

## Parallel Disturbance Force Compensator for Electrical Machines

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**Abstract:** The objective of the research is to diminish unwanted forces generated by rotation and unbalanced rotor mass on the rotor of an electrical machine. These forces, dependent on rotational speed, cause vibration that, when occurring in the machine's natural frequency, causes severe problems. Extra windings are built in the stator of the machine, and they are supplied with current to create an opposite force to the vibration. The main task is to develop a new controller to the system, in order to continuously provide the needed voltage input to the new actuator.

The system was first modeled for finite element model (FEM) software, and based on FEM simulations a simplified state-space model was identified. Separate models for the rotor mechanics and for the actuator were created for convenience. Input to the actuator model was voltage given by the controller, and the output was the compensating force to the rotor. The rotor model mapped total input force of rotor to displacement, vibration. There was an internal feedback from rotor displacement to actuator, which was taken into account in the actuator model.

Because the source of vibration is well known, the problem was attacked at the very source. A compensator was designed for balancing the forces in the rotor. The forces were not measured and remained thus unknown, but they could be estimated. The adaptive compensator was designed so that other controllers can be used parallel, without having to make any changes to the compensator.

Keywords: Electrical machines; Rotor dynamics; Active vibration control; Convergent control; Adaptive control.

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### 1. INTRODUCTION

Vibration control has become a more and more important topic in technological applications of today's industrial community. Ranging from tower buildings to industrial machines and further to e.g. modern adaptive materials the need for passive, semi-active and active vibration damping methodologies is continuously growing, (Inman (2006)). The digitalization process in the technology field gives a direct emphasis on developing new active damping methods to prevent oscillations in different environments. The application area studied in this paper is electrical machines, especially rotor vibrations in the so-called critical speed and its harmonics.

In rotating machinery mechanical or electrical phenomena can create harmonic disturbances, which cause harmful rotor vibrations. The simplest way to explain that is to use

a simple rotor model, the Jeffcott rotor (Jeffcott (1919)), where an unbalanced mass creates a resonance peak in the frequency response of the rotor, occurring at the so-called critical frequency. It is known that in induction machines 40 – 50 % of the faults are caused by bearings, which are damaged because of wear and effects of the rotor vibration, (Negrea (2006)). Therefore the electrical machines are usually driven at subcritical speeds to avoid rotor oscillations and to protect the bearings.

In active vibration control an external control force is used in the system in order to dampen the vibrations. Mechanical constructions are usually needed to implement the new controller. For example, in small-power electrical machines a new force actuator is implemented by a supplementary winding installed in the stator slots, (Chiasson (2005)).

In order to design a new actuator (extra windings implemented in the stator slots of the rotor) careful modelling is needed to characterize the electromagnetic interactions of such a construction. The exact modelling from the first principles of the force generation of the new actuator has been reported in (Laiho et al. (2007a); Laiho et al. (2007b)). The related simulation results have been described in (Zenger et al. (2007)).

Active control algorithms can to some extent be designed with traditional control methods, but the harmonics of the main vibration frequencies and the load disturbance may need more sophisticated control methods (Tammi (2007), Daley et al. (2006)). Different control methods can be designed by using first-principle models of both the the actuator and the mechanical rotor model. To that end, classical LQ(G) control, (Glad and Ljung (2000); Skogestad and Postlethwaite (1996)), robust and adaptive control methods (Zhou and Doyle (1998)), QFT-based control (Quantitative Feedback Theory, Houppis et al. (2006)), convergent control and repetitive control methods (Daley et al. (2006); Tammi (2007); Knospe et al. (1997)) have been reported.

In this paper a new adaptive control algorithm for active vibration control of rotor is proposed. An observer is constructed to estimate the modes of the actuator-rotor system, and an algorithm for controlling the current to the extra winding is designed, to produce a counterforce acting against the disturbance force. Adaptation is used to suppress the inaccuracy caused by estimation errors. Several such adaptive algorithms can operate in parallel without disturbing each other. Each one will suppress vibration at one frequency, i.e. at the critical speed and higher harmonics.

The structure of the paper is as follows. The process and its model is shortly described in Section 2. Identification results and comparison of the model to an exact FE model of the process are discussed in Section 3 to show the accuracy of the low-order state-space model constructed to the system. The new adaptive controller is presented in Section 4, and its operation is verified by FEM simulations in Section 5. Conclusions are given in Section 6.

