Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Explicit Model Predictive Control for Systems with Linear Parameter-Varying State Transition Matrix

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Abstract: In this paper we propose a closed-loop min-max MPC algorithm based on dynamic programming, to compute explicit control laws for systems with a linear parameter-varying state transition matrix. This enables the controller to exploit parameter information to improve performance compared to a standard robust approach where no uncertainty knowledge is used, while keeping the benefits of fast online computations. The off-line computational burden is similar to what is required for computing explicit control laws for uncertain or nominal LTI systems. The proposed control strategy is applied to the controlled Hénon map to draw a comparison, in terms of complexity and control performance, with a controller based on a piecewise affine approximation.

Keywords: Control of constrained systems; Linear parameter-varying systems; Optimal control theory;

1. INTRODUCTION

Linear Parameter-Varying (LPV) systems are defined as linear systems, where the system matrices are not constant but depend on some time-varying parameters, Apkarian et al. (1995). Contrary to systems with parametric uncertainties, the current values of the parameters are assumed to be known. The parameters lie in a bounded set, such that the LPV system describes a *family* of linear systems. The variation of the parameters within a bounded set might be arbitrarily fast or restricted by a certain rate of variation. The LPV framework constitutes a useful theoretical foundation for gain-scheduling which is a common procedure in many industrial applications. It allows the embedding of nonlinear systems into a family of linear systems, and thus to some extend the application of linear control techniques to nonlinear systems.

During the last years, a lot of research effort has been spent to investigate Model Predictive Control (MPC) schemes for discrete-time LPV systems. There exists a vast amount of literature, and different approaches have been suggested such as interpolation-based MPC, Pluymers et al. (2005), scheduling quasi-min-max MPC, Lu and Arkun (1999, 2000), or MPC with a parameter dependent terminal weighting matrix, Lee and Won (2006). A common trait among these approaches is the use of a quadratic objective function, and most of them rely on Linear Matrix Inequalities (LMI) based state-feedback techniques.

Around the millennium, the application of multi-parametric programming to model predictive control was initialized and resulted in many publications during the following years, see e.g. Bemporad and Morari (1999); Pistikopoulos et al. (2000); Bemporad et al. (2002). In Borrelli (2003) it is shown how multi-parametric programming can be used to compute explicit solutions for optimal control of constrained linear and piecewise affine (PWA) systems. By computing explicit solutions to the optimal control problems, the computational effort of MPC can be moved from online to offline. Instead of solving an optimization problem at each time instance, the optimal input is obtained from a look-up table, which significantly reduces the online computational effort. When piecewise linear cost functions are employed, the computational complexity of the offline problem can be lowered by using multi-parametric programming in a dynamic programming fashion, Baotić et al. (2006); Christophersen (2007).

By making use of parametric programming techniques, different min-max MPC schemes were developed for explicit robust control of linear discrete-time systems with parametric uncertainty, i.e., when the parameter is bounded but *unknown*, see e.g. Bemporad et al. (2003); Sakizlis et al. (2004). It makes intuitively sense and was also shown e.g. in Lu and Arkun (2000) that knowledge of the current parameter allows for an improvement of control performance. The contribution of this paper is to enable this improvement of performance also for explicit model predictive control. A reformulation of LPV systems to LPV-A systems, where only the state transition matrix A is parameter-varying, can be used. For this kind of LPV systems we will show how explicit parameterized control laws can be computed and thus to move the computational effort offline. Our approach is a closed-loop min-max MPC strategy, and follows the philosophy in Löfberg (2003), in the sense that we parameterize the control law in the uncertainty.

The paper is structured as follows. In Section 2, the model predictive control problem for the LPV-A system is stated. The proposed algorithm for the computation of an explicit solution to this problem is presented in Section 3, followed by a verification procedure for stability (Section 4). Afterwards the application of the algorithm is shown in a numerical example (Section 5), providing the opportunity for a comparison to optimal control for piecewise affine (PWA) systems. Finally the conclusions are drawn.

