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Optimal Speed on Small Gradients - Consequences of a Non-Linear Fuel Map

Maria Ivarsson * **Jan Åslund** ** **Lars Nielsen** **

* *Scania CV AB, SE-151 87 Södertälje (e-mail: maria.ivarsson@scania.com)*
** *Linköpings Universitet, SE-581 83 Linköping, Sweden (e-mail: jaasl,lars@isy.liu.se)*

Abstract: Consequences of non-linearities in sfc, i.e. specific fuel consumption, of a heavy truck combustion engine are studied. A quasi-static analysis gives valuable insights into the intrinsic properties of the studied problem, i.e. minimization of fuel consumption on small gradients. Two objective functions are shown to give different optimal velocity trajectories on a constant gradient due to non-linearities in sfc. When a constraint is set to keep to a final time, switching between two speeds is optimal. Instead, if consumed time is part of the objective function, in addition to fuel consumption, keeping to one constant speed is optimal. However, the different optimal solutions still show similarities. For a certain significant non-linearity a specific speed range is shown to be unreachable, independent of objective function. Also under more realistic conditions, a dynamic analysis, implemented by a numerical optimizer, confirms that an alternating speed profile is optimal for the case of fixed final time.

Keywords: Fuel-optimal control; Non-linear engine control.

1. INTRODUCTION

1.1 Background

Consider the problem of driving a heavy truck on small gradients in a fuel optimal way, that also respects the trip time. Many researchers have looked into fuel optimal driving, and one early work is [SL77]. According to other previous works, eg. [FHN06] and [CM05], constant speed is optimal on roads with small gradients given that the engine torque is an affine function of speed and fueling. A small gradient, α, is defined to be a gradient in which the vehicle is able to keep a constant speed, as in [FHN06]. An extension to that work is to consider engines where the assumption of an affine engine map is not valid. One example of an engine map with non-linearities is shown in Figure 1 where the sfc, specific fuel consumption [g/kWh], of a combustion engine of a heavy truck is plotted. Intuitively the mapped data gives the appearance that certain points are more beneficial than others. To give some perspective to the fuel map, the tractor with trailer that will be studied in this paper, has a total mass of 40 000 kg, and it requires a torque of approximately 38% of the maximum torque on level road at the highest gear.

In a traditional combustion engine, without any electric control system, the sfc map typically has a convex shape. The characteristic peaks in sfc of modern engines, shown in Figure 1, emerge because of control strategies that today are possible to implement. After treatment of exhaust gases, optimization of gas flows and engine cooling control are now common for combustion engines of heavy trucks.

In this paper the fuel consumption for a given driving mission on small gradients is calculated for different velocity profiles. Fuel consumption and trip time are two important factors in the economy of a heavy truck long haulage driving mission. It has been noted in the predictive fuel and time optimal control achieved in simulations according to [HFN06], using a non-linear sfc map, that a significant adverse speed is never obtained. This is true regardless of how heavily total time of the driving mission is weighed into the objective function. The reason to this is looked into in the paper at hand. The consequences of the peaks and valleys in sfc will be studied as well as how the optimal velocity profile depends on the used objective function.

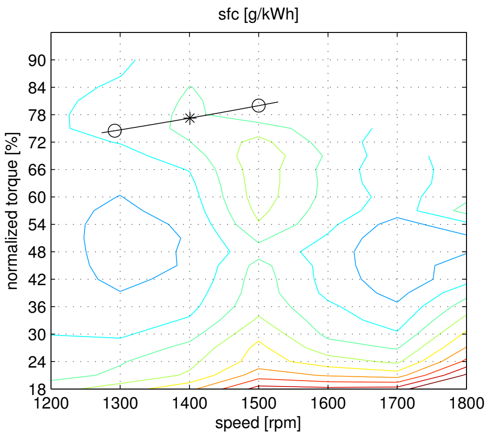

Fig. 1. Curves showing sfc [g/kWh] of a heavy truck combustion engine. The adverse speed 1500 rpm requires a higher sfc than 1300 or 1700 rpm.

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2. PROBLEM FORMULATION

2.1 Model

The truck model used here is a basic longitudinal model. However, it captures the important characteristics to be able to predict fuel consumption correctly. The foundation for the modeling work is found in [KN05, San01].

