Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Nonlinear Passivity Based Control Law with Application to Electropneumatic System

K. TURKI SAIED*/. M. SMAOUI*. D. THOMASSET*. F. MNIF**. N. DERBEL****

** Université de Lyon, Laboratoire Ampère, Institut National des Sciences Appliquées de Lyon, FRANCE*
(Tel: +334 72 43 60 33; e-mail:karima.turki@insa-lyon.fr).
*** Université de Sfax, Unité de Recherche MEChatronics Autonomous Systems, Ecole Nationale des Ingénieurs de Sfax, TUNISIE*

Abstract: This paper presents a synthesis of a nonlinear controller to an electropneumatic system. Nonlinear passivity based control law is applied to the system under consideration. First, the nonlinear model of the electropneumatic system is presented. It is transformed to be a nonlinear affine model and a coordinate transformation is then making possible the implantation of the nonlinear controller. A nonlinear control law is developed to track desired position. Experimental results are also presented and discussed.

Nomenclature

b	viscous friction coefficient (N/m/s)
k	polytropic constant
M	total load mass (kg)
p	pressure in the cylinder chamber (Pa)
q_m	mass flow rate provided from servodistributor to cylinder chamber (kg/s)
r	perfect gas constant related to unit mass (J/kg/K)
S	area of the piston cylinder (m^2)
T	temperature (K)
V	volume (m^3)
y, v, a	position (m), velocity (m/s), acceleration(m/s^2)
$\varphi(.)$	leakage polynomial function (kg/s)
$\psi(.)$	polynomial function (kg/s/V)
l	length of stroke (m)
F_{ext}	External force (N)
F_f	dry friction force (N)
u	control input
x	slide valve position

Subscript

D	dead volume
S	supply
N	chamber N
P	chamber P
d	desired

1. INTRODUCTION

Pneumatic control systems play important role in industrial automation due to their relatively small size, light weight, and high speed. One of the conspicuous trends is the need for the electropneumatic systems that can achieve precise tracking position control.

In recent years, research efforts have been directed toward meeting this requirement. Most of them have been in the field of feedback linearization (Bobrow et al., 1998). However, reasonably accurate mathematical models for the pneumatic system are required by the feedback linearization. A number of investigations have been conducted on fuzzy control algorithms (Li Ruihua et al., 2004), adaptive control (Errahimi et al., 2002) (Di Zhou et al., 2003), backstepping control (Smaoui et al., 2006a), sliding mode control (Laghrouche et al., 2006) and robust linear control (Mattei, 2001).

Passivity based control is a generic name given to a family of controller design techniques that achieve the control of objective via the route of passivation, that is, rendering the closed-loop system passive with a desired storage function (that usually qualifies as a Lyapunov function for the stability analysis). See the fundamental book (Ortega et al., 1998) and (Brogliato et al., 2007). Passivity based control, has been found convenient in some application, in particular for mechanical electrical and hydraulic systems. The main contribution of this paper consists in designing a single-input/single-output (SISO) passivity based control law for electropneumatic system in order to track the desired position.

The paper is organized as follows. The following section describes the model of the electropneumatic system and equations governing the motion of this plant have been put in a nonlinear affine form. In order to use the passivity techniques, a coordinate transformation has been proposed. Section 3 presents a systematic passivity based controller design, then the control algorithm is implemented on the electropneumatic system. Section 4 will be devoted to the experimental results whereas section 5 concludes the paper.

2. ELECTROPNEUMATIC SYSTEM MODELING

2.1. Physical model

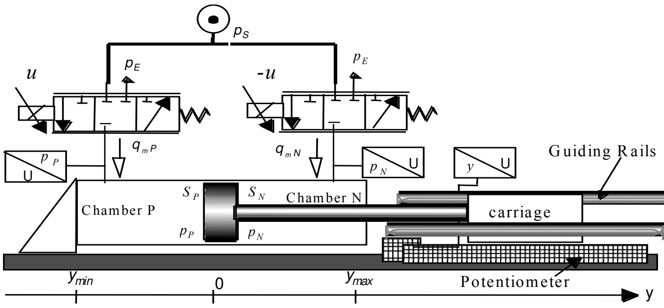

Fig.1. The electropneumatic system

The considered system (fig. 1) is a linear inline double acting electropneumatic servo-drive using a single rod controlled by two three-way servodistributors. The actuator rod is connected to one side of the carriage and drives an inertial load on guiding rails. The total moving mass is *17 kg*.