2. PROBLEM STATEMENT

The class of systems we consider is LPV-A systems, linear discrete-time systems with a parameter-varying state transition matrix, which are defined as

$$x_{k+1} = A(\theta_k)x_k + Bu_k \,. \tag{1}$$

The variables $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, and $\theta_k \in \mathbb{R}^{n_\theta}$ denote the state, the control input, and the time-varying parameter, respectively. Furthermore, the system is constrained, $u_k \in \mathbb{U}$ and $x_k \in \mathbb{X}$. The constraint sets $\mathbb{U}$ and $\mathbb{X}$ are assumed to be bounded polyhedrons,

$$u \in \mathbb{U} = \{u : E_u u \leq f_u\}\,, \tag{2a}$$
$$x \in \mathbb{X} = \{x : E_x x \leq f_x\}\,. \tag{2b}$$

Remark: For ease of notation, we restrict ourselves to separate constraints on the state and inputs in (2). It is straightforward to modify the presented algorithm in this paper to the case of mixed constraints, i.e. $E_x x + E_u u \leq f_{xu}$.

The parameter θ_k is measured online and *known to the controller*. Future values are however only known to be constrained to a simplex,

$$\sum_{j=1}^{n_\theta} \theta_k^j = 1, \quad \theta_k \geq 0\,. \tag{3}$$

The state transition matrix $A(\theta_k)$ is known to lie in a polytope with the description

$$A(\theta_k) = \sum_{j=1}^{n_\theta} A^j \theta_k^j\,, \tag{4}$$

where A^j denotes the jth vertex of the polytope. This polytopic description is a common assumption in the LPV framework, see e.g. Apkarian et al. (1995). For the control problem to make sense[1], it is assumed that system (1) is controllable and observable for all admissible θ_k, Silverman and Meadows (1967); Balas et al. (2003).

Remark: Note that LPV systems with varying input matrix $B(\theta)$ can be reformulated to LPV-A systems by the *Δu-formulation*, Barmish (1983); Blanchini et al. (2007),

$$\begin{pmatrix} x_{k+1} \\ u_{k+1} \end{pmatrix} = \begin{bmatrix} A(\theta_k) & B(\theta_k) \\ 0 & I \end{bmatrix} \begin{pmatrix} x_k \\ u_k \end{pmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta u_k\,. \tag{5}$$

[1] This assumption is not necessary for the actual computations of the proposed procedure.

The price to pay is an increase of the system dimension by the number of inputs, and the introduction of an input delay which counteracts the idea of the input to depend on the current parameter. But the Δu-formulation also exhibits advantages as the possibility to constrain and/or penalize input variations, and zero steady-state errors when tracking reference steps. For the rest of the paper we will assume that the Δu-formulation was possibly performed, and the system under consideration is the LPV-A system (1).

For this class of systems we want to find an explicit state feedback control law

$$u_k = \mu(x_k, \theta_k), \tag{6}$$

which makes use of the information of the current θ_k. To compute this control law (6) within a Model Predictive Control scheme, a cost function is to be minimized. According to standard MPC, our cost function is defined as

$$J = \|Px_{k+N}\|_p + \sum_{i=0}^{N-1} \|Qx_{k+i}\|_p + \|Ru_{k+i}\|_p\,, \tag{7}$$

where p denotes a piecewise linear norm, either the 1-norm or the ∞-norm. Piecewise linear norms enable a parametric solution to the stated problem using dynamic programming. For the minimization of the cost function (7) we have to consider the current as well as the unknown future parameter values, as the state trajectories are parameter-dependent.

3. MAIN RESULTS

A reasonable parametrization of the future control inputs is

$$u_k = \sum_j \theta_k^j u_k^j\,. \tag{8}$$

In order to simplify notation, we introduce the basis $U_k := \{u_k^1, \ldots, u_k^{n_\theta}\}$. This parametrization makes the input parameter-dependent and leads to the following system description

$$x_{k+1} = \sum_{j=1}^{n_\theta} \theta_k^j (A^j x_k + Bu_k^j)\,. \tag{9}$$

In a closed-loop min-max MPC approach, one would assume that the future control u_{k+1} is calculated optimally over the horizon $N-1$ not until x_{k+1} and θ_{k+1} are available. But as the future values of the parameters are unknown, all possible cases must be regarded in order to accommodate for the worst-case scenario. This way it is assured that the actual cost function will be less or equal to the computed one, no matter how the parameters evolve. The optimization problem to solve in closed-loop min-max MPC is thus

$$u_k(x_k, \theta_k) = \arg\min_{u_k} \min_{U_{k+1}} \max_{\theta_{k+1}} \cdots \min_{U_{k+N-1}} \max_{\theta_{k+N-1}} J\,. \tag{10}$$

