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High-fidelity Tracking Control of Electric Shaking Tables ⋆

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Abstract: This paper proposes a design method of tracking control for electric shaking tables via the stochastic optimal control theory with preview action based on a third-order model with respect to the jerk, acceleration, velocity and displacement of the table. The reaction force caused by the dynamic motion of a test structure subjected to seismic disturbances can be regarded as a random disturbance for the shaking table. The mission of our tracking controller is accurately to reproduce the earthquake ground motions using the derived third-order model and the preview information of the displacement of the ground motions. An augmented discrete-time stochastic system of an incremental state and an incremental tracking error corresponding to the shaking table is constructed. This system is subject to the sum of the reaction force of the test structure and its derivative which is modeled by a white Gaussian noise. Combining this system with a command generator for making the preview information, a stochastic optimal tracking control based on the noisy observation data is constructed for the resultant system.

1. INTRODUCTION

The importance of vibration test using shaking tables is increasing for fields of mechanical engineering, architectural engineering, civil engineering, and so on. In the use of the shaking tables, it is significant that the acceleration of the recorded (random) vibration, e.g., ground motion of earthquakes, is accurately reproduced by the motion of the table. The shaking tables driven by several voice coil motors have an advantage of linearity of input-output response. However, there is a difficulty in accurate tracking control using a feedback controller because of the unknown reaction force generated by the random vibration of the test structure mounted on the table. The reason why the reaction force is unknown is that the dynamics of test structures are unknown. It is a significant point how the reaction force is considered in design of tracking controllers of electric shaking tables.

In order to realize the high-fidelity reproduction of displacement, velocity and acceleration of the time histories, a lot of researchers have developed the controllers for the shaking tables to reproduce recorded earthquake ground motion. Matsuura et al. proposed a design method of spectrum control for response of shaking tables [1]. Twitchell and Symans have achieved to accurately reproduce earthquake ground motion using a controller based on the system identification for the systems of the shaking table and the test structures [2]. Multi-axis shaking tables have been investigated in the literatures [3, 4, 5]. Dai et al. proposed a nonlinear controller for hydraulic shaking tables which have nonlinearity of input-output response of hydraulic actuators [6]. Kuehn et al. investigated optimal controllers of hydraulic shaking tables with feedforward compensators to improve frequency response [7].

In this paper we present an approach of high-fidelity tracking control for electric shaking tables subject to unknown reaction force caused by dynamic motion of test structures using the stochastic optimal control theory with preview action. Researches on preview control theory have begun in 1970's. Tomizuka [8] and Kojima et al. [9] investigated the preview control of continuous-time systems. The preview controllers for the discrete-time systems have been discussed in [10, 11, 12].

The main purpose of this work is to develop optimal controllers for reproducing the displacement, velocity and (especially) acceleration of the shaking table, precisely. The dynamics of the electric shaking tables is described by two ordinary differential equations subject to the reaction force due to the motion of the test structures which can be regarded as random disturbance. Eliminating the electric current variable from these two dynamics, a third-order differential equation consisting of the jerk, acceleration, velocity and displacement terms corresponding to the table is obtained. This equation gives us a state- space model with respect to the displacement, velocity and acceleration. In order to use the discrete-time stochastic optimal control with preview action based on the noisy observation data, a discrete-time version of a tracking error system is constructed. An augmented discrete-time stochastic system consisting of an incremental tracking error system and an incremental state-space system is constructed. Combining the augmented system with a command generator for making the preview information, a stochastic optimal control is constructed for the resultant system.

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2. SYSTEM MODEL OF ELECTRIC SHAKING TABLE