The electropneumatic system model can be obtained using three physical laws: the mass flow rate through a restriction, the pressure behavior in a chamber with variable volume and the fundamental mechanical equation.

The pressure evolution law in a chamber with variable volume is obtained via the following assumptions (Shearer, 1956): i) the air is a perfect gas and its kinetic energy is negligible; ii) the pressure and the temperature are supposed to be homogeneous in each chamber; iii) the process is polytropic and characterized by coefficient k. Moreover, the electropneumatic system model is obtained by combining all the previous relations and assuming that the temperature variation is negligible and equal to the supply temperature.

The servodistributor dynamic has been approximated by a third order function where parameters have been experimentally identified. The two servodistributors are supposed identical. The state model is given by:

$$\begin{cases} \dfrac{d\ddot{x}_P}{dt} = -a_{11}\ddot{x}_P - a_{12}\dot{x}_P + k_1 u_{Pcal} \\ \dfrac{d\dot{x}_P}{dt} = \ddot{x}_P \\ \dfrac{dx_P}{dt} = \dot{x}_P \\ x_P = u \end{cases}$$

$$\begin{cases} \dfrac{d\ddot{x}_N}{dt} = -a_{11}\ddot{x}_N - a_{12}\dot{x}_N + k_1 u_{Ncal} \\ \dfrac{d\dot{x}_N}{dt} = \ddot{x}_N \\ \dfrac{dx_N}{dt} = \dot{x}_N \\ x_N = -u \end{cases} \quad (1)$$

The mechanical equation includes pressure force, friction, dry friction forces and an external constant force due to atmospheric pressure. The following equation gives the physical model of the above system:

$$\begin{cases} \dfrac{dy}{dt} = v \\ \dfrac{dv}{dt} = \dfrac{1}{M}\left[S_P p_P - S_N p_N - bv - F_f(v) - F_{ext}\right] \\ \dfrac{dp_P}{dt} = \dfrac{krT}{V_P(y)}\left[q_{mP}(u, p_P) - \dfrac{S_P}{rT} p_P v\right] \\ \dfrac{dp_N}{dt} = \dfrac{krT}{V_N(y)}\left[q_{mN}(-u, p_N) + \dfrac{S_N}{rT} p_N v\right] \end{cases} \quad (2)$$

where:

$$\begin{cases} V_P(y) = V_P(0) + S_P y \\ V_N(y) = V_N(0) - S_N y \end{cases}$$

with:

$$\begin{cases} V_p(0) = V_{DP} + S_P \dfrac{l}{2} \\ V_N(0) = V_{DN} + S_N \dfrac{l}{2} \end{cases}$$

are the piping volumes of the chambers for the zero position and $V_{D(P\,or\,N)}$ are dead volumes present on each extremities of the cylinder.

The dry friction forces F_f, which act on the moving part in presence of viscous friction, is a nonlinear model given by relation (Tustin, 1947):

$$F_f(v) = \left[F_s + (F_s - F_C)\exp(-c|v|)\right] sgn(v) \quad (3)$$

where F_s, F_c and c are the stiction friction, the Coulomb friction and the constant Coulomb friction.

Fig 2 shows the results of the friction model for low velocities. Outside of the small velocity region shown in Fig.2, the dry friction is dominated by the constant Coulomb friction value.

