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Backstepping Technique for tracking control of an underactuated surface vessel with unmeasured thruster dynamic

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Abstract: We consider in this paper the tracking control problem of an underactuated surface vessel moving on the horizontal plane. A reference feasible trajectory for the position and orientation of the surface vessel is planned so that it is consistent with vehicle dynamics. Using these reference values the dynamics of the vehicle is transformed to the error cascade structure. The proposed controller is designed using Lyapunov's direct method and the popular backstepping techniques to force the tracking error to globally exponentially stabilized at the origin. Extension to unmeasured thruster dynamics is also considered. Simulation results that validate the proposed tracking methodology are presented and discussed.

1. INTRODUCTION

This paper concentrates on the global tracking control of surface ships with only sway force and yaw moment available. The study is interested in designing a controller such that it makes the position (sway and surge) and orientation (yaw angle) of surface ships track the reference position and orientation generated by a virtual reference ship. Since the interested surface ships have fewer numbers of actuators than degrees of freedom to be controlled and the constraint on the acceleration [7] is nonintegrable, they are a class of underactuated systems with nonintegrable dynamics. [5] used a continuous time invariant state feedback controller to achieve global exponential position tracking under an assumption that the reference surge velocity is always positive. Unfortunately, the orientation of the ship was not controlled. [6] provided a high gain based semiglobal tracking result. [1] designed a global tracker based on a transformation of the ship tracking system into the so-called convenient form.

In this paper, we consider the tracking control problem of an underactuated surface vessel. Under realistic assumptions, we propose new tracking controller with the aid of the cascade structure of the closed loop system. We follow the same idea of our previous work to transform the system into a pure cascade form [4]. We show through some key properties of the model that the tracking problem of the resulting cascade system can be reduced to the tracking problem of a system consisting of third order chained form. Furthermore, the surface vessel usually operates in open sea subject to environmental disturbances. By exploiting the cascade structure of the underactuated surface vessel, we design a controller that compensate for the constant or slowly-varying bias of the disturbances. We also discuss the extension of our proposed controller to the case of unmeasured thruster dynamics.

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2. PROBLEM STATEMENT

Consider an underactuated surface vessel discussed in [8]. It has two propellers which are the force in surge and the control torque in yaw. the kinematics of the system can be written as

$$\dot{\eta} = \mathbf{R}\nu$$
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \tag{1}$$

where (x, y) denotes the coordinate of the mass center of the surface vessel in the earth fixed frame, ψ is the orientation of the vessel, and u, v and r are the velocities in surge, sway and yaw, respectively. We assume that the inertia, added mass and damping matrices are diagonal. The dynamics of the surface vessel can be written as

$$\begin{aligned} \dot{u} &= \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} \tau_1 \\ \dot{v} &= -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v \\ \dot{r} &= \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_2 \end{aligned} \tag{2}$$

where $m_{ii} > 0$ are given by the vessel inertia and the added mass effects, $d_{ii} > 0$ are given by the hydrodynamic damping, m_{ii} and d_{ii} are assumed to be constant. τ_1 and τ_2 are the surge control force and the yaw control moment, respectively.

Consider the system (1)-(2). Assume given a feasible bounded reference trajectory $(x_r, y_r, \psi_r, u_r, v_r, r_r)$ with reference input (τ_{1r}, τ_{2r}) satisfying

$$\begin{aligned} \dot{x}_d &= u\cos(\psi_d) - v\sin(\psi_d) \\ \dot{y}_d &= u\sin(\psi_d) + v\cos(\psi_d) \\ \dot{\psi}_d &= r_d \end{aligned}$$

$$\begin{aligned}
\dot{u}_d &= \frac{m_{22}}{m_{11}} v_d r_d - \frac{d_{11}}{m_{11}} u_d + \frac{1}{m_{11}} \tau_{1d} \\
\dot{v}_d &= -\frac{m_{11}}{m_{22}} u_d r_d - \frac{d_{22}}{m_{22}} v_d \\
\dot{r}_d &= \frac{m_{11} - m_{22}}{m_{33}} u_d v_d - \frac{d_{33}}{m_{33}} r_d + \frac{1}{m_{33}} \tau_{2d}
\end{aligned} \tag{3}$$