The model represents a stiff driveline with engine, transmission, final gear, wheels and chassis, based on Newton's second law of motion (1). The braked engine torque is denoted T_e, the vehicle speed, v, and the total driving resistance, F_{res}. The radius of the wheel is denoted r and the total transmission ratio i, which is assumed to be constant. This assumption is based on results from simulations according to [HFN06]. Thus,

$$m\dot{v} = \frac{i}{r}T_e - F_{res} \tag{1}$$

where F_{res} is a sum of rolling resistance (F_r), air drag (F_a) and gravitational force (F_g). These forces are modelled as

$$F_r(\alpha) = c_r mg \cos\alpha \tag{2}$$
$$F_g(\alpha) = mg \sin\alpha \tag{3}$$
$$F_a(N) = \frac{1}{2}c_w A_a \rho_a v^2 \tag{4}$$

where c_r and c_w are constants, m is the total mass of the vehicle, A_a is the maximum vehicle cross section area, ρ_a is the air density and g is the gravitational acceleration.

The vehicle speed [m/s], v, is a direct function of the engine speed [rpm], N, assuming no slip or elasticities

$$v = \frac{r2\pi}{i60}N \tag{5}$$

The consumed fuel mass per meter [mg/m], $\frac{1}{v}\dot{m}_f$, is a function of the engine fueling [mg/stroke], δ. The fueling is in the problem at hand chosen to be modeled as a function of sfc and T_e according to

$$\frac{1}{v}\dot{m}_f = \frac{\delta n_{cyl} i}{n_r r 2\pi} \tag{6}$$
$$\delta = \frac{\text{sfc} T_e n_r 2\pi}{n_{cyl}3600} \tag{7}$$

where the number of cylinders is denoted n_{cyl} and the revolutions per stroke is denoted n_r. The specific fuel consumption of the engine is a function of T_e and N as

$$\text{sfc} = f_{\text{sfc}}(T_e, N) \tag{8}$$

and is only valid for $T_e > 0$. The function $f_{\text{sfc}}(T_e, N)$ is either a model of the mapped data in Figure 1 or the engine map itself. In Section 3 only positive T_e is considered. In Section 4 the full range of torque, from drag torque to maximum torque is optimized over, and for $T_e < 10\%$, δ instead of sfc is received from mapped measured data.

The reduction of fuel consumption, Δm_f is defined as

$$\Delta m_f = \frac{m_{f,v_0} - m_f}{m_{f,v_0}} \tag{9}$$

where m_{f,v_0} is the fuel consumption for a constant speed trajectory and m_f is the fuel consumption of the alternative speed profile.

2.2 Objective functions

Two different objective functions are used for optimization in this paper. Objective function I is commonly used, eg. in [FHN06]. Objective function II gives a simpler optimization and is used in eg. [HFN06].

Objective function I The first objective function is according to (10) with constraints (11) and (12)

$$J = \int_0^s \frac{1}{v}\dot{m}_f ds \tag{10}$$
$$t_f = \int_0^s \frac{1}{v} ds = \frac{s}{v_0} \tag{11}$$
$$s = \int_0^s ds \tag{12}$$

where (10) represents total amount of consumed fuel and (11) is the constraint that ensures that the final time, t_f, will be kept.

Objective function II The second objective function is according to

$$J = \int_0^s \left(\frac{1}{v}\dot{m}_f + \beta\frac{1}{v}\right) ds \tag{13}$$

where the integrals from (10) and (11) are combined. Instead of having the final time, t_f, as a constraint it is here weighted into the objective function with a weighing constant, β.

The advantage of this objective function is that it reduces the number of states by one and the fact that low-order systems are less time consuming to optimize. However, objective function (13) will only give the optimal trade-off between fuel consumption and time (given a certain β).

3. QUASI-STATIC ANALYSIS USING SFC MODEL

The quasi-static analysis in this section assumes that the changes in speed are instantaneous, which implies that neither acceleration nor deceleration is included. This is a good approximation if the studied road stretch is long. The road inclination is assumed to be constant, and the driving scenario of a 0.94% road gradient will be used as an example.