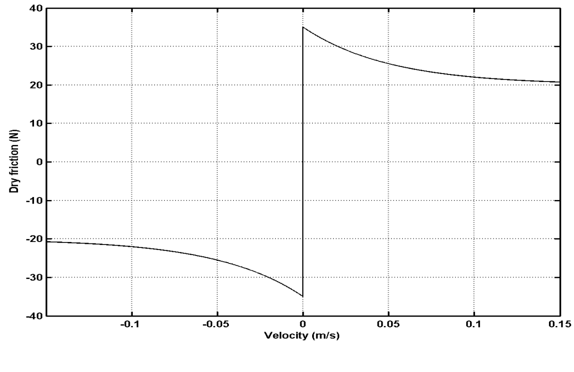

Fig. 2. Dry friction model

The main difficulty for model (2) is related to the knowledge of the mass flow rates q_{mP} and q_{mN}.

2.2. Control model

The dynamics of the servodistributors may be neglected. The benchmark band-pass is about $10Hz$ whereas the servodistributor band-pass is about $200Hz$. In such case, the servodistributors model can be reduced to two static relations between the mass flow rates $q_{mP}(u, p_P)$ and $q_{mN}(-u, p_N)$, where u is the input voltage, p_P and p_N are the output pressures.

To establish a mathematical model of the power modulator flow stage, some research works present approximations based on physical laws (Araki, 1981) by modeling of the geometrical variations of the restriction areas of the servodistributor. Some authors presented an experimental-based characterization model (Richard et al., 1996).
In this paper, the results of the global experimental method giving the static characteristics of the flow stage (Sesmat et al., 1996) have been used. The global characterization corresponds to the static measurement of the output mass flow rate q_m, which depends on the input control u and the output pressure p, for constant source and exhaust pressure. The global characterization has the advantage of obtaining simply, by projection of the characteristic series $q_m(u, p)$ on different planes:

- The mass flow rate characteristics series (plane $p - q_m$)
- The mass flow gain characteristics series (plane $u - q_m$)
- The pressure gain characteristics series (plane $u - p$)

(Belgharbi et al., 1996) have developed analytical models for both simulation and control purposes. The flow stage characteristics were approximated characteristics by polynomial functions affine in control such that:

$$q_m(u, p) = \varphi(p) + \psi(p, sgn(u))\ u \tag{4}$$

$\varphi(p)$ in (4) is a polynomial function of the pressure whose evolution corresponds to the mass flow rate leakage, and does not depend of the input control. $\psi(p, sgn(u))$ is a polynomial function both of the pressure and of the input control *sgn,* because the behavior of the mass flow rate characteristics is clearly different for the inlet ($u > 0$) and the exhaust ($u < 0$). The polynomial functions $\varphi(p)$, $\psi(p, u > 0)$, $\psi(p, u < 0)$ have degrees equal to five. The maximum mass flow rate error between static measurement and polynomial approximation is less then 10 %. Fig. 3 shows the function of $\psi(p, sgn(u))$.

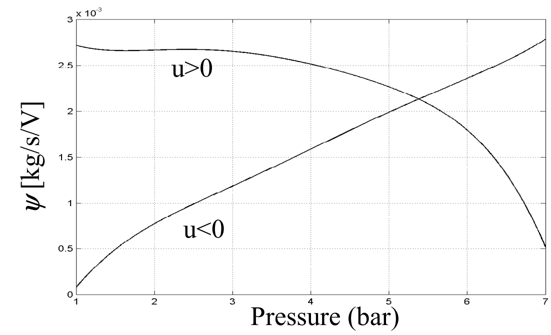

Fig. 3. The function $\psi(p, sgn(u))$

The dry friction forces F_f (3), cannot be used for the synthesis of the passivity control law because this model contains the function $sgn(v)$ which makes it not suited for analytical developments. Moreover, F_s and F_c are not easily measurable. Thus, to simplify the control model (5), this function has been neglected:

$$\begin{cases} \dfrac{dy}{dt} = v \\ \dfrac{dv}{dt} = \dfrac{1}{M}\left[S_P p_P - S_N p_N - bv - F_{ext}\right] \\ \dfrac{dp_P}{dt} = \dfrac{krT}{V_P(y)}\left[q_{mP}(u, p_P) - \dfrac{S_P}{rT} p_P v\right] \\ \dfrac{dp_N}{dt} = \dfrac{krT}{V_N(y)}\left[q_{mN}(-u, p_N) + \dfrac{S_N}{rT} p_N v\right] \\ z = y \end{cases} \tag{5}$$

where z is the output.