To facilitate the controller design, we apply a state and an input transformation to system(1)-(2) as described in [4], leads to the following cascade system:

$$\begin{aligned}
\dot{z}_1 &= -\frac{B}{A} z_1 - \frac{B}{A} \alpha + \left(Z_2 - \frac{v}{B} \right) r \\
\dot{v} &= -Bv + B\left(z_1 + \alpha\right) r \\
\dot{Z}_2 &= \alpha r \\
\dot{z}_3 &= r \\
\dot{\alpha} &= \tau_\alpha \\
\dot{r} &= \tau_r
\end{aligned} \tag{4}$$

where $A = \frac{m_{11}}{m_{22}}$, $B = \frac{d_{22}}{m_{22}}$ and the state transformation

$$\begin{cases}
z_1 &= x \cos\psi + y \sin\psi \\
Z_2 &= z_2 + \dfrac{v}{B} \\
z_2 &= -x \sin\psi + y \cos\psi \\
z_3 &= r \\
\alpha &= -z_1 - \dfrac{A}{B} u \\
\tau_\alpha &= \dfrac{B}{A}(\mathbf{z}_1 + \alpha) - r(Z_2 - \dfrac{v}{B}) - \dfrac{A}{B}\tau_u \\
\tau_u &= \dfrac{m_{22}}{m_{11}} vr - \dfrac{d_{11}}{m_{11}} u + \dfrac{1}{m_{11}} \tau_1 \\
\tau_r &= \dfrac{m_{11} - m_{22}}{m_{33}} uv - \dfrac{d_{33}}{m_{33}} r + \dfrac{1}{m_{33}} \tau_2
\end{cases} \tag{5}$$

Similarly, by the transformation

$$\begin{cases}
z_{1d} &= x_d \cos\psi_r + y_d \sin\psi_d \\
Z_{2d} &= z_{2d} + \dfrac{v_d}{B} \\
z_{2d} &= -x_d \sin\psi_d + y_d \cos\psi_d \\
z_{3d} &= r_d \\
\alpha_d &= -z_{1d} - \dfrac{A}{B} u_d \\
\tau_{\alpha_d} &= \dfrac{B}{A}(\mathbf{z}_{1d} + \alpha_d) - r_d(Z_{2d} - \dfrac{v_d}{B}) - \dfrac{A}{B}\tau_{ud} \\
\tau_{ud} &= \dfrac{m_{22}}{m_{11}} v_d r_d - \dfrac{d_{11}}{m_{11}} u_d + \dfrac{1}{m_{11}} \tau_{1d} \\
\tau_{r_d} &= \dfrac{m_{11} - m_{22}}{m_{33}} u_d v_d - \dfrac{d_{33}}{m_{33}} r_d + \dfrac{1}{m_{33}} \tau_{2d}
\end{cases}$$

we have

$$\begin{aligned}
\dot{z}_{1d} &= -\frac{B}{A} z_{1d} - \frac{B}{A} \alpha_{\mathbf{d}} + \left(Z_{2d} - \frac{v_d}{B} \right) r_d \\
\dot{v}_d &= -Bv_d + B\left(z_{1d} + \alpha_{\mathbf{d}}\right) r_d \\
\dot{Z}_{2d} &= \alpha_{\mathbf{d}} r_d \\
\dot{z}_{3d} &= r_d \\
\dot{\alpha_{\mathbf{d}}} &= \tau_\alpha \\
\dot{r}_d &= \tau_d
\end{aligned} \tag{6}$$

Next, we define the tracking errors

$$\begin{aligned}
&e_1 = z_1 - z_{1d} \;\; e_2 = v - v_d \;\; e_3 = Z_2 - Z_{2d} \\
&e_4 = z_3 - z_{3d} \;\; e_5 = \alpha - \alpha_d \quad e_6 = r - r_d
\end{aligned}$$