Here we propose a *dynamic programming* procedure to solve (10) by iterating backwards in time. For more details on dynamic programming, see Bertsekas (1995). We start at the prediction horizon N with the initial cost function

$$J_N^*(x_{k+N}) = \|Px_{k+N}\|_p\,. \tag{11}$$

Then at each iteration we use (9) to replace x_{k+i+1} in J^*_{i+1}. As θ_{k+i} is unknown at time instance k, we consider the worst case, which leads to

$$J^*_i(x_{k+i}) = \min_{U_{k+i}} \max_{\theta_{k+i}} J_i(x_{k+i}, \theta_{k+i}, U_{k+i}) \tag{12}$$

with

$$J_i(x_{k+i}, \theta_{k+i}, U_{k+i}) = \|Qx_{k+i}\|_p + \|Ru_{k+i}\|_p + J^*_{i+1}(x_{k+i+1}) . \tag{13}$$

In order to determine the worst case, we just have to check the vertices, since J_i is a convex function of θ_{k+i}. This leads to the following optimization problem

$$J^*_i(x_{k+i}) = \min_{U_{k+i}} t \tag{14a}$$

s.t. for all $1 \le j \le n_\theta$

$$\|Qx_{k+i}\|_p + \|Ru^j_{k+i}\|_p + J^*_{i+1}(A^j x_{k+i} + Bu^j_{k+i}) \le t , \tag{14b}$$

$$x_{k+i} \in \mathbb{X} , \qquad u^j_{k+i} \in \mathbb{U} . \tag{14c}$$

Basically, the unknown future parameters are dealt with by performing an epigraph reformulation, i.e., a constraint is introduced for each vertex of the parameter simplex. Hence the objective will end up in a constraint, what limits us to objectives which are representable by piecewise affine functions, since we have no efficient methods for multi-parametric programming for problems with quadratic constraints. By using piecewise linear norms instead of quadratic norms, the cost functions J^*_i are piecewise linear functions of the state x_{k+i} and the input u_{k+i}, such that in every iteration the optimization problem (14) can be formulated as a multi-parametric linear program (mp-LP) and solved parametrically with respect to x_{k+i}. In contrary to the closed-loop min-max MPC approach for uncertain systems, the future inputs are parameterized as $u_{k+i} = \sum_j \theta^j_{k+i} u^j_{k+i}$, i.e., the future inputs are functions of the future parameters.

Using the terminology of the robust optimization community, in every step of the proposed dynamic programming procedure, we are solving the *Affinely Adjustable Robust Counterpart* (AARC) of an uncertain linear program in a parametric fashion. By restricting the input matrix B to be constant, this AARC is of *fixed recourse*, ensuring computational tractability, see e.g. Ben-Tal et al. (2004).

The last step of the iteration differs from the previous steps. As the parameter θ_k is measured and known, this information can, and should, be taken into account instead of considering the worst case. Hence, we are looking for a multi-parametric solution, with respect to x_k and θ_k, of the following optimization problem

$$J^*(x_k, \theta_k) = \min_{u_k} \|Ru_k\|_p + J^*_1(x_k, \theta_k) . \tag{15}$$

Unfortunately, (15) is a bilinear function in x_k and θ_k (according to (9)), and bilinear constraints appear in the minimization problem, which prevents a standard multi-parametric solution strategy. One way around this is to parameterize the parametric problem not in the measured state x_k, but in the *uncontrolled successor state*, which is

$$z_k = (\sum_{j=1}^{n_\theta} \theta^j_k A^j) x_k . \tag{16}$$

Instead of using (9) to substitute x_{k+1} in (15), we substitute by

$$x_{k+1} = z_k + Bu_k , \tag{17}$$

to obtain a piecewise affine function in z_k and u_k. The final step problem can now be solved using a standard multi-parametric linear program, thus leading to an explicit control law. Online, all we have to do is to compute the uncontrolled successor state, which is completely determined by the measured state and parameter, and evaluate the look-up table to obtain the optimal control input u_k.