Consider an uniaxial electric shaking table illustrated in Fig.1. The shaking table consists of a square sliding platform and is attached to a voice coil motor. We assume that a test structure whose dynamics and physical parameters are unknown is mounted on the table. The shaking table is driven by the voice coil motor so that the shaking table reproduces historical earthquake records. The reaction force caused by the random vibration of the test structure excited by the shaking table can be regarded as a random disturbance for the table. The dynamics of the electric shaking table subjected to the random disturbance due to the motion of the test structure are given by the following stochastic systems:

$$m\ddot{x}(t) + \mu\dot{x}(t) + \kappa x(t) = K_f\, i(t) + d(t) \tag{1}$$

$$L\dot{i}(t) + Ri(t) = v(t) - K_e\dot{x}(t), \tag{2}$$

where $x(t)$ denotes the displacement of the table from the reference axis; $i(t)$ the electric current flowing in the circuit of voice coil; $v(t)$ the control voltage; $d(t)$ represents the reaction force generated by the motion of test structure. m denotes the total mass of the table with the voice coil motor; μ and κ the damping coefficient and the stiffness corresponding to the rubber diaphragm covering the voice coil. L and R represent the inductance and the resistance of the voice coil; K_f and K_e the thrust and the back electromotive force coefficients of the voice coil motor.

From the equation (1), the electric current $i(t)$ is expressed by

$$i(t) = \frac{1}{K_f}\left\{m\ddot{x}(t) + \mu\dot{x}(t) + \kappa x(t) - d(t)\right\}. \tag{3}$$

Substituting (3) into (2) in order to eliminate $i(t)$, we have the following third-order differential equation with respect to $x(t)$:

$$Lm\,\dddot{x}(t) + (L\mu + Rm)\ddot{x}(t) + (L\kappa + R\mu + K_fK_e)\dot{x}(t)$$
$$+ R\kappa x(t) = K_f v(t) + L\dot{d}(t) + Rd(t) \tag{4}$$

This equation consists of the position $x(t)$, its velocity, acceleration and jerk, i.e. $\dot{x}(t)$, $\ddot{x}(t)$ and $\dddot{x}(t)$. The second and third terms in the right hand side represent the disturbance corresponding to the reaction force due to the vibration of the test structure.

The shaking table reproducing ground motions of recorded earthquakes excites the test structure. The reaction force generated by the vibration of the test structure acts on the shaking table as sum of forces proportion to the acceleration of each story. Since, these acceleration have wide spectrum, it is reasonable for us to regard the reaction force $L\dot{d}(t) + Rd(t)$ as a white Gaussian noise.

In this paper, the disturbance terms in R.H.S. of (4), $L\dot{d}(t) + Rd(t)$ is replaced by the white Gaussian noise $\gamma(t)$ with $\mathcal{E}\{\gamma(t)\} = 0$ and $\mathcal{E}\{\gamma(t)\gamma(\tau)\} = W_\gamma\delta(t-\tau)$ for the sake of simplicity, where $\delta(\cdot)$ denotes the Dirac's delta function.

Define a state vector $z(t) = [\ddot{x}(t), \dot{x}(t), x(t)]^{\mathrm{T}}$. (1) and (2) can be expressed by the following state-space model:

$$dz(t) = Az(t)dt + Bv(t)dt + Gdw_\gamma(t), \tag{5}$$

where $w_\gamma(t) = \int_0^t \gamma(\tau)d\tau$; the system matrices are given by

$$A := \begin{bmatrix} -\dfrac{L\mu + Rm}{Lm} & -\dfrac{L\kappa + R\mu + K_fK_e}{Lm} & -\dfrac{R\kappa}{Lm} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B := \begin{bmatrix} \dfrac{K_f}{Lm} \\ 0 \\ 0 \end{bmatrix}, \quad G := \begin{bmatrix} \dfrac{1}{Lm} \\ 0 \\ 0 \end{bmatrix}.$$

The third term in the right hand side of (5) represents the disturbance which is generated by the random vibration of the test structure mounted on the table.

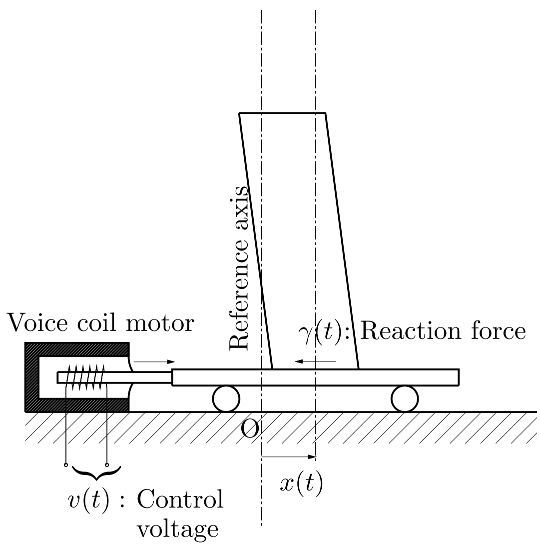

Fig. 1. Electric shaking table with unknown test structure.