The error dynamic is then given as follow

$$\sum\nolimits_1 : \begin{cases}
\dot{e}_1 &= -\dfrac{B}{A} e_1 - \dfrac{B}{A} e_5 + (e_3 e_6 + r_d e_3 - Z_{2d} e_6) \\
&- \dfrac{1}{B}(e_2 e_6 + r_d e_2 - v_d e_6) \\
\dot{e}_2 &= -Be_2 + B(e_5 e_6 + r_d e_5 + \alpha_d e_6 \\
&+ e_1 e_6 + r_d e_1 - z_{1d} e_6)
\end{cases}$$

$$\sum\nolimits_2 : \begin{cases}
\dot{e}_3 &= e_5 e_6 + r_d e_5 + \alpha_d e_6 \\
\dot{e}_4 &= e_6 \\
\dot{e}_5 &= \tau_u - \tau_{ud} \\
\dot{e}_6 &= \tau_r - \tau_{rd}
\end{cases}$$

The control problem is then transformed into a stabilization problem of the error dynamics.

Proposition 1 *The states of $\sum_1$ can be made bounded and exponentially convergent if the states of $\sum_2$ are bounded and exponentially convergent*

proof For $\sum_1$, consider the following Lyapunov function

$$V_1 = 0.5 e^\top \Delta e \tag{7}$$

where $e = (e_1, e_2)^\top$ and $\Delta = diag(B, \frac{1}{B})$. It's derivative along the solutions of $\sum_1$ gives

$$\begin{aligned}
\dot{V}_1 &= Be_1\dot{e}_1 + \frac{1}{B} e_2 \dot{e}_2 \\
&= -\frac{B^2}{A} e_1^2 - e_2^2 - \frac{B^2}{A} e_1 e_5 + Be_1(e_3 e_6 + r_d e_3 - Z_{2d} e_6) \\
&\quad -e_1 e_2 e_6 - e_1 e_2 r_d + v_d e_6 e_1 + e_2 e_5 e_6 + e_2 r_d e_5 \\
&\quad +\alpha_d e_2 e_6 + e_2 e_1 e_6 + r_d e_1 e_2 - z_{1d} e_2 e_6
\end{aligned} \tag{8}$$

Since $(x_d, y_d, u_d, v_d, r_d)$ and (τ_{1d}, τ_{2d}) are bounded and subsequently are the variables $(z_{1d}, Z_{2d}, \alpha_d)$, it follows, by taking norms, $\dot{V}$ satisfies

$$\dot{V}_1 \le -a_1 V_1 + a_2(t)\sqrt{V_1} \tag{9}$$

where

$$\begin{aligned}
a_1 &= 2min\{B, \frac{B}{A}\} \\
a_2(t) &= \sqrt{2B}\Big(\Big|\frac{B^2}{A} e_5\Big| + \Big|Be_3 e_6\Big| + \Big|Br_d e_5\Big| \\
&\quad +\Big|BZ_{2d} e_6\Big| + \Big|v_d e_6\Big| + \Big|\alpha_d e_6\Big| + \Big|e_6 e_5\Big|\Big)
\end{aligned}$$

Note that by assumption (Proposition 1) on $\sum_2$, we have that $a_2(t)$ is bounded and exponential convergent. Performing the change of variables $\varphi(t) = \sqrt{V_1}$ to obtain a linear differential inequality and using the fact that $\dot{\varphi} = \frac{\dot{V}_1}{2\sqrt{V_1}}$, it follows, when $V_1 \neq 0$, that

$$\dot{\varphi} \le -\frac{a_1}{2}\varphi + \frac{1}{2} a_2(t) \tag{10}$$

When $V_1 = 0$, it can be shown [3], that the upper right-hand derivative $D^+\varphi$ satisfies $D^+\varphi \le \frac{a_2(t)}{2}$ and consequently inequality (10) is satisfied for all values of V_1. Thus applying the comparison lemma [3], φ satisfies

$$\varphi(t) \leq \varphi(t_0)e^{-\frac{a_1}{2}(t-t_0)} + \frac{1}{2}\int_{t_0}^{t} e^{-\frac{a_1}{2}(t-\theta)}|a_2(\theta)|d\theta \quad (11)$$