The algorithm finishes with a piecewise affine control law, not defined over a set of current states x_k, but of feasible uncontrolled successor states z_k,

$$\mathbb{Z}_f = \{z : E_z z \le f_z\} . \tag{18}$$

However, it might be interesting to compute a region which tells us which actual initial states are feasible, moreover, which initial states are *admissible*, i.e., for all parameter values the uncontrolled successor state is feasible. This way we guarantee that a solution exists for all initial states of the admissible state set, independent from the parameter value. For a fixed state x_k, the set of all possible uncontrolled successor states (16) is a polytope, where A^j determines the jth vertex. From convexity reasons it is sufficient to check if all vertices of this polytope lie in the polytope (18), such that the set of admissible initial states can be determined by

$$x \in \mathbb{X}_f = \{x : \begin{bmatrix} E_z A^1 \\ \vdots \\ E_z A^{n_\theta} \end{bmatrix} x \le \begin{bmatrix} f_z \\ \vdots \\ f_z \end{bmatrix} \} . \tag{19}$$

4. STABILITY

Note that the proposed procedure does not guarantee stability a-priori, what is a classical issue of finite horizon MPC. However, there are known variations of the model predictive control scheme which can be employed, for example *dual mode MPC* or the (overly conservative) *terminal equality constraint*. For an overview of these methods see the prominent survey paper Mayne et al. (2000). A procedure more appropriate for min-max MPC was proposed recently in Lazar et al. (2007). Since the mentioned variations generally result in a decreased feasible space and/or loss of performance, we favor an a-posteriori analysis by means of a set-theoretic reachability analysis, see e.g. Blanchini (1994, 1995).

The reachability analysis is performed in the space of the uncontrolled successor state, where the LPV-A system (1) under the PWA control law $u_k = \mu(z_k)$ corresponds to the uncertain closed-loop system

$$z_{k+1} = A(\theta_{k+1})\{z_k + B\mu(z_k)\} . \tag{20}$$

For the reachability analysis, a positive invariant target set $\mathbb{P}^{(0)}$ containing the reference values is employed. When regulating to the origin, the region containing the origin can be used as target set, if the invariance (and stability) of this region can be checked e.g. by using the cost function as Lyapunov function. Note that all other reference values in the state space correspond to a set of reference values in z-space, such that instead of stability only ultimate boundedness to a target set can be certified. Important is the property of invariance of the target set:

$$z_k \in \mathbb{P}^{(0)} \Rightarrow z_{k+1} \in \mathbb{P}^{(0)} \quad \forall \theta_{k+1} . \tag{21}$$

In order to compute the set of stable initial states, we iterate backwards in time:

$$\mathbb{P}^{(r)} := \{z_k \in \mathbb{Z}_f : z_{k+1} \in \mathbb{P}^{(r-1)} \quad \forall\, \theta_{k+1}\}\,. \tag{22}$$

The iteration terminates when $\mathbb{P}^{(r)} = \mathbb{P}^{(r-1)}$. All uncontrolled successor states $z \in \mathbb{P}^{(r)}$ are controlled to the target set $\mathbb{P}^{(0)}$ by construction. The algorithms needed to establish (21) and (22) for the system (20) boil down to polytopic manipulations and can be adapted from the algorithms given in the references mentioned above.