Using (4), the nonlinear affine model is then given by :

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ z = y \end{cases} \tag{6}$$

where x, $f(x)$ and $g(x)$ in R^4, u in R, and:

$$x = (y, v, p_P, p_N)^T \tag{7}$$

$$f(x) = \begin{pmatrix} v \\ \dfrac{1}{M}\left[S_P p_P - S_N p_N - bv - F_{ext}\right] \\ \dfrac{krT}{V_P(y)}\left[\varphi(p_P) - \dfrac{S_P}{rT} p_P v\right] \\ \dfrac{krT}{V_N(y)}\left[\varphi(p_N) + \dfrac{S_N}{rT} p_N v\right] \end{pmatrix} \tag{8}$$

$$g(x) = \begin{pmatrix} 0 \\ 0 \\ \dfrac{k.r.T}{V_P(y)}.\psi(p_P, sgn(u)) \\ -\dfrac{k.r.T}{V_N(y)}.\psi(p_N, sgn(-u)) \end{pmatrix} \tag{9}$$

In the next section we will develop a passivity control law using (6).

3. PASSIVITY BASED CONTROL

3.1 Background

Passivity based control (Ortega et al., 1998) is a recursive procedure, which enables a control law to be derived for a nonlinear system, associated to the appropriate Lyapunov function, which guaranties passivity. The class of systems to be studied in this work is systems can be transformed into the strict feedback form shown in (10):

$$\begin{cases} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\ \vdots & \\ \dot{x}_{r-1} &= f_{r-1}(x_1,\dots,x_{r-1}) + g_{r-1}(x_1,\dots,x_{r-1})x_r \\ \dot{x}_r &= f_r(x_1,\dots,x_r) + g_r(x_1,\dots,x_r)u \\ z &= h(x_1) \end{cases} \tag{10}$$

where $x_i, \forall i \in [1, \quad r]$, z et u are the state, the output and the input of system. $g_i(x) \neq 0\ \forall x, \forall i \in [1, \quad r]$.

Suppose the output of the system, z, is to track some desired value of z and the tracking error is defined as $e = z - z_d$. For clarity of exposition, assume that $z = h(x_1) = x_1$.

Create r separate error dynamics as follow:

$$e_i = x_i - x_{id} \qquad \forall i \in [1 \quad r] \tag{11}$$

Differentiating each error equation in (11) once gives:

$$\begin{aligned} \dot{e}_i &= \dot{x}_i - \dot{x}_{id} = f_i + g_i x_{i+1} - \dot{x}_{id}, \quad \forall i \in [1, \quad r-1] \\ \dot{e}_r &= \dot{x}_r - \dot{x}_{rd} = f_r + g_r u - \dot{x}_{rd} \end{aligned} \tag{12}$$

Equation (12) may be written as:

$$\begin{aligned} \dot{e}_i &= f_i + g_i e_{i+1} + g_i x_{(i+1)d} - \dot{x}_{id}, \quad \forall i \in [1, \quad r-1] \\ \dot{e}_r &= f_r + g_r u - \dot{x}_{rd} \end{aligned} \tag{13}$$

Define the desired values of the states and the input of system as:

$$\begin{aligned} x_{(i+1)d} &= \frac{1}{g_i}(-f_i + \dot{x}_{id} - k_i e_i), \quad \forall i \in [1, \quad r-1] \\ u &= \frac{1}{g_r}(-f_r + \dot{x}_{rd} - k_r e_r) \\ k_i &> 0, \forall i \in [1, \dots r] \end{aligned} \tag{14}$$