## 2. PROCESS MODELING

A small 30kW two-pole cage induction motor is considered. The parameters shown in Table 1 correspond to a real test machine, which is currently being built. The machine has an extended rotor shaft, which brings critical speed (42.0Hz) close to the nominal speed (49.5Hz). A four-pole winding with 40 turns in each phase is distributed in the stator slots to build the control winding, (Laiho et al. (2007a)). A layout of the control configuration is shown in Fig.1. The main winding controlling the rotating rotor is driven by frequency converter, and an extra input is used to supply 3-phase current to the new winding, i.e. the new actuator. The disturbance  $f_{ex}$  at the input of the rotor is caused by the unbalancing forces, which are compensated by the control force  $f_c$  produced by the actuator. The internal feedback from the two-dimensional rotor movement to the induction machine block is caused by the fact that the unsymmetrical position of the rotor (in the air gap) distorts the magnetic field, which has to

Table 1. Test machine data

Parameter	Value
supply frequency [Hz]	50
rated voltage (rms) [V]	400
connection	delta
rated current [A]	50
rated power [kW]	30
number of phases	3
number of parallel paths	1
number of poles	2
rated slip [%]	1.0
rotor mass (rotor core and shaft) [kg]	55.80
rotor shaft length [mm]	1560
bearing vertical stiffness [MN/m]	500
bearing horizontal stiffness [MN/m]	100
radial air-gap length [mm]	1.0
critical speed [Hz]	42.0
nominal speed [Hz]	49.5

be taken into account in the extra winding (second input in the actuator model). Starting from first-principle laws

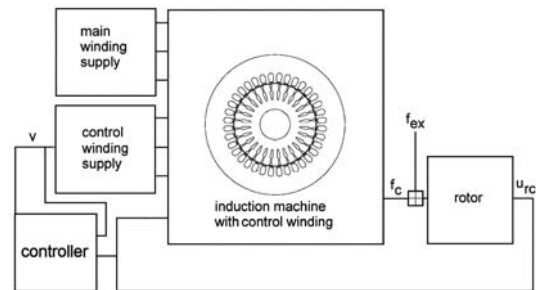


Fig. 1. Control configuration in the test machine

of electrodynamics a differential model of the actuator and the rotor were constructed. An FE model of the 'real' system was built to help in the validation of the results and to generate black box data (Arkkio (1987)). The next phase was to manipulate the physical models in such way that they could be presented by standard transfer function matrices. Alternatively, an augmented state-space representation that includes the actuator, disturbance and rotor models in a single composed model can be used. The state-space representation was constructed by selecting the state-variables from the physical model that yielding an exact model of the system with some couplings between inputs. The other approach is to identify the system from the data achieved from FEM-simulations, which describe the system behavior very accurately. By using identification data the model can be simplified such that the couplings between inputs disappear, but the model still describes the system well enough. When the reduced model is ready, it is validated by simulations against the FE-model. That is very slow to simulate, so a Simulink-model with approximately the same behavior was also constructed.

The new actuator is driven directly from a dSpace system. The rotor unbalance force can be modelled by a two-dimensional ( $x$  and  $y$  directions) input entering in the rotor model input. The rotor model is a generalized form of the basic Jeffcott-rotor model. The actuator model can be expressed in the form (1) and the rotor model of the machine in the form (2), which together can be changed

into a general LTI state-space form (3). For details, see (Laiho et al. (2007a)).

$$\begin{aligned} \frac{di}{dt} &= A_{em}i + B_{em}v + S_{em}\dot{u}_{rc} + Q_{em}u_{rc} \\ f_c &= C_{em}i + P_{em}u_{rc} \end{aligned} \quad (1)$$

In (1) the vector  $i$  contains currents in the rotor and stator,  $v$  the control voltages (two-dimensional),  $u_{rc}$  the rotor center position and  $f_c$  the control force generated to the rotor. In the mechanical model, (2) the term  $\eta$  is the modal coordinate vector,  $f_{ex}$  is the disturbance force, and  $\Phi_{rc}$ ,  $\Sigma$  and  $\Omega$  are matrices related to the generalized eigenfrequencies and damping coefficients of the rotor.