5. NUMERICAL EXAMPLE

This section consists of a numerical example, demonstrating the application of the proposed algorithm and comparing it to MPC for piecewise affine systems. We will examine the application of the proposed procedure by computing explicit control laws for the Hénon map. An LPV model and a PWA model is derived to investigate when a parameter-varying model could be of use. The Hénon map is a nonlinear second-order system and a popular example for chaotic systems, Hénon (1976). It is defined as

$$\begin{aligned} x_{k+1}^{[1]} &= -a(x_k^{[1]})^2 + x_k^{[2]} + 1\,, \\ x_{k+1}^{[2]} &= bx_k^{[1]}\,, \end{aligned} \tag{23}$$

where the superscript in brackets indicates the element of the state vector. When the coefficients are $a = 1.4,\ b = 0.3$, the system has an unstable fixed point at

$$\bar{x} = (-1/4 + \frac{\sqrt{609}}{28})\,[1\ 0.3]^T \approx [0.63\ 0.19]^T\,.$$

Already small deviations from this fixed point lead to chaotic behavior, and the system moves along a so-called chaotic attractor. A chaotic attractor has the property that during an infinite amount of time, the system is getting arbitrary close to every point on the attractor.

In order to control the Hénon map, we introduce an input to obtain the controlled Hénon map,

$$\begin{aligned} x_{k+1}^{[1]} &= -a(x_k^{[1]})^2 + x_k^{[2]} + 1 + u\,, \\ x_{k+1}^{[2]} &= bx_k^{[1]} + cu\,, \end{aligned} \tag{24}$$

The input coefficient in the controlled Hénon map (24) is set to $c = 0.1$. A linear controller can stabilize this system to the fixed point from a surrounding domain of attraction, Vincent (1997). For points outside the domain of attraction, linear control can make the system completely unstable.

In the following, two methods are used to compute explicit control laws for the controlled Hénon map: One is to model the Hénon map by an LPV model and apply the method described above. As an alternative approach the Hénon map is approximated by a PWA model and the associated optimal control law is computed.

LPV model of the Hénon map

If we want to compute an explicit controller with the proposed method, we have to bring the Hénon map to the form (1). Due to the affine term in (24), this is not directly possible. However, the proposed algorithm easily extends to systems with affine terms in the state prediction. The Hénon map can thus be written as

$$x_{k+1} = \begin{bmatrix} -ax_k^{[1]} & 1 \\ b & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ c \end{bmatrix} u_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{25}$$

For the definition of a suitable parameter θ_k, which varies in $[0, 1]$, we have to declare an interval of admissible $x_k^{[1]}$, which turns out to be a trade-off. On the one hand we want the state transition matrix $A(\theta_k)$ to vary as little as possible to mitigate the introduced conservatism of the approach, on the other hand we want to make the domain as large as possible. As the chaotic attractor lies in $[-1.5, 1.5]$, this interval was chosen and leads to the description

$$x_{k+1} = \begin{bmatrix} 3a\theta_k - 1.5a & 1 \\ b & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ c \end{bmatrix} u_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{26}$$

$$\theta_k = (1.5 - x_k^{[1]})/3\,. \tag{27}$$

Note that the first entry of the state transition matrix can take values in the interval $[-2.1, 2.1]$. The parameter causes such a severe change of dynamics, that a robust min-max MPC scheme, assuming uncertain parameter θ_k, failed to stabilize (26) to $\bar{x}$.

PWA model of the Hénon map

It is also possible to obtain a piecewise affine approximation of (24) and compute the optimal control for this system. Piecewise affine (PWA) systems are defined as

$$\begin{aligned} x_{k+1} &= A^j x_k + B^j u_k + F^j, \\ y_k &= C^j x_k + D^j u_k, \end{aligned} \qquad \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in \mathbb{D}^j\,, \tag{28}$$

where the state-input space is partitioned into polyhedral regions, and each region $\mathbb{D}^j$ is associated with different system equations. The computation of optimal control laws for PWA systems with multi-parametric programming and dynamic programming is well known and can be found e.g. in Borrelli (2003); Christophersen (2007). For this approach, the quadratic term in (24) has to be approximated by a piecewise affine function. The quadratic term as well as a possible piecewise affine approximation with 5 affine terms is shown in Fig. 1. The used approximation is

$$-a(x^{[1]})^2 \approx \begin{cases} 2.5ax^{[1]} + \frac{25a}{16}, & x^{[1]} < -0.94, \\ 1.26ax^{[1]} + 0.4a, & -0.94 \le x^{[1]} < -0.32, \\ 0, & -0.32 \le x^{[1]} < 0.32, \\ -1.26ax^{[1]} + 0.4a, & 0.32 \le x^{[1]} < 0.94, \\ -2.5ax^{[1]} + \frac{25a}{16}, & 0.94 \le x^{[1]}\,. \end{cases} \tag{29}$$