The output of the shaking table can be obtained by measuring its position, velocity and/or acceleration, i.e. $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$, respectively. The observation $y(t) \in \mathbb{R}^P$ is given by the following system:

$$y(t) = Cz(t) + D\beta(t), \tag{6}$$

where $\beta(t) \in \mathbb{R}^P$ denotes the observation noise modeled by the white Gaussian noise with $\mathcal{E}\{\beta(t)\} = 0$ and $\mathcal{E}\{\beta(t)\beta^{\mathrm{T}}(\tau)\} = W_\beta\delta(t-\tau)$; $C \in \mathbb{R}^{P\times 3}$ and $D \in \mathbb{R}^{P\times P}$.

In order to employ the discrete-time preview optimal controller, the continuous-time stochastic system described by (5) and (6) is converted into a discrete-time stochastic system. The discrete-time system with the time interval ΔT can be described by

$$z(k+1) = A_dz(k) + B_dv(k) + G_dw_\gamma(k) \tag{7}$$

$$y(k) = Cz(k) + Dw_\beta(k), \tag{8}$$

where k denotes the time-step, i.e., $z(k) := z(t_k)$; $w_\gamma(k)$ and $w_\beta(k)$ the white Gaussian noise corresponding to $\gamma(t)$ and $\beta(t)$, respectively. Their means and covariance are given by $\mathscr{E}\{w_\gamma(k)\} = 0$, $\mathscr{E}\{w_\gamma(k)w_\gamma^{\mathrm{T}}(j)\} = W_\gamma\delta_{kj}$, $\mathscr{E}\{w_\beta(k)\} = 0$, $\mathscr{E}\{w_\beta(k)w_\beta^{\mathrm{T}}(j)\} = W_\beta\delta_{kj}$, δ_{kj} the Kronecker delta and the system matrices are given by

$$A_d := \exp\{A\Delta T\}$$

$$B_d := \int_0^{\Delta T} \exp\{A\tau\}d\tau\, B$$

$$G_d := \int_0^{\Delta T} \exp\{A\tau\}d\tau\, G.$$

3. AUGMENTED SYSTEM

As a reference vector, we consider a vector consisting of the acceleration, velocity and displacement data of the shaking table expressed by $r(k) := [r_a(k), r_v(k), r_d(k)]^{\mathrm{T}}$ where $r_a(k), r_v(k)$ and $r_d(k)$ denote the reference signal with respect to the acceleration, velocity and displacement of the table, respectively. This reference signal is assumed to be previewable, i.e., at each time step k, future values $r(k+1), \cdots, r(k+h)$ are available as well as the present and the past values of the reference vector. The controller is designed so that the state $z(k)$ tracks the reference vector $r(k)$. In order to reduce the tracking error defined by

$$e(k) := r(k) - z(k) \equiv [e_a(k), e_v(k), e_d(k)]^{\mathrm{T}}, \quad (9)$$

the LQI control technique is applied [10, 11], where $e_a(k)$, $e_v(k)$ and $e_d(k)$ represent the tracking errors of acceleration, velocity and displacement of the table, respectively.

The incremental representation of the error $e(k)$ can be expressed by

$$\tilde{e}(k+1) := e(k+1) - e(k). \quad (10)$$

Substituting (9) into (10), we have

$$\tilde{e}(k+1) = \tilde{r}(k+1) - \tilde{z}(k+1), \quad (11)$$

where $\tilde{r}(k) = r(k) - r(k-1)$.