$$\begin{aligned} \ddot{\eta} + 2\Sigma\Omega\dot{\eta} + \Omega^2\eta &= \Phi_{rc}^T f_c + \Phi_{rc}^T f_{ex} \\ u_{rc} &= \Phi_{rc}\eta \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} \xi \\ \eta \\ i \end{pmatrix} &= \begin{bmatrix} -2\Omega\Sigma & \Phi_{rc}^T P_{em}(t)\Phi_{rc} & \Phi_{rc}^T C_{em} \\ I & 0 & 0 \\ S_{em}\Phi_{rc} & Q_{em}\Phi_{rc} & A_{em} \end{bmatrix} \begin{pmatrix} \xi \\ \eta \\ i \end{pmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ B_{em} \end{bmatrix} v + \begin{bmatrix} \Phi_{rc}^T \\ 0 \\ 0 \end{bmatrix} f_{ex} \\ u_{rc} &= [0 \ \Phi_{rc} \ 0] \begin{pmatrix} \xi \\ \eta \\ i \end{pmatrix} \end{aligned} \quad (3)$$

### 3. IDENTIFICATION AND SIMULATION

The identification data was created by feeding zero mean pseudo random signals limited to 1V in amplitude into the control winding of the FE-model. Pseudo random signal was chosen because of its properties. The signal can be limited to a desired range, while randomness guarantees that the whole frequency range is being used, making the signal rich enough for identification purposes. In Fig.

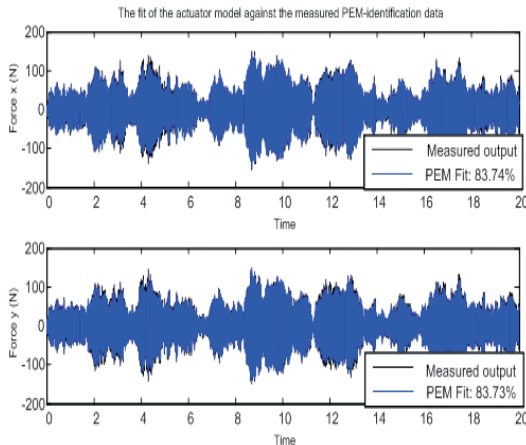


Fig. 2. Force output of the model

2 the force output generated by the actuator is shown. Both the estimated force (PEM) and the one obtained by FE simulation have been presented in the same figure, although the curves cannot be distinguished from each other. The fit is quite good, which is a measure of the accuracy of the identification. Actually, the model of the system is quite complicated, because the output signal of the rotor (i.e. vibration) has an effect on the actuator, which must be taken into account by an internal feedback

signal. That is because vibration causes an unsymmetrical movement of the rotor with respect to the air-gap, which causes an extra magnetic field distortion. In Figs. 3 and

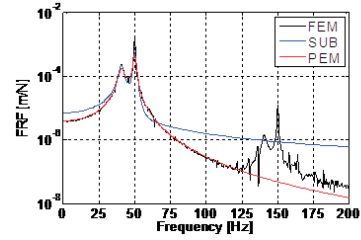


Fig. 3. Gain plots of the model

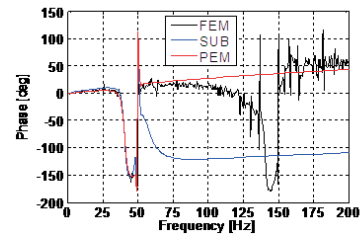


Fig. 4. Phase plots of the model

4 the frequency responses of the identified state-space model (obtained by prediction error method PEM and subspace identification SUB) and those obtained by FEM-simulation are presented. It is noticed that the match is reasonably good at lower frequencies, but there is clearly unmodelled dynamics in higher frequencies. Also, there is a larger error in phase.

More on modelling, identification and simulation of the process has been reported in (Zenger et al. (2007)), (Laiho et al. (2007a)) and (Laiho et al. (2007b)).

### 4. THE NEW ADAPTIVE ALGORITHM

Because the models used are linear, the control signal that compensates the sinusoidal disturbance force has to be sinusoidal also and in the same frequency. Any harmonics created by nonlinearities in the actuator with sinusoidal control signal can and must be neglected. Therefore, the compensator can focus on one frequency only. The harmonics can be dealt with by another controller parallel to the first compensator.

