2\ell + 1)$.

Then the sigma points are transformed through the function F defined in (9)

$$\chi^*_{k|k-1} = F(\chi_{k-1}, U_{k-1}) \tag{14}$$

and a priori state estimate and a priori state covariance are given by

$$\hat{X}_k^- = \sum_{i=0}^{2\ell} W_i \left(\overset{*}{\chi}_{k|k-1}\right)_i \quad (15)$$

$$P_{X_k}^- = \sum_{i=0}^{2\ell} W_i \left((\overset{*}{\chi}_{k|k-1})_i - \hat{X}_k^-\right)\left((\overset{*}{\chi}_{k|k-1})_i - \hat{X}_k^-\right)^T + R_\omega \quad (16)$$

The sigma points should be augmented with additional points derived from the matrix square root of the process noise covariance R_V, in order to incorporate the effect of process noise on the sigma points. This requires changing the size of augmented sigma points from $\ell \times (2\ell+1)$ to $\ell \times (4\ell+1)$ and recalculating the weights W_i, accordingly.

$$\chi_{k|k-1} = [\overset{*}{\chi}_{k|k-1} \quad \left(\overset{*}{\chi}_{k|k-1}\right)_0 + \sqrt{(\ell+\kappa)R_\omega} \quad \left(\overset{*}{\chi}_{k|k-1}\right)_0 - \sqrt{(\ell+\kappa)R_\omega}\,] \quad (17)$$

Therefore the augmented sigma point $\chi_{k|k-1}$ is an $\ell \times (4\ell+1)$ matrix which can be transformed through the measurement function defined in (10) as follows

$$\mathrm{Y}_{k|k-1} = H(\chi_{k|k-1}) \quad (18)$$

$$\hat{Y}_k^- = \sum_{0}^{4\ell} W_i \left(Y_{k|k-1}\right)_i \quad (19)$$

and the predicted measurement covariance matrix P_{Y_k} and the cross covariance matrix $P_{X_k Y_k}$ are calculated accordingly

$$P_{Y_k} = \sum_{i=0}^{4\ell} W_i \left((Y_{k|k-1})_i - \hat{Y}_k^-\right)\left((Y_{k|k-1})_i - \hat{Y}_k^-\right)^T + R_V \quad (20)$$

$$P_{X_k Y_k} = \sum_{i=0}^{4\ell} W_i \left((X_{k|k-1})_i - \hat{X}_k^-\right)\left((Y_{k|k-1})_i - \hat{Y}_k^-\right)^T. \quad (21)$$

Now we can estimate a posteriori state estimation through the following equation where K_k is the Kalman gain defined in (23)

$$\hat{X}_k = \hat{X}_k^- + K_k(Y_k - \hat{Y}_k^-) \quad (22)$$

$$K_k = P_{X_k Y_k} P_{Y_k}^{-1} \quad (23)$$

and a posterior estimate of error covariance is calculated using the following equation.

$$P_{X_k} = P_{X_{k|k-1}} - K_k P_{Y_k} K_k^T. \quad (24)$$

□

The system dynamics for the acoustic level gauge is presented in (8) and the measurement equation is as follows

$$\begin{bmatrix} Y_{n+1} \\ Y_{n+2} \end{bmatrix} = \begin{bmatrix} \lambda_{n+1} \\ \lambda_{n+2} \end{bmatrix} + \begin{bmatrix} V_{n+1} \\ V_{n+2} \end{bmatrix} \quad (25)$$

where $X_{n+1} = \begin{bmatrix} \lambda_{n+1} \\ \lambda_{n+2} \end{bmatrix}$ and $\begin{bmatrix} V_{n+1} \\ V_{n+2} \end{bmatrix}$ is the measurement noise vector. In addition, P and R_V are 2×2 matrices.

It can be seen that (8) and (25) are in the proper forms of (9) and (10) and consequently they can be applied to the UKF to improve the accuracy of the measurement process.

3. SIMULATION & EXPERIMENTAL RESULTS

Simulation and experimental tests aimed at validating the proposed method were carried out for different values of L. Simulations show that the estimation has good results not only for low noise measurements of λ, but also for very noisy measurements as shown in Fig. 3.