and consequently

$$\begin{aligned}\left(\left\| \begin{matrix} e_1(t) \\ e_2(t) \end{matrix} \right\|\right) &\leq \left(\left\| \begin{matrix} e_1(t_0) \\ e_2(t_0) \end{matrix} \right\|\right)e^{-\frac{a_1}{2}(t-t_0)} \\ &+ \frac{1}{2\sqrt{a_1}}\int_{t_0}^{t} e^{-\frac{a_1}{2}(t-\theta)}|a_2(\theta)|d\theta \\ &\leq \left(\left\| \begin{matrix} e_1(t_0) \\ e_2(t_0) \end{matrix} \right\|\right)e^{-\frac{a_1}{2}(t-t_0)} \\ &+ \frac{1}{\sqrt[3]{a_1}}[1-e^{-\frac{a_1}{2}(t-t_0)}]\sup_{t\geq t_0}|a_2(t)| \quad (12)\end{aligned}$$

Since $a_1 > 0$ and by assumption that $a_2(t)$ is bounded and converges exponentially to zero, then it follows from (12) that there exists $\sigma_0 > 0$ and a γ class-$\mathcal{K}$ function such that

$$\left(\left\| \begin{matrix} e_1(t) \\ e_2(t) \end{matrix} \right\|\right) \leq \gamma\Big(\|e_{\sum_1}(t_0), e_{\sum_2}(t_0)\|\Big)e^{-\sigma_0(t-t_0)} \quad (13)$$

where $e_{\sum_1} = (e_1, e_2)^\top$ and $e_{\sum_2} = (e_3, e_4, e_5, e_6)^\top$. □

3. CONTROLLER DESIGN

Noting the result in Proposition 1, it is only needed to design a stabilizing control law for $\sum_2$. In the following, a nonlinear control law is proposed for $\sum_2$. For subsystem $\sum_2$, we have the following result.

Lemma 1 For subsystem $\sum_2$, if $(x_r, y_r, \psi_r, u_r, v_r, r_r)$ is bounded and r_r satisfies

$$\int_0^t r_r^2(\theta)d\theta \geq \delta t \quad \forall t \geq 0 \quad (14)$$

where δ is a positive constante, the control inputs

$$\begin{aligned}\tau_\alpha &= \tau_{\alpha_d} - k_3\dot{r}_d e_3 - k_3 r_d(e_5 e_6 + r_d e_5 + \alpha_d e_6) \\ &\quad - k_5 e_5 - (k_3 k_4 - 1)r_d e_3 - e_3 e_6 \\ \tau_r &= \tau_{rd} - (k_1 + k_2)e_6 - k_1(k_1 + k_2)e_4\end{aligned} \quad (15)$$

ensure $e_{\sum_2}$ globally exponentially converges to zero, where control parameters $k_i > 0, (1 \leq i \leq 4)$. By proposition 1 and Lemma 1 we have the following theorem.

Theorem 1 *For systems $\sum_1$ and $\sum_2$, if $(x_r, y_r, \psi_r, u_r, v_r, r_r)$ is bounded and r_r satisfies (14), control law (15) ensure $e_i > 0, (1 \leq i \leq 6)$ globally exponentially converge to zero, where control parameters $k_i > 0, (1 \leq i \leq 4)$.*

The exponential convergence rate of subsystem $\sum_2$ can be adjusted by the controller gains $k_i (1 \leq i \leq 4)$. However the convergence rate of subsystem $\sum_1$ is governed by the vessel parameters d_{22}, m_{11}, m_{22} and the convergence rate of sates of subsystem $\sum_2$ that is $(e_3, e_4, e_5, e_6)^\top$.