Since the control signal is assumed to be sinusoidal and its frequency is known, only amplitude and phase are left as variables. Amplitude and phase are most conveniently presented in frequency domain, where they can be represented as just one complex number. The biggest benefit of doing calculations in the complex plane is that the compensating control signal or signals should there be almost constant.

To compensate forces in the rotor, the actuator output force should have the same amplitude as the disturbance force but in 180 degrees phase shift. Estimates for both forces are given by the linear estimator, but the estimates still need to be converted to frequency domain. Calculating the Fourier transformation and taking the first component is what is needed here, but such algorithms are too heavy

and thus not suitable. The transformation should be done in real time. There is a way around this problem, when the geometry and mechanics of the system are studied closer.

The initial assumption was that the vibration and disturbance force are generated by an unbalanced mass in the rotor. Such system can always be reduced to a system where there is just one dot like mass object somewhere on the rotor. As the rotor rotates, a support force is needed to keep this extra mass object rotating with the rotor. The force that counters the support force pulls the center of the rotor outwards radially. Now as the rotor rotates, the force rotates with it. It can be concluded that the disturbance force is not only sinusoidal in  $x$ - and  $y$ -directions, as assumed by the estimator, but also circular. In reality, this is not quite so simple since the center of rotor is moving as well due to the effect of the force.

If the disturbance force is circular, the force amplitudes for  $x$ - and  $y$ -directions are equal having a 90 degrees phase shift. This means that only one complex number is needed to represent both directions. Now it is possible to utilize the analogy between complex numbers and polar coordinate system.

By assigning the force in  $x$ -direction as the real component and the force in  $y$ -direction as the imaginary component, the disturbance force can then be presented in complex form accordingly. Because complex numbers can be represented in both Cartesian and polar form, the action can be regarded as moving to a polar coordinate system. Next, to get rid of the rotation, one simply needs to change the frame of reference. To move to a coordinate system that is rotating with the rotor, the time dependent part of phase needs to be removed. In complex plane this can be done by multiplying the complex number with the exponent function (4)

$$D(f, t) = e^{-j2\pi ft} (d_x(t) + jd_y(t)) \quad (4)$$

The result is a force rotating with the frequency  $f$ . The formula (4) looks a lot like Fourier-transformation except that there is no integral. That is why the result is still a function of time. The disturbance force was assumed to be circular, but only an estimate is available. The estimate can contain higher harmonics and measurement noise. Also, if the force estimate were not circular but elliptic, the transformation would create more harmonics. In Fourier transformation, the integral removes all other harmonics, but here the integral is not possible due to real-time requirements.

Instead of integrating the harmonics they can be filtered. Since it was assumed that the disturbance force is circular, and therefore constant in the complex plane, a simple constant gain filter

$$D(k+1) = D(k) + K_d(D(t, f) - D(k)) \quad (5)$$

(5) will be enough. As mentioned earlier, the control signal should also be constant in complex plane so it can be filtered in the same way as well. Due to filtering, the time dependency still remaining in the force estimate can be partially ignored.

There is not much that can be assumed about the other force in the rotor, the actuator output force, but, based on

claims above, same transformation can be applied to it as well (6). The objective of the compensator is to have the actuator force and the disturbance force cancel each other out; so, the actuator force should at least converge towards a constant value. Also actuator force is filtered (7).

$$Y_a(t, f) = e^{-j2\pi ft} (y_{a,x}(t) + jy_{a,y}(t)) \quad (6)$$

$$Y_a(k+1) = Y_a(k) + K_{ya}(Y_a(t, f) - Y_a(k)) \quad (7)$$

Besides the filtered complex force estimates, one more thing is needed for controlling the system. One needs to know how the control signal changes the forces in the rotor. To accomplish this, a transfer function from control signal input to actuator force output is needed. That can be acquired from actuator's state-space model. The internal feedback through rotor does not have to be taken into account here.

The actuator transfer function will also be converted to complex plane. Transfer functions are already in the frequency domain but since the controller is thought to operate on one frequency only, the transfer function reduces into a complex matrix. The matrix is calculated by replacing the Laplace variable  $s$  in the transfer function with imaginary unit  $j$  times the angular frequency of the rotor.

$$M_a = G_{actuator}(j\omega) = G_{actuator}(j2\pi f) \quad (8)$$

With the actuator matrix, it is possible to calculate a control signal that in ideal case would cancel forces in the rotor in steady state. Although the disturbance force was assumed to be circular, it doesn't mean that the control signal is also circular. That is why the  $x$ - and  $y$ -directions need to be separated again.