Let us introduce an incremental state $\tilde{z}(k) = z(k) - z(k-1)$, an incremental control input $\tilde{v}(k) = v(k) - v(k-1)$ and an incremental disturbance $\tilde{w}_\gamma(k) = w_\gamma(k) - w_\gamma(k-1)$, where $\mathscr{E}\{\tilde{w}_\gamma(k)\} = 0$ and $\mathscr{E}\{\tilde{w}_\gamma(k)\tilde{w}_\gamma(j)\} = 2W_\gamma\{\delta_{kj} - \delta_{|k-j|,1}\}$. The incremental state is described by

$$\tilde{z}(k+1) = A_d\tilde{z}(k) + B_d\tilde{v}(k) + G_d\tilde{w}_\gamma(k). \quad (12)$$

Substituting this system with the relation (10) into (11), the following recurrence equation with respect to $e(k)$ is obtained:

$$e(k+1) = e(k) - A_d\tilde{z}(k) - B_d\tilde{v}(k) + \tilde{r}(k+1) - G_d\tilde{w}_\gamma(k). \quad (13)$$

Let us define an augmented state vector $\zeta(k) := [e^{\mathrm{T}}(k), \tilde{z}^{\mathrm{T}}(k)]^{\mathrm{T}}$. The augmented system with respect to $\zeta(k)$ is described by

$$\zeta(k+1) = A_z\zeta(k) + B_z\tilde{v}(k) + D_z\tilde{r}(k+1) + G_z\tilde{w}_\gamma(k), \quad (14)$$

where the system matrices are given by

$$A_z = \begin{bmatrix} I & -A_d \\ 0 & A_d \end{bmatrix}, \quad B_z = \begin{bmatrix} -B_d \\ B_d \end{bmatrix}$$

$$D_z = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad G_z = \begin{bmatrix} -G_d \\ G_d \end{bmatrix}.$$

4. STATE ESTIMATOR

In order to control the system given by (14), the state estimate $\zeta(k)$ based on the set of observation data $Y_k = \{y(j)|0 \le j \le k\}$ is required. The estimate $\hat{\zeta}(k)$ is defined by

$$\hat{\zeta}(k) = \begin{bmatrix} \mathcal{E}\{e(k)|Y_k\} \\ \mathcal{E}\{\tilde{z}(k)|Y_k\} \end{bmatrix} = \begin{bmatrix} r(k) - \hat{z}(k) \\ \hat{z}(k) - \hat{z}(k-1) \end{bmatrix}, \quad (15)$$

where $\mathcal{E}\{\cdot|*\}$ denotes the mathematical conditional expectation. This equation suggests that the estimate $\hat{\zeta}(k)$ can be calculated based on $\hat{z}(k)$ and $r(k)$, where $\hat{z}(k) := \mathcal{E}\{z(k)|Y_k\}$ which is the output of the following Kalman filter,

$$\hat{z}(k+1) = (I - KC)A_d\hat{z}(k) + (I - KC)B_dv(k) + Ky(k+1), \quad \hat{z}(0) = 0 \quad (16)$$

$$K = PC^{\mathrm{T}}[CPC^{\mathrm{T}} + W_\beta]^{-1} \quad (17)$$

$$P = A_dPA_d^{\mathrm{T}} + G_dW_\gamma G_d^{\mathrm{T}} - PC^{\mathrm{T}}[CPC^{\mathrm{T}} + W_\beta]^{-1}CP. \quad (18)$$

5. PREVIEW TRACKING STOCHASTIC OPTIMAL CONTROL

The cost functional of the preview tracking stochastic optimal control to reduce the tracking error of the shaking table is given by the cost functional:

$$J(\tilde{v}) = \lim_{T\to\infty} \frac{1}{T}\mathscr{E}\{\Psi_{0,T}(\zeta, \tilde{v})|Y_T\}, \quad (19)$$

where

$$\Psi_{j,k}(\zeta, \tilde{v}) := \frac{1}{2}\sum_{\ell=j}^{k} \{\zeta^{\mathrm{T}}(\ell)Q\zeta(\ell) + r_0\tilde{v}^2(\ell)\} \quad (20)$$

$$Q := \begin{bmatrix} Q_e & 0 \\ 0 & 0 \end{bmatrix}, \quad Q_e := \mathrm{diag}\{q_a, q_v, q_d\} \quad (21)$$

and q_a, q_v and q_d are the nonnegative scalars; and r_0 the positive scalar.