4. ROBUSTNESS ISSUES

The aim of this section is to discuss a simple way to design a controller to handle disturbances caused by wave drift, currents and mean wind forces. When disturbances are present, the dynamic part of the surface vessel (2) can be written as

$$\begin{aligned}\dot{u} &= \frac{m_v}{m_u}vr - \frac{d_u}{m_u}u + \frac{1}{m_u}\tau_1 + \tau_{udis} \\ \dot{v} &= -\frac{m_u}{m_v}ur - \frac{d_v}{m_v}v + \tau_{vdis} \\ \dot{r} &= \frac{m_{uv}}{m_r}uv - \frac{d_r}{m_r}r + \frac{1}{m_r}\tau_3 + \tau_{rdis}\end{aligned}$$

where τ_{udis}, τ_{vdis} and τ_{rdis} being the disturbances acting on the surge, sway and yaw axes respectively. We first deal with the disturbance component on the sway direction τ_{vdis}. The idea is to handle τ_{vdis} through designing an observer to estimate τ_{vdis} as

$$\begin{aligned}\dot{\hat{v}} &= k_{x_1}\tilde{v} - Bur - B\hat{v} + \hat{\tau}_{vdis} \\ \dot{\hat{\tau}}_{vdis} &= k_{x_2}\tilde{v}\end{aligned} \quad (16)$$

where $\tilde{v} = v - \hat{v}$. Clearly, the estimate errors $\tilde{v}$ and $\tilde{\tau}_{vdis}$ are asymptotically exponentially stable if all roots of the characteristic polynomial $H(s) = s^2 + k_{x_1}s + k_{x_2}$ associated with the system

$$\begin{bmatrix} \dot{\tilde{v}} \\ \dot{\tilde{\tau}}_{vdis} \end{bmatrix} = \begin{bmatrix} -k_{x_1} & 1 \\ -k_{x_2} & 0 \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\tilde{\tau}}_{vdis} \end{bmatrix}$$

have strictly negative real parts. Hence the desired sway dynamic to track should be like below:

$$\dot{v}_d = -Au_d r_d - Bv_d + \hat{\tau}_{vdis} \quad (17)$$

It is now straightforward to conclude the following lemma which will be useful for establishing the convergence of the closed-loop system.

Lemma 2 : *Suppose that the ocean current disturbance is constant. Consider the observer system (16), where the gains k_{x_1} and k_{x_2} are chosen such that the observer system is asymptotically stable. Then, the variables $\tilde{v}, \tilde{\tau}_{vdis}, \hat{v}, \hat{\tau}_{vdis}, \dot{\hat{\tau}}_{vdis}, v_d, \dot{v}_d$ are bounded. Moreover, the the errors $\tilde{v}$ and $\tilde{\tau}_{vdis}$ converge to zero as t goes to infinity*

The error tracking cascade form together with disturbances being introduced and the observer (16) is written as bellow

$$\sum_1^{'} : \begin{cases} \dot{e}_1 = -\frac{B}{A}e_1 - \frac{B}{A}e_5 + (e_3e_6 + r_de_3 - Z_{2d}e_6) \\ \quad -\frac{1}{B}(e_2e_6 + r_de_2 - v_de_6) \\ \dot{e}_2 = -Be_2 + B(e_5e_6 + r_de_5 + \alpha_de_6 \\ \quad +e_1e_6 + r_de_1 - z_{1d}e_6) + \tilde{\tau}_{vdis} \end{cases}$$
$$\sum_2^{'} : \begin{cases} \dot{e}_3 = e_5e_6 + r_de_5 + \alpha_de_6 + \frac{\tilde{\tau}_{vdis}}{B} \\ \dot{e}_4 = e_6 \\ \dot{e}_5 = \tau_\alpha - \tau_{\alpha_d} - \frac{A}{B}\tau_{udis} \\ \dot{e}_6 = \tau_r - \tau_{rd} + \tau_{rdis} \end{cases} \tag{18}$$

Similarly as in section 3, a stabilizing control for subsystem $\sum_2^{'}$ is needed to ensure the stabilization of the cascade system. See the statement of the theorem below which proof is omitted due to space limitation.