Now that the signal properties for the force estimates in the rotor have been derived, the actuator transfer function matrix, defined in (8), can be used and a control law can be written. In ideal case, the best possible control is achieved by multiplying the negative value of disturbance estimate with the inverse of the actuator matrix.

$$U_{ideal} = M_a^{-1}(-D_\omega) \quad (9)$$

The ideal control gives a control signal that in steady state produces a force that cancels the disturbance force - that is in theory. This control is not very robust since there is actually no feedback. Control can be made more robust and adaptive by introducing a feedback from the actuator output force estimate.

Assuming that the force estimates are good, a controller that makes the force estimates cancel each other would be both ideal and robust; ideal meaning that the disturbance force could be completely compensated. To the forces to be compensated, the negative of the actuator force should equal the disturbance force, and their difference can be used for control. That difference is actually the sum of the actuator force and the disturbance force and therefore the total force on the rotor of the machine, as illustrated in Fig. 5. From the figure it can be seen that if one wishes to control the total force on rotor to zero, the current control signal would need to be changed such that the added part would generate a force opposite to the current rotor force. The signal that generates such force can be calculated using transfer function as in (9), but now it is

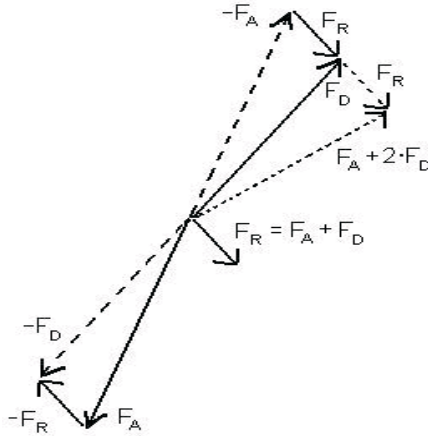


Fig. 5. Force diagram used to design the control law.  $F_A$  is the actuator force,  $F_D$  the disturbance force, and  $F_R$  the total rotor force.

not the actuator transfer function that is enough. Here the internal feedback from rotor displacement to the actuator force must be taken into account. Also, if there are any other feedbacks, they must be taken into account as well.

Until now all dynamics of the system have been forgotten. There might be a delay before the effect of control is seen in the estimates. To fix that the controller must be slowed down. Therefore a small constant factor needs be added to make the controller take smaller steps towards the right control signal. This also adds filtering which was needed to complete the Fourier transformations and justify the use of transfer functions. The fact that it is done after the transfer functions were used does not matter since all operations done in between have been linear.

With the filtering added this converging control law becomes

$$U_{cc}(k+1) = U_{cc}(k) + K_u M_{sys}^{-1} (-Y_{a,\omega}(k) - D_\omega(k)) \quad (10)$$

where  $M_{sys}$  is the complex system matrix calculated as in (8) from transfer function from control input to actuator output force with all feedbacks included, and  $K_u$  is the damping or filtering constant. Note that (10) is actually the formula for convergent control (Tammi (2007)), the only difference being that it is controlling rotor force estimate and not the actual process. If the rotor force estimates were calculated by converting measured displacements into complex variables and using complex matrix for rotor's transfer function, one would get the original convergent control in (Tammi (2007)), for which stability is proved.

The combination of the control laws (9) and (10) can be written in the form

$$U(k+1) = U(k) - K_u (M_a^{-1} (D_\omega(k) + K_{ud}(D_\omega(k) + Y_{a,\omega}(k))) + U(k)) \quad (11)$$

A filter is added to the control law to keep the convergent part slow enough and dampen the harmonics that have been neglected from the beginning.  $K_u$  is the filter gain and  $K_{ud}$  is the additional damping for the convergent part. The minus signs have been moved in front of  $K_u$ .

So, the combined control law (11) first tries to directly compensate the disturbance force and then makes the force estimates cancel each other. But the actuator output force doesn't just depend on the control. There was that internal feedback, which means that part of the actuator force depends only on the displacement. What the controller should do is to make control signal part of the force that cancels the disturbance.