As the information of preview part, the reference signal of the displacement, $\{r_d(k+1), r_d(k+2), \cdots, r_d(k+h)\}$, is employed. In order to update the preview information, the discrete-time system given by (14) can be augmented by a command generator system which models the preview part of the system

$$\tilde{x}_r(k+1) = A_r\,\tilde{x}_r(k), \quad (22)$$

where $\tilde{x}_r(k) := x_r(k) - x_r(k-1)$, $x_r(k) = [r_a(k+1), r_v(k+1), r_d(k+1), \cdots, r_d(k+h)]^{\mathrm{T}}$ and

$$A_r := \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{(h+2)\times(h+2)}.$$

Now define a new vector $\xi(k) = [\zeta^{\mathrm{T}}(k), \tilde{x}_r^{\mathrm{T}}(k)]^{\mathrm{T}}$, we have a state-space system given by

$$\xi(k+1) = A_c\xi(k) + B_c\tilde{v}(k) + G_c\tilde{w}_\gamma(k), \qquad (23)$$

where

$$A_c := \begin{bmatrix} A_z & G_r \\ 0 & A_r \end{bmatrix}, \quad B_c := \begin{bmatrix} B_z \\ 0 \end{bmatrix}$$

$$G_c := \begin{bmatrix} G_z \\ 0 \end{bmatrix}, \quad G_r := [\, D_z \ 0 \ \cdots \ 0 \,].$$

With the state vector $\xi(k)$, the cost functional given by (19) and (20) can be expressed in terms of the augmented system as:

$$J(\tilde{v}) = \lim_{T\to\infty} \frac{1}{T}\mathscr{E}\{\Psi_{0,T}(\xi, \tilde{v})|Y_T\} \qquad (24)$$

$$\Psi_{j,k}(\xi, \tilde{v}) = \frac{1}{2}\sum_{\ell=j}^{k}\left\{\xi^{\mathrm{T}}(\ell)Q_c\xi(\ell) + r_0\tilde{v}^2(\ell)\right\}, \qquad (25)$$

where $Q_c :=$ Block diag$\{Q, 0\}$.

The optimal control to minimize the cost functional expressed by (24) and (25) is calculated via the LQG control theory for discrete-time systems with full-observed information as

$$\tilde{v}^o(k) = -(R + B_c^{\mathrm{T}}\hat{\Pi}B_c)^{-1}B_c^{\mathrm{T}}\hat{\Pi}A_c\hat{\xi}(k) \qquad (26)$$

$$\equiv -[K_e, K_z, K_r]\hat{\xi}(k), \qquad (27)$$

where $\hat{\xi}(k) := [\hat{\zeta}^{\mathrm{T}}(k), \tilde{x}_r^{\mathrm{T}}(k)]^{\mathrm{T}}$; $K_e \in \mathbb{R}^{1\times 3}$, $K_z \in \mathbb{R}^{1\times 3}$ and $K_r \in \mathbb{R}^{1\times(h+2)}$ represent gain matrices; $\hat{\Pi}$ is the solution of the following matrix Riccati type algebraic equation:

$$\hat{\Pi} = Q_c + A_c^{\mathrm{T}}\hat{\Pi}A_c - A_c^{\mathrm{T}}\hat{\Pi}B_c(R + B_c^{\mathrm{T}}\hat{\Pi}B_c)^{-1}B_c^{\mathrm{T}}\hat{\Pi}A_c. \qquad (28)$$

The optimal control can be described using the definition of the incremental state by

$$v^o(k) = -K_e\sum_{j=1}^{k}\hat{e}(j) - K_r x_r(k) - K_z\hat{z}(k) + K_r x_r(0) + v^o(0). \qquad (29)$$

The first term in R.H.S. of (29) represents the integrator with respect to the tracking error. The second term is the feedforward control corresponding to the preview information. The third term is the feedback control based on the state. $v^o(k)$ depends on the initial values $x_r(0)$ and $v^o(0)$.