Subsisting (14) into (13) leads to a chain of interconnected error dynamics:

$$\begin{aligned} \dot{e}_i &= -k_i e_i + g_i e_{i+1}, \quad \forall i \in [1, \quad r-1] \\ \dot{e}_r &= -k_r e_r \end{aligned} \tag{15}$$

Now, consider the following positive definite Lyapunov function:

$$\phi_i = \frac{1}{2}e_i^2, \quad \forall i \in [1, \quad r-1] \tag{16}$$

Differentiating equation (16) gives:

$$\dot{\phi}_i = -k_i e_i^2 + g_i e_{i+1} e_i \qquad \forall i \in [1, \quad r-1] \tag{17}$$

Consider $g_i e_{i+1}$ as the input, and e_i as the output. And rewrriting (17) result in:

$$\underbrace{g_i e_{i+1}}_{Input} \underbrace{e_i}_{output} = \dot{\phi}_i + \underbrace{k_i e_i^2}_{\geq 0} \qquad \forall i \in [1, \quad r-1] \tag{18}$$

Therefore, it is evident that the relationship between e_i and e_{i+1} is strictly passive and hence BIBO (Bounded Input Bounded Output) stable for any $i \in [1 \quad r-1]$. The serial interconnection of strictly passive elements is also strictly passive (Andrew et al., 2001). From the rth error dynamics equation:

$$\dot{e}_r = -k_r e_r \tag{19}$$

3.2 Passivity based control synthesis of pneumatic system

To use passivity based control, a coordinate transformation for the pneumatic system (5) is proposed as follow as:

$$\begin{cases} \dot{y} = v \\ \dot{v} = a \\ \dot{a} = f_a + g_a u \\ z = y \end{cases} \tag{20}$$

where (21) :

$$f_a = \frac{S_p k\, rT}{MV_p(y)}\left[\varphi(p_p) - \frac{S_p}{rT} p_p v\right] - \frac{S_n k\, rT}{MV_n(y)}\left[\varphi(p_n) + \frac{S_n}{rT} p_n v\right] - \frac{b}{M}a$$

$$g_a = \frac{k\, rT}{M}\left[\frac{S_p \psi(p_p, sign(u))}{V_p(y)} + \frac{S_n \psi(p_n, sign(-u))}{V_n(y)}\right]$$

Create 3 separate error dynamics as follow:

$$\begin{aligned} e_y &= y - y_d \\ e_v &= v - v_d \\ e_a &= a - a_d \end{aligned} \tag{22}$$

Define the desired values of the states and the input of system as:

$$\begin{aligned} v_d &= \dot{y}_d - k_y e_y \\ a_d &= \dot{v}_d - k_v e_v \qquad k_y, k_v\ et\ k_a > 0 \\ u &= \frac{1}{g_a}(-f_a + \dot{a}_d - k_a e_a) \end{aligned} \tag{23}$$

where $\psi(.) > 0$ and $\varphi(\cdot) \neq 0$ over the physical domain.

3.3 Experimental results

Before the implementation of the control law (23) in the electropneumatic system, the co-simulation was used. This technique consists in using jointly, the software developed by the researchers in modeling, and the software dedicated for system control. Thus, the physical model of electropneumatic system (1 and 2) was treated by AMESim, and the control law (23) was developed on Simulink. A satisfactory simulation results are obtained. Then, the control law is implemented using a Dspace 1104 controller boar with the dedicated digital signal processor. The controller require measurements of acceleration for feedback. However, accelerometer is seldom used in practical drive systems, because of the complexity they add to the overall process as they are mounted to the load in displacement. For this, a robust differentiator via high order sliding mode (Smaoui et al., 2005) is used to estimate the acceleration. In order o assume the system convergence, the gains must be only positive. The gains $k_y = 40$, $k_v = 20$ et $k_a = 200$ have been tuned in order to minimize the position tracking error. These values ensure good static and dynamic performance. Some experiment results are provided here to demonstrate the effectiveness of the passivity controller.