A quasi-static analysis using a model of the sfc-map gives valuable insights into the intrinsic properties of the problem. It turns out that objective function I gives rise to a solution where switching between two speeds is optimal, if the sfc-map is significantly non-linear. Even though objective function II leads to one optimal speed in stationary conditions, also here two velocities play a crucial role in defining an unreachable velocity range. Further on, it is seen that if there are no significant non-linearities in the sfc-map, then the solution turns into that these two velocities are the same, i.e. constant velocity is optimal as known prior to this work, see eg. [FHN06] and [CM05], and there is no longer any speed range that is unreachable. Finally, optimization is performed with the same conditions, except that mapped sfc from measured data is used instead of the sfc model. That last subsection is a validation that the sfc model is good enough to use in an analytical optimization.

3.1 Model based optimization

In the stationary conditions that are considered in this section the acceleration, $\dot{v}$, in (1) is equal to zero. The equations (1) - (5) thereby result in torque as a quadratic polynomial of speed. The sfc model that is chosen to approximate the mapped sfc is a rational function. The sfc is assumed to be a function of speed, N, alone, i.e. $f_{\mathrm{sfc}}(T_e, N) = f_{\mathrm{sfc}}(N)$. This assumption is based on the fact that the torque, T_e, is a function of speed only and that the solution to the problem will render only small variations of torque. The function $\mathrm{sfc}(N)$ is symmetric around N_{sfc_0} (i.e. the speed when the maximum sfc, sfc_0, is achieved) according to

$$\mathrm{sfc}(N) = \frac{C_d}{1 + C_i(N - N_{\mathrm{sfc}_0})^2} + (\mathrm{sfc}_0 - C_d) \tag{14}$$

where C_d, C_i and sfc_0 are constant model parameters for a certain constant road gradient.

This model interprets, for a given driving scenario, an intersection of the engine map. For the studied driving scenario, the intersection is along the thick line in Figure 1 for an interesting speed range.

Objective function I In this subsection the objective function in use is according to (10). If (6) and (7) are used for describing the consumed fuel mass per meter, it is realized that minimizing fuel consumption is equal to minimizing the integral of sfc multiplied with T_e over the distance (since $\frac{i}{r3600}$ is constant). The control variables are thus $u = [N_1 \quad N_2]$ where N_1 and N_2 will be used to propel the truck over the distances s_1 and s_2 respectively. Objective function I can accordingly be rewritten to

$$J = \mathrm{sfc}(N_1)T_e(N_1)s_1 + \mathrm{sfc}(N_2)T_e(N_2)s_2 \tag{15}$$

$$t_f = \frac{s_1}{v_1} + \frac{s_2}{v_2} = \frac{s}{v_0} \tag{16}$$

$$s = s_1 + s_2 \tag{17}$$

where s_1 and s_2 are non-negative.

The problem can be formulated as an optimal control problem, see eg. [BH75]. Hence, the objective function is augmented with the constraints using Lagrange method resulting in

$$\begin{aligned} H = {} & \mathrm{sfc}(N_1)T_e(N_1)s_1 + \mathrm{sfc}(N_2)T_e(N_2)s_2 + \\ & + \lambda_1\left(\frac{s}{v_0(N_0)} - \frac{s_1}{v_1(N_1)} - \frac{s_2}{v_2(N_2)}\right) + \\ & + \lambda_2\left(s - s_1 - s_2\right) - \mu_1 s_1 - \mu_2 s_2 \end{aligned} \tag{18}$$

where λ_1, λ_2, μ_1 and μ_2 are Lagrange multipliers and $\mu_i \geq 0$ if $s_i = 0$, $\mu_i = 0$ if $s_i > 0$ for $i = 1, 2$. The necessary conditions for a stationary value are

$$\frac{\partial H}{\partial N_i} = 0 \tag{19}$$

$$\frac{\partial H}{\partial s_i} = 0 \tag{20}$$

$$\frac{\partial H}{\partial \lambda_i} = 0 \tag{21}$$

$$\frac{\partial H}{\partial \mu_i} = 0 \tag{22}$$

where (22) is relevant only when the inequality constraints are active (i.e. $s_i = 0$, $i = 1, 2$). When the inequality constraints are inactive (19) – (20) are according to

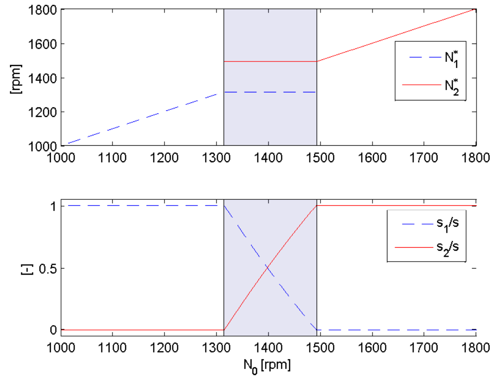

Fig. 2. Optimal speeds and distances for varying N_0 for the driving scenario $\alpha = 0.94\%$ (N_{sfc_0}=1400 rpm). The shaded area shows the unreachable speed range in optimal driving.