6. NUMERICAL SIMULATION

6.1 Simulation Setup

We consider an electric shaking table and a scale models of five stories which is the test structure. The scale model is corresponding to a 10[m] high structure, i.e., its scale is set as 1/10. The dynamics of the shaking table and the scale model are given by

$$m\ddot{x}(t) + \mu\dot{x}(t) + \kappa x(t) = K_f i(t) - \sum_{\ell=1}^{5} m_\ell\ddot{x}_\ell(t) - \sum_{\ell=1}^{5} m_\ell\ddot{x}(t) \qquad (30)$$

$$m_1\ddot{x}_1(t) + (c_1 + c_2)\dot{x}_1(t) + (k_1 + k_2)x_1(t) - c_2\dot{x}_2(t) - k_2x_2(t) = -m_1\ddot{x}(t) \qquad (31)$$

$$\begin{aligned} &m_\ell\ddot{x}_\ell(t) + (c_\ell + c_{\ell+1})\dot{x}_\ell(t) + (k_\ell + k_{\ell+1})x_\ell(t) \\ &\quad -c_\ell\dot{x}_{\ell-1}(t) - c_{\ell+1}\dot{x}_{\ell+1}(t) \\ &\quad -k_\ell x_{\ell-1}(t) - k_{\ell+1}x_{\ell+1}(t) = -m_\ell\ddot{x}(t) \\ &\qquad (\ell = 2, 3, 4) \end{aligned} \qquad (32)$$

$$m_5\ddot{x}_5(t) + c_5(\dot{x}_5(t) - \dot{x}_4(t)) + k_5(x_5(t) - x_4(t)) = -m_5\ddot{x}(t), \qquad (33)$$

where $x_\ell(t)$, $(\ell = 1, \cdots, 5)$ denotes the displacement of each story from the center of the shaking table.

Comparing (30) with (1), the disturbance term for the shaking table on which the five stories model is mounted can be expressed by

$$d(t) = -\sum_{\ell=1}^{5} m_\ell\ddot{x}_\ell(t) - \sum_{\ell=1}^{5} m_\ell\ddot{x}(t). \qquad (34)$$

The reaction force acting at the shaking table generated by the random vibration of the scale model consists of the sum of the reaction forces caused by each story and the total mass of the scale model.

In this simulation the physical parameters of the electric shaking table are set as $m = 17.6$[kg], $\mu = 600$[Ns/m], $\kappa = 5000$[N/m], $L = 2.96 \times 10^{-4}$[H], $R = 0.7[\Omega]$, $K_f = K_e = 34.4$[N/A]. The parameters of the scale model are $k_1 = 7776.6$[N/m], $k_2 = 5604.4$[N/m], $k_3 = 4440.8$[N/m], $k_4 = 2537.8$[N/m], $k_5 = 1621.7$[N/m], $c_1 = 660$[Ns/m], $c_2 = 619$[Ns/m], $c_3 = 556$[Ns/m], $c_4 = 469$[Ns/m], $c_5 = 340$[Ns/m]. Furthermore the mass of each story is set as $m_k = 1.625$[kg]$(k = 1, \cdots, 5)$. As an earthquake acceleration data, we used the Tokachi-oki earthquake (1968, M7.9 recorded at Hachinohe, Fig.2) and Miyagi-oki earthquake (1978, M7.4, recorded at Tohoku University, Fig.6) whose time axis is compressed to $1/\sqrt{10}$ and the high-pass filter has been used to block the trend of the reference signal. The reference trajectory has been generated by integrating the acceleration record twice. The system parameters are set as $W_\gamma = 140$, $W_\beta = \mathrm{diag}\{1 \times 10^{-9}, 1 \times 10^{-9}\}$ and

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

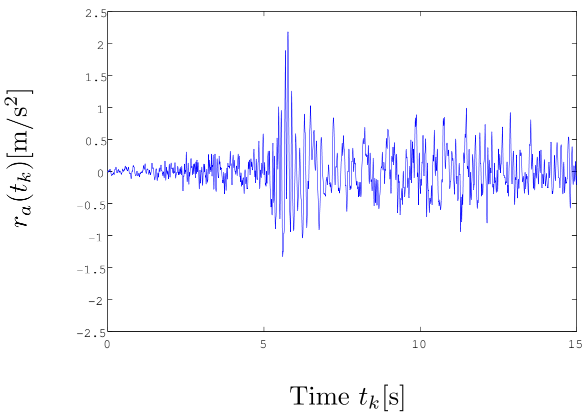

Fig. 2. Reference signal of acceleration $r_a(t_k)$ (Tokachi-oki earthquake).