Figure (4) shows the position, the desired position, the position error and the control input u. The maximum dynamic position tracking error is about $1.5\ mm$. In steady state, the average position error is about $0.06\ mm$. The chattering phenomena in the control law are undesirable and seem considerably to decrease the lifetime of some components.

On the same experimental set-up and in the same conditions, has been implemented:

- The sliding mode controller (Smaoui et al., 2006 b), in this case, the error in steady sate is about $0.11\ mm$.
- The linear control (Smaoui et al., 2006 b), in this case, the error in steady sate is about $0.21\ mm$.
- The backstepping control (Smaoui et al., 2006 b), in this case, the error in steady sate is about $0.1\ mm$.

Thus, the tracking performances obtained by passivity based control are good in regard of precedents one.

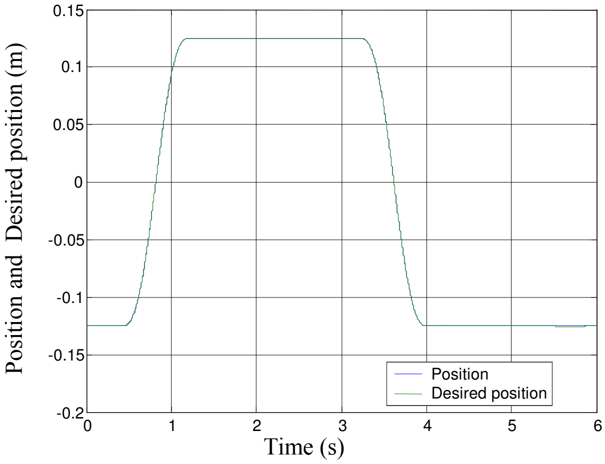

Fig. 4.a : Position and Desired Position (m)

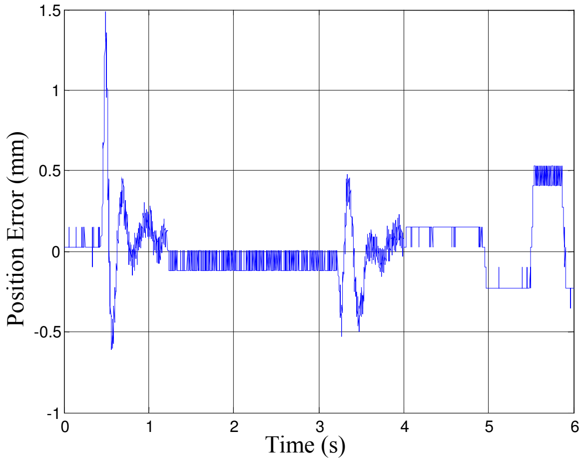

Fig. 4.b : Position Error (mm)

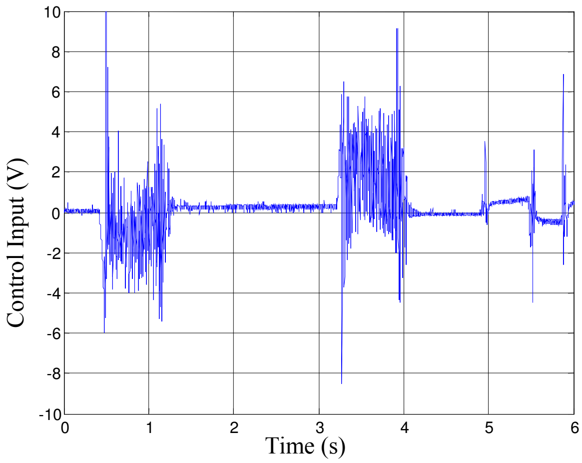

Fig. 4.c : Control input (V)

4. CONCLUSIONS

This paper has successfully demonstrated the application of a SISO passivity based controller to control the position of an electropneumatic system. Firstly, the mathematical model of the electropneumatic system was introduced. Then, the theoretical background for the controller synthesis was described in detail. Experiments were carried out to test the effectiveness of the proposed controller. Satisfactory control performance has been obtained by the passivity based controller. Future work will focus on the passivity based control for the MIMO (multi-input, multi-output) systems applied to electropneumatic system.