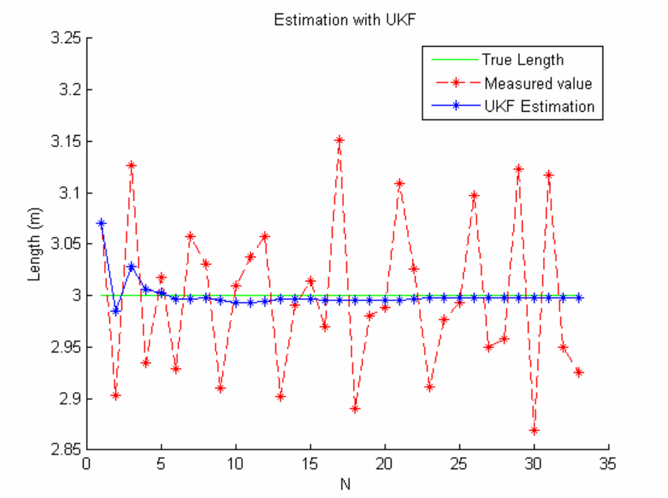

Fig. 3(a)

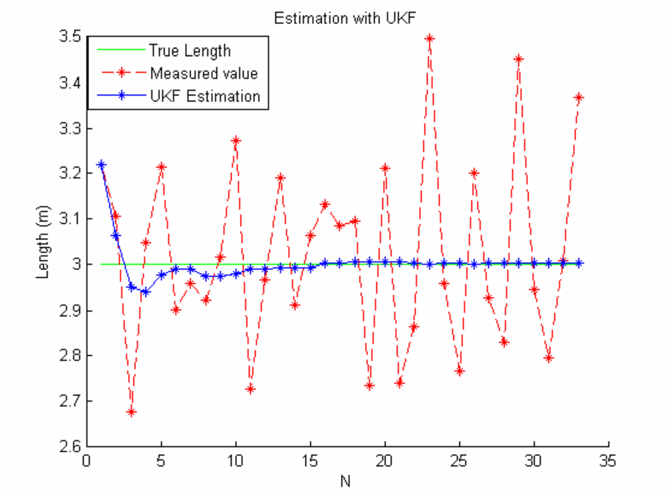

Fig. 3(b)

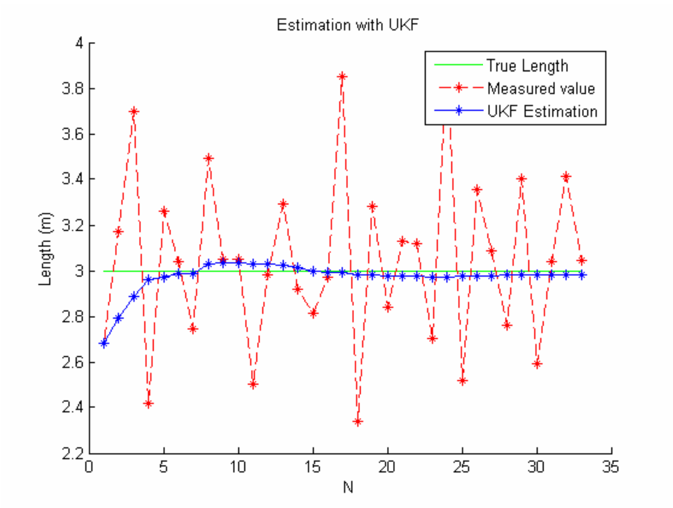

Fig. 3(c)

Fig. 3. Simulation results for a 3-meter long tube in the presence of a zero mean white noise added to the measured resonant wavelengths. Simulation has been accomplished for different measurement noise covariances which are 1.5e-7, 2.9e-6 and 9.8e-6 for Fig. 3(a), Fig. 3(b) and Fig. 3(c), respectively.

An advantage of using UKF is its independence from large transient errors in measured wavelengths. For instance, if a temporary sound source is located near the microphone, then the controller will not find the true acoustic resonance because the sound source can change the location of the resonance wavelength or it can make an impulse which may be distinguished as a resonant wavelength by the controller. Fig. 4 shows a case that the three initial measurements have large errors (which are not zero mean), but it is seen that the UKF can eliminate the effect of these errors. The UKF can also eliminate the effect of these large errors, while these noisy wavelengths are anywhere in the frequency range.