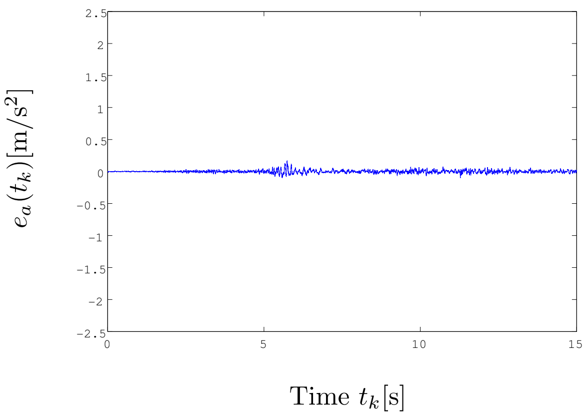

Fig. 3. Tracking error of acceleration $e_a(t_k)$ (Tokachi-oki earthquake).

which means an acceleration and a displacement sensors are installed for the observation system. The controller was designed with the parameters $Q_e = \text{Blockdiag}\{100, 9.7 \times 10^7, 10\}$, $r_0 = 10$, $h = 120$.

6.2 Results

Fig.2 shows the reference signal with respect to the acceleration of the table (Tokachi-oki earthquake). The tracking error of the acceleration is depicted in Fig.3. In this case, the tracking error is quite small. The tracking errors of the velocity and the displacement were also sufficiently small (the behaviors of the tracking errors of the velocity and displacement are omitted). The control voltage and the behavior of the fifth floor of the test structure are shown in Figs.4 and 5.

Figs.6-9 show the numerical results in the case that the Miyagi-oki earthquake was used as the reference signal. The tracking error of the acceleration is depicted in Fig.7. In this case, the tracking error is also quite small. The control voltage and the behavior of the fifth floor of the test structure are shown in Figs.8 and 9.

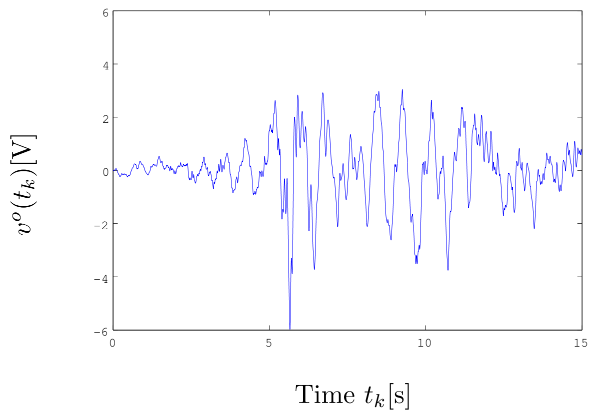

Fig. 4. Control voltage (Tokachi-oki earthquake).

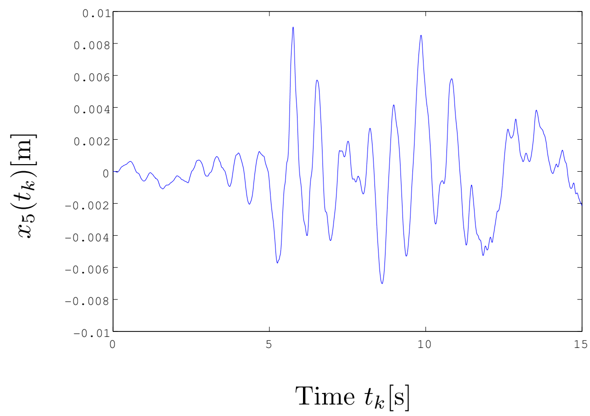

Fig. 5. Behavior of the 5th floor (Tokachi-oki earthquake).