Theorem 2 *Consider the nonlinear invariant system $\sum_{Cascade+Obs}$ composed by the interconnected system (18), the current observer (16), and the control law*

$$\begin{aligned} \tau_\alpha &= \tau_{\alpha_d} - k_3\dot{r}_de_3 - k_3r_d(e_5e_6 + r_de_5 + \alpha_de_6) \\ &\quad -k_5e_5 - (k_3k_4 - 1)r_de_3 - e_3e_6 - \frac{A}{B}\hat{\tau}_{udis} \\ \tau_r &= \tau_{rd} - (k_1 + k_2)e_6 - k_1(k_1 + k_2)e_4 - \hat{\tau}_{rdis} \end{aligned} \tag{19}$$

where k_1, k_2, k_3 and k_4, are positive constants. Let the update law for the unknown disturbances components τ_{udis} and τ_{rdis} be given as

$$\begin{aligned} \dot{\hat{\tau}}_{udis} &= \gamma_{01}proj(e_5, \hat{\tau}_{udis}) \\ \dot{\hat{\tau}}_{rdis} &= \gamma_{02}proj(e_6, \hat{\tau}_{udis}) \end{aligned} \tag{20}$$

where γ_{01} and γ_{02} are the adaptation gains and the operator proj represents the Lipschitz projection algorithm [10]. Let $\mathcal{X}_{Cascade+Obs} : [t_0, \infty) \to R^7$, $t_0 \geq 0, \mathcal{X}_{Cascade+Obs} := (e_1, e_2, e_3, e_4, e_5, e_6, \tilde{\tau}_{vdis})^\top$ be a solution of $\sum_{Cascade+Obs}$ and and $\mathcal{D} \subset R^7$ the domain in which the closed-loop system is forward complete. Then, for every initial condition $\mathcal{X}_{Cascade+Obs}(t_0) \in \mathcal{D}$ the control signals and the solution $\mathcal{X}_{Cascade+Obs}(t)$ are bounded. Furthermore, for every initial condition $\mathcal{X}_{Cascade+Obs}(t_0) \in \mathcal{D}$ the error tracking variables $(e_1, e_2, e_3, e_5, e_6)^\top$ converges to zero as $t \to \infty$.

5. UNMEASURED THRUSTER DYNAMIC EXTENSION

The purpose of this section is to show that the controller proposed previously can be directly extended to a dynamic model of underactuated ships with unmeasured thruster dynamics, which has often been exclusive from earlier works [6]. Here, the the thruster dynamics are described by the following equation:

$$\dot{\tau} = -A^{-1}\tau + \Psi\chi \tag{21}$$

where $\tau = [\tau_1, \tau_2]^\top$, $A = diag(a_i)$ is a diagonal matrix of positive known actuator time constants $(a_i > 0)_{i=1,2}$. $\Psi = diag(\Psi_1, \Psi_2)$ is a diagonal matrix of positive known constants, $\chi = [\chi_1, \chi_2]^\top$ is a vector of commanded actuator inputs.

Now assuming that τ is unmeasured, In order to reconstruct the unmeasured states τ, introduce the global exponentially observer of the form

$$\dot{\hat{\tau}} = -A^{-1}\hat{\tau} + \Psi\chi \tag{22}$$

Obviously, the observation error $\tilde{\tau} = \hat{\tau} - \tau$ satisfy

$$\dot{\tilde{\tau}} = -A^{-1}\tilde{\tau} \tag{23}$$

With this in mind, we can design a combined controller/observer with χ as the control input given at the top of the next page by invoking a simple application of one step "backstepping" technique [9].

6. SIMULATION

In this section, we carry out some computer simulations to demonstrate the performance of our tracking controllers and to validate our constructive methodology for underactuated ships. The simulation is performed on a model of a monohull ship having one propeller and one rudder, see Fig. 1. The propeller provides the surge force, and the rudder is used to generate the yaw moment. The ship has the length of $38m$, mass of 118×10^3 kg. The different parameters of the ship are listed bellow:

$$\begin{aligned} m_{11} &= 120 \times 10^3 \; m_{22} = 177 \times 10^3 \\ m_{33} &= 636 \times 10^5 \; d_{11} = 215 \times 10^2 \\ d_{22} &= 147 \times 10^3 \; d_{33} = 803 \times 10^4 \end{aligned}$$

Fig. 1. A 38 m monohull ship. Courtesy http://www.austal.com/products.