With this approximation a piecewise affine model of the Hénon map can be derived, consisting of five regions with different dynamics. The approximation would become more accurate by using more affine terms. However, with a more complex PWA model, the complexity of the explicit controller rapidly grows, and the computations become intractable already for very short prediction horizons.

Comparison of optimal control laws

For both models of the nonlinear system the optimal control laws were computed that minimize the cost function (7). In both cases, the weight matrices $Q = I_2, R = 0.1$ and a prediction horizon of $N = 4$ was chosen. The terminal cost P was selected to be equal to Q, and the 1-norm was used in the cost. Instead of penalizing the weighted

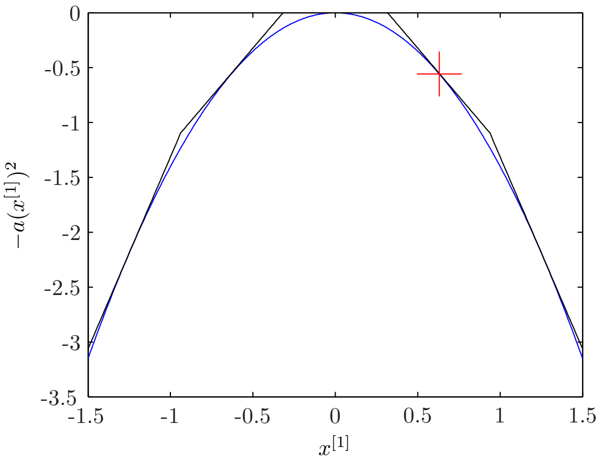

Fig. 1. PWA approximation of the quadratic term of the Hénon map.

model type	controller regions	computation time [3]
PWA	344	181 secs
LPV	93	24 secs

Table 1. Comparison of optimal control laws.

1-norm of the state, the difference to the fixed point $\bar{x}$ was penalized, which trivially can be incorporated in both algorithms.

The Multi-parametric Toolbox (MPT) and YALMIP were used to compute the explicit control laws, Kvasnica et al. (2004); Löfberg (2008).[2] The complexity of the resulting control laws can be seen in Table 1. The explicit control law for the LPV model was computed faster[3] and resulted in less regions than the explicit control law for the PWA model. This is due to transitions between the regions of the PWA model, which have to be handled in a combinatorial fashion. The most time in the PWA case was spent to remove redundant regions from overlapping partitions. This phenomenon of overlapping partitions does not appear in the LPV case. Moreover, the cost function and feasible state set of the controller for the LPV model are convex, which allows a faster evaluation of the lookup table online, Borrelli (2003). The ultimate boundedness of the controlled Hénon map under explicit LPV-MPC to a target region in z-space has been verified for the box $[-1.5, 1.5]^2$ following the reachability analysis in Section 4.

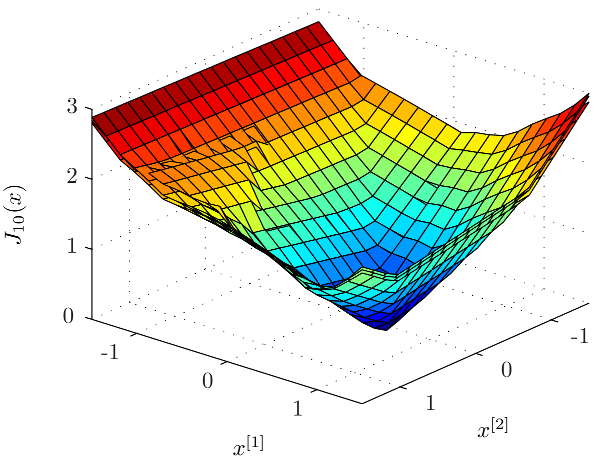

Fig. 2. Actual 10-steps-cost of LPV, PWA and optimal nonlinear control.