$$\begin{aligned} & \mathrm{sfc}'(N_i)T_e(N_i) + \mathrm{sfc}(N_i)T_e'(N_i) + \\ & \qquad + \lambda_1 \frac{1}{v_i(N_i)^2} v_i'(N_i) = 0 \end{aligned} \tag{23}$$

$$\mathrm{sfc}(N_i)T_e(N_i) - \frac{\lambda_1}{v_i(N_i)} - \lambda_2 = 0 \tag{24}$$

for $i = 1, 2$. The optimal control N_1^* and N_2^* is solved from (23) – (24). If, instead, k control variables, i.e. $u = [N_1 \quad N_2 \quad N_3 \quad \ldots \quad N_k]$, would have been used, the solution to the $2k$ equations would still result in only two optimal speeds, equal to N_1^* and N_2^*. The same solution is found for all final times, $t_f = \frac{s}{v_0(N_0)}$, giving a mean average speed, N_0, according to, $N_1^* \leq N_0 \leq N_2^*$.

When the optimal speeds have been found, the corresponding distances, s_1 and s_2, can be determined from the constraints (21). To fulfill the constraints, the distances, s_1 and s_2, are adjusted. When $N_0 \to N_i^*$ then $s_i \to s$.

A valid stationary point for the studied driving scenario is N_1^* =1314 and N_2^* =1492 rpm. The fuel consumption is reduced by -0.72%, if the road is travelled with N_1^* for 51.5% of the road stretch and N_2^* for 48.5% of the road stretch instead of constant speed, N_0=N_{sfc_0}=1400 rpm. It can be shown that this solution is a minimum, generating the lowest fuel consumption.

In Figure 2 it is shown how the optimal control, N_1^* and N_2^*, depends on N_0. It can be concluded that if the non-linearity is significant enough (as it is in the studied engine map), then the speed range (N_1^*,N_2^*) becomes unreachable for optimal driving. To fulfill the constraint of final time, the bottom subfigure shows how the distance on each speed is adjusted.

Objective function II By combining (13) and (6) – (7) objective function II is rewritten to

$$J = \int_0^s \left(C_0 \mathrm{sfc} T_e + \beta \frac{1}{v}\right) ds \tag{25}$$

where $C_0 = \frac{i}{r3600}$. The variables sfc, T_e and v are all functions of N. This objective function results in one optimal speed, in the stationary conditions that are considered. The stationary solutions of (25) are given by $\frac{dJ}{dN} = 0$. Since stationary conditions are considered, the

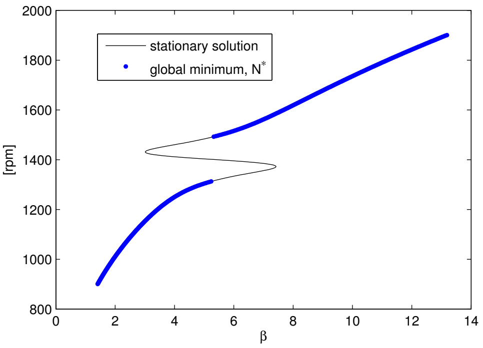

Fig. 3. Stationary solutions, $\hat{N}$, and global minimum, N^*, of objective function II at 0.94% inclination.

stationary solutions to the objective function can also be found by studying the minimum of the integrand of the objective function (denoted I, i.e. (25) is $J = \int_0^s Ids$). Thereby, the necessary condition is $\frac{dI}{dN} = 0$ which gives

$$C_0\left(sfc'(N)T_e(N) + sfc(N)T_e'(N)\right) - \\ - \beta\frac{1}{v(N)^2}v'(N) = 0 \quad (26)$$

The stationary solution of (26), $\hat{N}$, is a function of β if studied locally, see Figure 3. For a given β far from the non-linearity there is one single stationary solution. But, if β is close to the non-linearity there exists three solutions to (26). One of these stationary solutions is a local maximum and two are local minima. This is studied by differentiating (26) with respect to β which leads to

$$\frac{d^2I}{dN^2}\frac{d\hat{N}}{d\beta} = \frac{1}{v^2}v' \quad (27)$$

Since $\frac{1}{v^2}v'$ is always positive, both $\frac{d^2I}{dN^2}$ and $\frac{d\hat{N}}{d\beta}$ must always be either positive or negative. This means that whenever the derivative of the function $\hat{N}(\beta)$ in Figure 3 is positive, there is a local minimum of the objective function.