For simulation uses, we make the following choice of initial conditions for reference system (3) $[\, x_d(0)\ y_d(0)\ \psi_d(0)\ u_d(0)\ v_d(0)\ r_d(0)\,] = [\,0\ 0\ 0\ 15\ 0\ 0.5\,]$. We pick the following initial condition for the real system: $[\, x(0)\ y(0)\ \psi(0)\ u(0)\ v(0)\ r(0)\,] = [\,9.15\ -17.7\ 27.25^0\ 24\ 0\ 0.5\,]$. based on Section 3 the control parameters are taken as $k_1 = 20$, $k_2 = 30$, $k_3 = 10$, $k_4 = 50$.

In simulation we use an explicit expression of τ_1 and τ_3 directly computed from (19). Fig 2-9 are simulation results. Fig. 2-7 show the given desired trajectories and the response of each state. They show that the state of the closed loop system converge to the desired trajectories. In Fig.7 the natural logarithm of the norm

$$\rho = \sqrt{x_e^2 + y_e^2 + \psi_e^2 + r_e^2 + u_e^2 + v_e^2}$$

is shown. We see that it is upper bounded by a decreasing straight line, and this explains the exponential convergence

$$\chi_2 = \frac{1}{m_{11}\Psi_1}\Big((\frac{d_{11}}{m_{11}} + \frac{1}{a_1})\widehat{\tau}_1 + (\frac{d_{11}m_{22}}{m_{11}} - d_{22} - \frac{d_{33}m_{22}}{m_{33}})vr + (\frac{m_{22}(m_{11}-m_{22})}{m_{33}}v^2 - m_{11}r^2 - \frac{d_{11}^2}{m_{11}})u + \frac{m_{22}}{m_{33}}\widehat{\tau}_2 v + m_{11}(\dot{\alpha}_2 + \sigma\overline{y}_2 - \tilde{\mathbf{c}}_2\widetilde{u}_2)\Big) \tag{24}$$

$$\chi_1 = \frac{m_{33}}{\Psi_2}\Big[(\frac{d_{33}}{m_{33}^2} + \frac{1}{a_2 m_{33}})\widehat{\tau}_2 - \big[(\frac{m_{11}-m_{22}}{m_{33}})(\frac{m_{22}}{m_{11}}v^2 + \frac{m_{11}}{m_{22}}u^2) + (\frac{d_{33}}{m_{33}})^2 \big] r + (\frac{m_{11}-m_{22}}{m_{33}})\big[\frac{d_{33}}{m_{33}} + (\frac{d_{11}}{m_{11}} + \frac{d_{22}}{m_{22}}) \big] uv - \sigma\widetilde{u}_1 + \dot{\alpha}_3 - \widetilde{\mathbf{c}}_3\widetilde{\tau}_r \Big] \tag{25}$$

of the tracking error. Fig. 8 shows the desired trajectory and the actual trajectory of the vessel in X -Y plane.

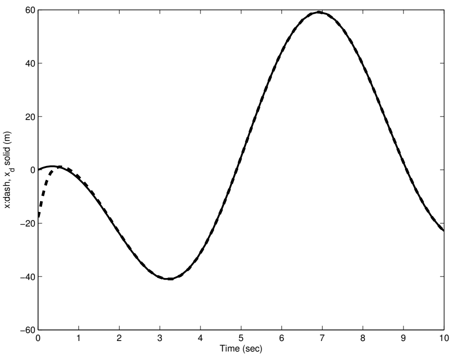

Fig. 2. Response of x and x_d.

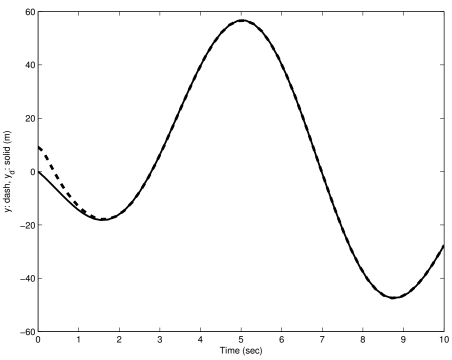

Fig. 3. Response of y and y_d.