[2] A complete implementation of the example can be found at http://control.ee.ethz.ch/~joloef/wiki/pmwiki.php?n=Examples.Examples

[3] All computations were performed on a 3 GHz Pentium 4.

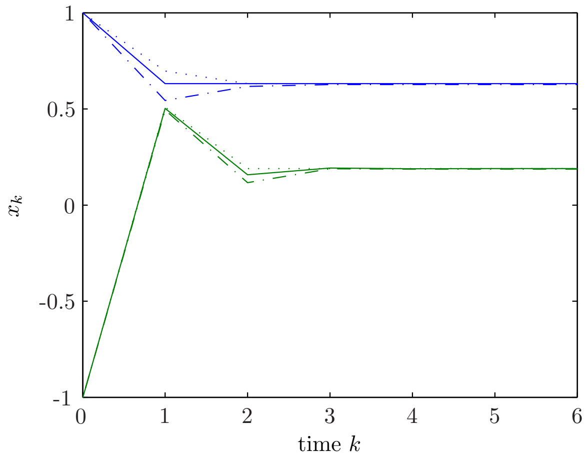

Fig. 3. State evolution from $x_0 = [1, -1]^T$ under LPV (—), PWA (– ·) and optimal nonlinear (· · ·) control.

Both control laws were tested in simulations by controlling the system from 400 initial points x_0, uniformly distributed over the box $[-1.5, 1.5]^2$, to the fix-point $\bar{x}$. Since the approximation with a PWA model leads to a nonzero steady-state input $\bar{u}_{\text{PWA}}$, this was substracted from the control signal of the PWA controller during simulations. The actual simulated costs accumulated over 10-steps can be seen in Fig. 2. The truly optimal solution, based on solving the optimal control problem for the nonlinear system model, is also shown. The solution was computed using the global branch-and-bound based solver available in YALMIP. The differences are hardly visible, relative to the optimal control, the LPV control exhibits an average cost increase of 2.3 % and the PWA control of 3.9 %.

For a closer look, the state evolution of the system under LPV-, PWA- and optimal control, starting from $x_0 = [1, -1]^T$, is shown in Fig. 3. Indeed, the PWA control has a small steady-state error due to modeling errors. These errors would vanish, if the region including the fixed point would be a tangent in the fixed point. However, this is only possible for the regulation to certain points, but not for tracking of reference values.

6. CONCLUSIONS

In this paper, a method was proposed to compute explicit control laws for LPV-A systems, linear time-discrete systems with a parameter-varying state transition matrix. LPV-A systems are a subclass of LPV systems, but the inherent restriction is not as severe as it might seem, because a Δu-formulation can be employed to shift parameter variations from the input matrix to the state transition matrix. For these LPV-A systems a parametrization of the input was used in a dynamic programming approach similar to min-max MPC for uncertain systems. Our approach fits between two different approaches to approximately solve optimal control problems, which are the robust and the PWA approach.

A drawback of explicit control laws is that the number of controller regions grows exponentially with the prediction horizon and the states. As the suggested approach is based on multi-parametric programming, it participates in this drawback, and is thus only tractable for systems of a limited size. However, the final step tends to reduce the number of regions and the resulting number of regions is typically smaller than the number of regions one would obtain when solving the robust min-max MPC problem (assuming no knowledge of the uncertainty).

Our procedure enables an alternative to constrained optimal control of piecewise affine systems, when PWA models are approximating nonlinear systems. As an example, optimal control laws were computed for an LPV model and a PWA model of the nonlinear Hénon map. The resulting explicit control laws were compared by means of control performance and complexity. Though conservatism is introduced in the LPV approach by considering the worst case for future parameter values, no approximation errors are introduced as in the approximation by a PWA model and the cumbersome incorporation of region transitions in the prediction is omitted.

ACKNOWLEDGEMENTS

We want to thank Miroslav Barić for fruitful discussions.

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