The thick lines in Figure 3 show the global minimum, N^*. The discontinuity in speed that appears in the global minimum means that it is impossible to achieve a speed within that range when using objective function II. There is no β that can weigh up for the increased fuel consumption for speeds in this gap. The unreachable speed range for objective function II is denoted (N_A, N_B).

3.2 Analyzing model based optimization

Similarities between objective functions The unreachable speed ranges that arise with objective function I (N_1^*, N_2^*) and II (N_A, N_B) are analyzed here. For objective function II, if $f(N) = \text{sfc}(N)T_e(N)$, the equations that give N_A and N_B can be summarized as

$$C_0 f'(N_A) - \\ -\beta\frac{1}{v_A(N_A)^2}v_A'(N_A) = 0 \quad (28)$$

$$C_0 f'(N_B) - \\ -\beta\frac{1}{v_B(N_B)^2}v_B'(N_B) = 0 \quad (29)$$

$$C_0 f(N_A) + \frac{\beta}{v_A(N_A)} = C_0 f(N_B) + \frac{\beta}{v_B(N_B)} \quad (30)$$

where (28) and (29) aim to find two stationary solutions (N_A and N_B) for the same β and the last equation says that these solutions shall give the same objective function value.

The same set of equations (but with $N_1^* = N_A$ and $N_2^* = N_B$) is achieved if (23) – (24) is rewritten with $f(N_i) = \text{sfc}(N_i)T_e(N_i)$, $\beta = -\lambda_1 C_0$ and λ_2 is eliminated. This means that the unreachable speed ranges for the different objective functions are equal. Anyhow, the behavior of the systems will differ depending on the chosen objective function. The solution of objective function II is always a stationary speed, that will not give a final time comparable with any speed within the adverse speed range, (N_A, N_B). Objective function I requires a certain final time, which leads to a solution that switches between two speeds, N_1^* and N_2^*, if the final time is comparable to a mean velocity, N_0, between N_1^* and N_2^*.

Characteristics of sfc This subsection aims to answer the question '*How large must the non-linearity of sfc be to generate an unreachable velocity range*'. The optimal way of driving is calculated based on the sfc model. The non-linearity in sfc is described by the model parameter ratio $\frac{C_d}{\text{sfc}_0}$. This is chosen as an important ratio since $C_d = \text{sfc}_0 - \text{sfc}(|N - N_{\text{sfc}_0}| \to \infty)$. The model parameter C_i is also of great interest and it will be shown that the critical parameter ratio $\left(\frac{C_d}{\text{sfc}_0}\right)_{crit}$ that gives an unreachable speed range is a function of C_i.

The exact value of the critical parameter ratio is calculated by requiring necessary conditions for stationary solutions of objective function II, $\frac{dI}{dN} = 0$. With a decreasing $\frac{C_d}{\text{sfc}_0}$, the maximum of the cost function (close to N_{sfc_0}) becomes a minimum, i.e. the nonlinearity turns into being insignificant. This happens when the model parameter ratio has decreased to its critical value.

For this critical parameter setting, the three solutions shown in Figure 3 become a triple root (shown in Figure 4). Continuity is achieved in the function, $\hat{N}(\beta)$. If the equation $\frac{dI}{dN} = 0$ has a triple root it follows that also the second and the third derivative of the cost function has to be equal to zero. Summarized, the equations to be solved are according to

$$\frac{dI}{dN} = 0,\ \frac{d^2I}{dN^2} = 0,\ \frac{d^3I}{dN^3} = 0 \quad (31)$$

In these equations there are five unknowns (C_d, C_i, sfc_0, N and β) for a given driving mission with a constant gradient. N and β are solved for and the critical ratio $\left(\frac{C_d}{\text{sfc}_0}\right)_{crit}$ is presented as a function of C_i.