The control signal given is still complex and needs to be converted to time domain. This can be done by adding the rotation angle that was earlier removed back again by multiplying with exponent function. Because the control signal was chosen to be a cosine wave, the time domain signals are simply the real parts of the complex variables.

$$u(kT) = \text{Re}(U(k)e^{j2\pi ft}) \quad (12)$$

Like already mentioned, the compensator is not influenced by any other feedbacks the system might have. It is possible to add another controller e.g. to dampen the higher harmonics that the compensator might create. The control signal from that controller must be fed to the estimator that the compensator is using, but the control signal of the compensator must not be fed to the other controller. The estimator used by the controller does of course see this signal, so it can be used for the other controller as well. The idea and assumption was that the controller will remove the disturbance force and it will remain invisible to the other controller. The other controller will only see the disturbance force disappear.

## 5. RESULTS

The algorithm was tested by FEM simulations, because the actual test machine was not ready. Some results can be seen in Figs. 6-9. It is seen that the estimation algorithm converges to the correct disturbance value and that the control algorithm is able to reduce the amount of vibration considerably.

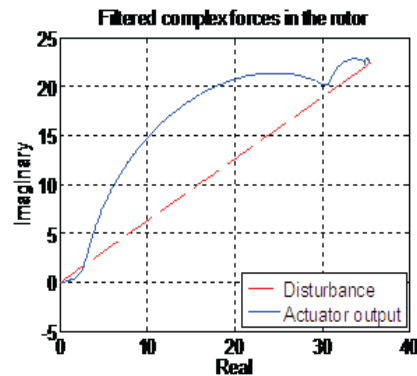


Fig. 6. Complex rotor force estimates with the parameters used. Actuator output is shown negative (49.5Hz).

## 6. CONCLUSION

A new adaptive vibration control algorithm was presented in the paper. The algorithm is somewhat similar to the well-known convergent control law, the difference being that several such controllers tuned to different frequencies



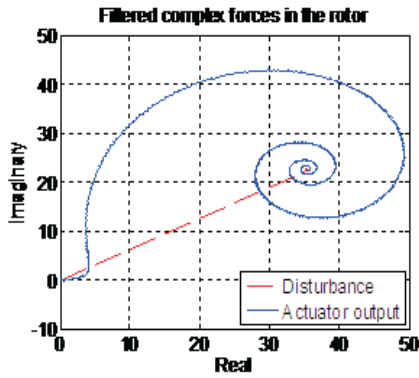


Fig. 7. Complex rotor force estimates multiplied by 10. Actuator output is shown negative (49.5Hz).

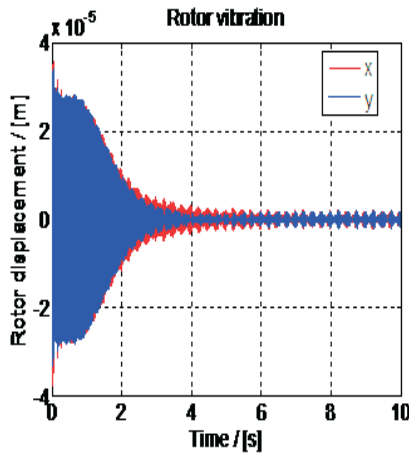


Fig. 8. Rotor displacement. The compensator was activated at 0.5s. The vibration level drops from 2.8E-5m to 2.0E-6m, which is a bit over 7%

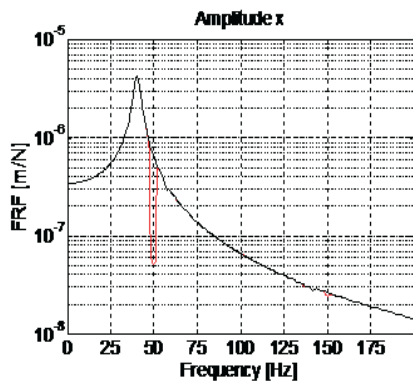


Fig. 9. Frequency response of rotor mechanics from force to displacement. Amplitude in x-direction. The effect of the vibration controller at the critical frequency is clearly visible.

can operate in parallel without disturbing each other. Also, no main controller is needed, which would make a modified plant to be then controlled by the convergent controller. The new algorithm works alone or together with similar algorithms in parallel. The performance was demonstrated by FEM simulations, and practical tests will be carried out, when the real test machine is ready.

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