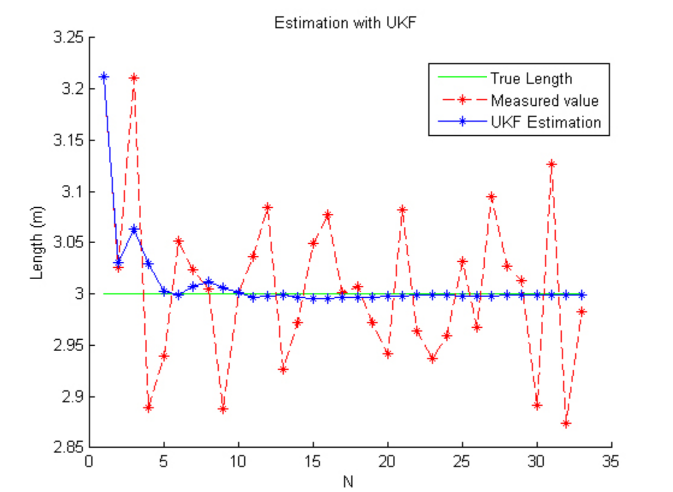

Fig. 4. A simulation result for a 3-meter long tube while some measured wavelengths have large transient error.

Fig. 5 shows the experimental result for a 5.3-meter long tube. In this case, the frequency range of the transmitted signal is from 1480 *Hz* to 2480 *Hz*. It is seen that there are approximately 30 resonant wavelengths in this frequency range. The minimum and maximum measured lengths are 5.212 *m* and 5.406 *m*, respectively, which yield to 2 % error in length measurement; but the minimum and maximum values of estimated lengths are 5.294 and 5.305 for $N>10$ (after transient). So it is seen that the estimated lengths have just 0.11% error.

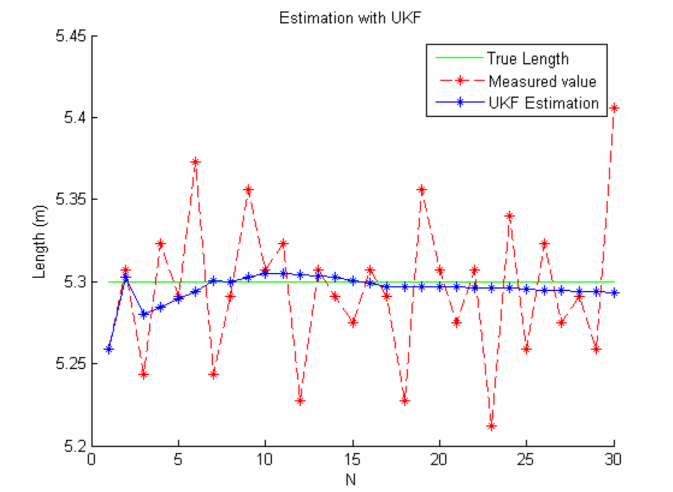

Fig. 5. Experimental result for a 5.3-meter long tube.

Several experiments have been performed for each value of L to find a maximum estimation error (worst case) in each case and to show the efficiency of the proposed method. The maximum estimation error is calculated according to the maximum difference between the true length and the estimated length for each L (after the transient) as shown in table 1. It should be noted that in most cases, the estimation error is smaller that the error given in table 1.

Table 1. Experimental results for different values of L

Experiment No.	True L (cm)	Max. estimation error (cm)	Relative error (%)
1	170	0.40	0.23
2	200	0.31	0.15
3	230	0.45	0.19
4	260	0.30	0.11
5	290	0.31	0.11
6	320	0.39	0.12
7	350	0.44	0.13
8	380	0.48	0.12
9	410	0.42	0.10
10	440	0.34	0.07
11	470	0.60	0.13
12	500	0.33	0.07
13	530	0.60	0.11
14	560	0.57	0.10
15	590	0.90	0.15

It can be seen that the maximum value of error for this method (in the case that low-cost microphone, speaker and temperature sensor have been used) is 0.9 *cm* for $L = 590\,cm$ which is accurate enough for many applications.

4. CONCLUSIONS

A level measurement method based on acoustic resonance has been proposed in this paper. Since the environment noise and the noise in measurements through speaker, microphone, parasitic reflection, etc affect the amplitude of the received signal, a nonlinear filter called the "Unscented Kalman filter" was used which greatly improved the accuracy of the measured level. As a result, this method can be used for level measurement especially in noisy environments or in the cases that low-cost instruments such as cheap microphone and speaker have been used.

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