REFERENCES

Andrew, A. and Rui L. (2000). Systematic Control of a Class of Nonlinear Systems with Application to Electrohydraulic Cylinder Pressure Control. *IEEE Transactions on Control Systems Technology*, **Volume 6 n°4**, 623-634.

Araki, K. (1981). Effects of valve configuration on a pneumatic servo. *International Fluid Power Symposium*, 271-290.

Belgharbi, M., Thomasset, D., Scavarda, S. and Sesmat, S. (1999). Analytical model of the flow stage of a pneumatic Servodistributor for simulation and nonlinear control. *The Sixth Scandinavian International Conference on Fluid Power, SICFP'99*, 847-860.

Brogliato, B., Lozano, R., Maschke, B. and Egeland, O. (2007). *Dissipative systems analysis and control. Theory and application*., (Springer London, second edition), London.

Bobrow, J.E. and McDonell, B.W. (1998). Modeling identification and control of a pneumaticcally actuated force controllable robot. *IEEE Transactions on Robotics and Automation*. **Volume 14 n°5**, 732-742.

Di Zhou, Tie long Shen, Tamura, K., Nakazawa, T. and Henmi, N. (2003). Adaptive control of a pneumatic valve with unknown parameters and disturbances. *SICE 2003 Annual Conference*, **Volume 3**, 2703 – 2707.

Errahimi, F., Cherrid, H., M'Sirdi, N.K. and Aberkane, H. (2002). Robust adaptative control and observer for a robot with pneumatic actuators. *Robotica*, **Volume 20**, 167-173.

Laghrouche, S., Smaoui, M., Plestan, F. and Brun, X. (2006). Higher order sliding mode control based on optimal approach of an electropneumatic actuator. *International Journal of Control.* **Volume 79 n°2**, 119-131.

Ruihua, L., Weixiang, S. and Qingyu, Y. (2004). Multi-region fuzzy tracking control for a pneumatic servo squeezing forces system. *Intelligent Control and Automation, WCICA 2004. Fifth World Congress.* **Volume 5**, 4504 – 4507.

Mattei, M. (2001). Robust regulation of the air distribution into an arc heater. *Journal of process control*. **Volume 11**, 285-297.

Ortega, R., Loria, A., Nicklasson, P.J. and Sira-Ramirez, H. (1998). *Passivity based control of Euler Lagrange Systems: Mechanical, Electrical and Electromechanical application*. (Springer Edition).

Richard, E. and Scavarda, S. (1996). Comparison between linear and nonlinear control of an electropneumatic servodrive. *Journal of Dynamic Systems, Measurement, and Control*. **volume 118**, 245-252.

Sesmat, S. and Scavarda, S. (1996). Static characteristics of a three way servovalve. *12th Aachen Conference on Fluid Power Technology, Aachen, Germany.* 643-652.

Shearer, J.L. (1956). Study of pneumatic processes in the continuous control of motion with compressed air. *Parts I and II. Trans. Am. Soc. Mech. Eng*. **volume 78**, 233-249.

Smaoui, M. Brun, X. and Thomasset, D. (2005). A Robust Differentiator-Controller Design for an Electropneumatic System. *44th IEEE Conference on Decision and Control and European Control Conference CDC-ECC Seville, Espagne.*

Smaoui, M. Brun, X. and Thomasset, D. (2006 a). A study on tracking position control of an electropneumatic system using backstepping design. *Control Engineering Practice*. **Volume 14 n°8**, 923-933.

Smaoui, M., Brun, X. and Thomasset, D. (2006 b). Systematic Control of an Electropneumatic System: Integrator Backstepping And Sliding Mode Control. *IEEE Trans. on control syst. Technology*. **volume 14 n°5**, 905-913.

Tustin, A. (1947). The effect of backlash and speed-dependent friction on the stability of closed-cycle control systems. *Journal of the Institution of Electrical Engineers*, 94, 143-151.