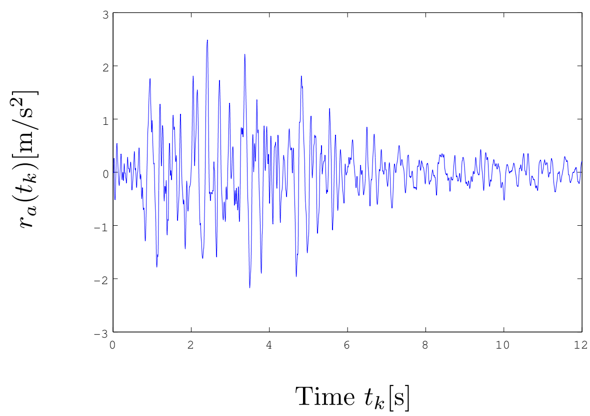

Fig. 6. Reference signal of acceleration $r_a(t_k)$ (Miyagi-oki earthquake).

7. CONCLUSIONS

In this paper we have presented a design method of high-fidelity preview tracking stochastic optimal control for electric shaking tables that is subject to unknown reaction force due to random vibration of test structures mounted on. In order to reduce the tracking error of the acceleration, the third-order mathematical model has been

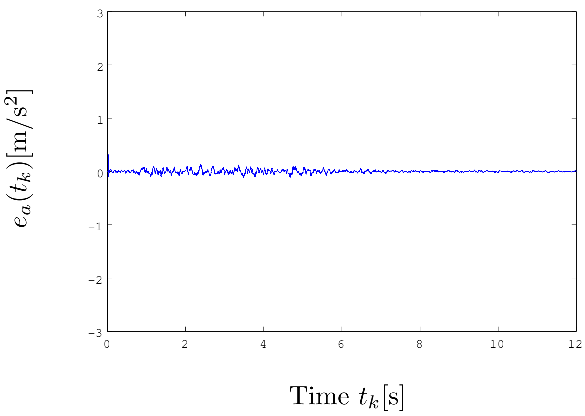

Fig. 7. Tracking error of acceleration $e_a(t_k)$ (Miyagi-oki earthquake).

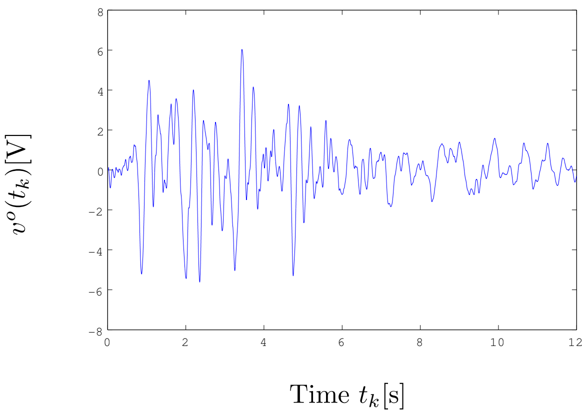

Fig. 8. Control voltage (Miyagi-oki earthquake).

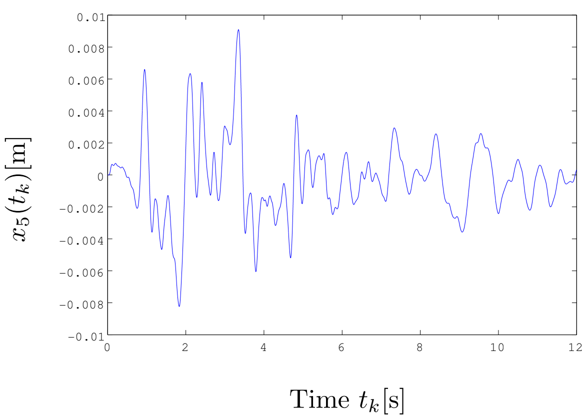

Fig. 9. Behavior of the 5th floor (Miyagi-oki earthquake).

obtained by eliminating the electric current variable. The proposed tracking controller is designed for this third-order model. The resultant preview controller has been derived based on the augmented system consisting of the error system, the incremental state system and the command generator system. The results of several numerical simulations indicated that the proposed controller showed high-fidelity tracking performance by giving weight to the tracking error of velocity.

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