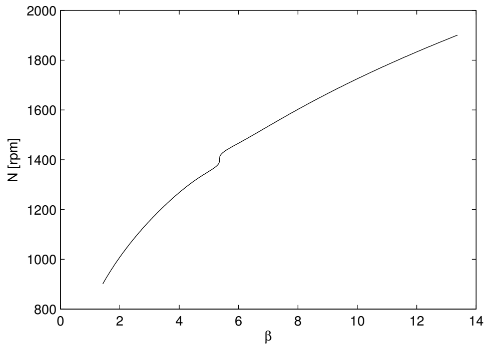

Fig. 4. Stationary solution of objective function II, at 0.94% inclination if $\frac{C_d}{\mathrm{sfc}_0} = \left(\frac{C_d}{\mathrm{sfc}_0}\right)_{crit}$.

The critical parameter ratio $\left(\frac{C_d}{\mathrm{sfc}_0}\right)_{crit}$ is dependent on e.g. the load except for C_i. The higher the load, i.e. α, the lower is the critical ratio. The dependency of the critical ratio to C_i for the studied driving scenario is according to Figure 5. If the model parameters are above the curve, the non-linearity is significant and constant speed close to the non-linearity is not optimal.

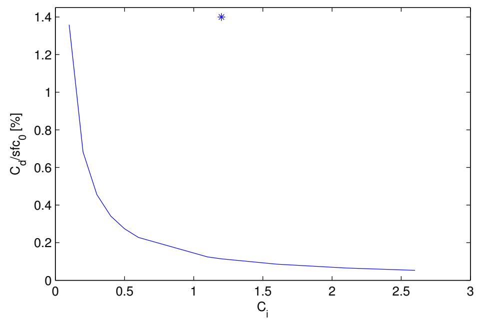

Fig. 5. The curve shows the critical model parameter settings. The $*$ above the curve points out the model parameters, $\frac{C_d}{\mathrm{sfc}_0}$ and C_i of the studied driving scenario.

3.3 Optimization based on use of engine map

In this subsection the engine map itself is used instead of a model to describe the sfc. Objective function I is used and the optimization is implemented by the use of Matlab [Mat07] and its function *fmincon*.

The solution is N_1^*=1292 and N_2^*=1500 rpm for the driving mission $\alpha = 0.94\%$, which is quite close to the result that was obtained when the sfc model was used. The fuel reduction of -0.76% is also similar to what was expected when the sfc model was used. In Figure 1 it is shown how the sfc and T_e changes by driving the truck with the optimal speeds N_1^* and N_2^* (marked by $\circ$) instead of constant speed N_0 (marked by $*$), a speed that gives a high sfc. Hence, the fact remains that it is not optimal to drive with a constant speed that gives a significantly higher sfc compared to the sfc in the surroundings of the engine map.

The same optimal control, N_1^* and N_2^*, is achieved, for a given driving mission, for all required final times that correspond to a constant average speed, N_0, within the speed range (N_1^*, N_2^*). This is analogous to the result shown in Figure 2, confirming that the sfc model is relevant to use in the analytical optimization.

4. DYNAMIC ANALYSIS USING MAPPED DATA

In this section, objective function I is utilized, i.e. the final time is constrained. Accelerations and decelerations are not neglected and there are no restrictions on the speed trajectories, except restrictions that originate from the dynamics of the system or the resolution of the control variables. To include all possible working points, mapped data is used instead of a model, and a numerical optimization is performed. Similarly as in Subsection 3.3 the non-linear optimizer *fmincon* is used.

The control variables are gathered in a vector, $\mathbf{u}$, which includes the n discrete torque variables, with a certain resolution, Δs, over the road stretch with a total distance s, ($\sum_1^n \Delta s = s$). The control vector, $\mathbf{u}$, also includes the velocity at the start of the driving mission.

The altitude profile used in Figure 6 follows the standards of the Swedish Road Administration for vertical curves, see [okV04]. An optimization is performed for a 2100 m driving mission starting with an uphill gradient ($\alpha = 0.94\%$) followed by a downhill gradient ($\alpha = -0.94\%$), with a resolution of $\Delta s = 30$ m. The alternating speed profile decelerates in uphill gradients and accelerates in downhill gradients. At the top of the altitude profile, the speed is at its lowest level. The fuel consumption reduction in this example is $\Delta m_f = -2.57\%$.