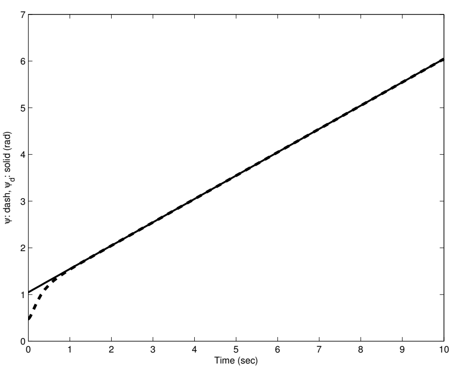

Fig. 4. Response of ψ and ψ_d.

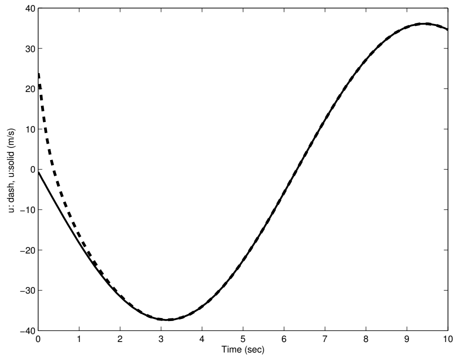

Fig. 5. Response of u and u_d.

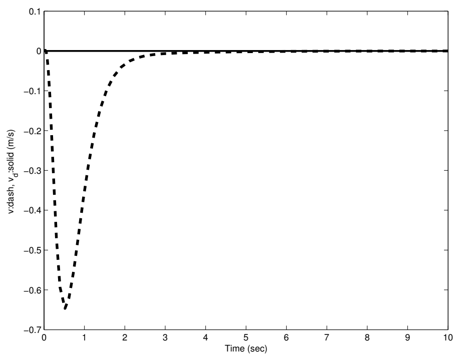

Fig. 6. Response of v and v_d.

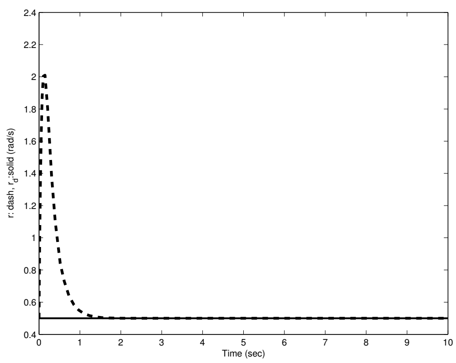

Fig. 7. Response of r and r_d.

7. CONCLUSION

The problem of trajectory tracking has been investigated for underactuated ships with only a surge force and a yaw moment. The tracking controller design is developed with the aid of backstepping technique and cascade structure

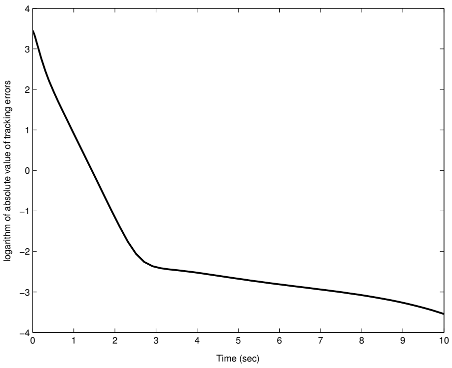

Fig. 8. Logarithm of absolute value of tracking errors.

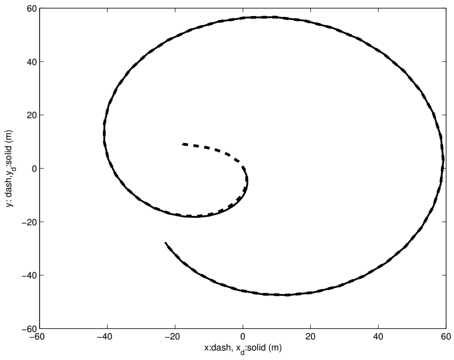

Fig. 9. Trajectory in X - Y plane.

property of the resulted system. An important feature of our tracing controller is that they can easily made robust against unmeasured thruster dynamics, whose presence has often been ignored in previous studies.
Future work will be focused on the issue of practical importance like robust adaptive tracking with parametric uncertainties.

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