In [HIÅN07] an MPC control was implemented in a truck and test runs were made over a road segment between Södertälje and Norrköping, Sweden. This road stretch has significant hills and fuel consumption reduction could therefore be achieved by reducing the need for braking and gear shifting. Anyhow, one part of this road stretch (south of Nyköping) mainly has small gradients. A 3350 m part of that road segment has been simulated both with constant speed and optimal control (which has been computed with

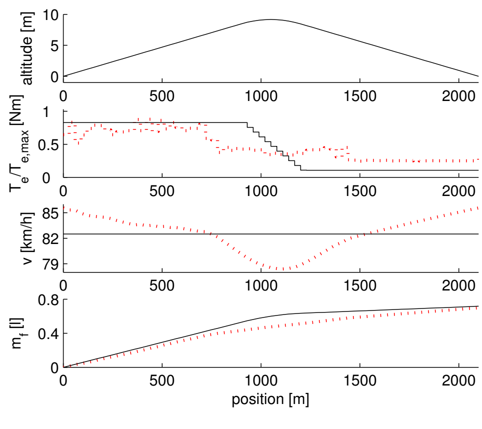

Fig. 6. Dotted - optimal control. Solid - constant speed control. $\Delta m_f = -2.57\%$ in a $\alpha = \pm 0.94\%$ 2100 m road stretch

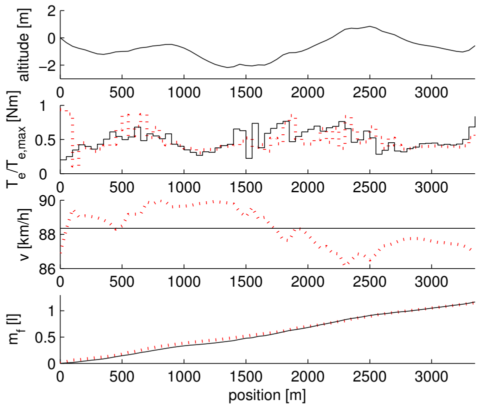

Fig. 7. Dotted - optimal control. Solid - constant speed control ($\Delta m_f = -1.1\%$)

Δs=50 m) and the results are shown in Figure 7. Even in this real road segment, the strategy of accelerating in negative gradients and decelerating in positive gradients can be seen in the optimal control.

This section has shown that it is possible to save fuel when more realistic conditions are considered. A general result from the dynamic analysis is that it is beneficial to vary the speed around an adverse speed (if the requirement is to finish the driving mission in a set time that correlates to this adverse speed). Hence, the dynamic analysis correlates to the quasi-static results.

5. CONCLUSIONS

It has been shown how the non-linearities in sfc of modern engines influence the fuel optimal way of driving a heavy truck on small gradients. Two objective functions have been studied. In a quasi-static analysis a model of the sfc is used to analyze driving missions with constant gradient. Objective function I, that minimizes the fuel consumption while keeping to a constraint on the final time, gives that it is optimal to switch between two speeds, N_1^* and N_2^*, on each side of the sfc-peak, if the non-linearity in sfc is significant enough. It is also seen that the solution (N_1^* and N_2^*) persists even if the required final time is changed. Instead, the distances, s_1 and s_2, traveled with each speed are adjusted to reach the end of the distance in the required final time. If the required final time correlates to a constant speed that is not in the speed range (N_1^*, N_2^*), then the optimal control is constant speed.

Objective function II, that weighs in both fuel consumption and final time in the criterion, generates a stationary optimal speed on constant gradients regardless of the appearance of the sfc map. For this objective function, the weighing constant β determines the optimal speed, N^*. However, even for this objective function, there is a speed range (N_A, N_B) around significant non-linearities in sfc that can never be reached. There is no β that can weigh up for the increased fuel consumption within this speed range. The optimal speed will either be above or below the speed range. The unreachable speed ranges of the two objective functions are found to correlate to each other according to $N_A = N_1^*$ and $N_B = N_2^*$.

The characteristics of sfc has also been studied, and in particular at what point a non-linearity in sfc gives a speed range that is unreachable in optimal driving, i.e. a significant non-linearity. The optimal control changes if the non-linearity becomes significant, as described above.

A dynamic analysis has also been performed by the use of a numerical optimizer that utilizes objective function I and mapped data from engine measurements. The same characteristics of optimal control is achieved by this optimizer. If there exists significant non-linearities in sfc around a required speed, an alternating speed trajectory turns out to be beneficial.

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