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An algebraic approach for behavioral model decomposition

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Abstract: This paper presents a constrained decomposition methodology for discrete-event models. Partial models are obtained and used for model-based fault detection and isolation. The constraints of the decomposition ensures that the resulting partial model is decoupled from a given subset of inputs. The constrained decomposition method is formulated using pair algebra concepts as an iterative algorithm for easy implementation.

Keywords: Algebraic approaches, decomposition methods, decoupling, discrete-event systems.

1. INTRODUCTION

This paper presents a decomposition methodology for deterministic behavioral models. The objective is to obtain a partial model decoupled from a given subset of inputs while remaining coupled with respect to another selected inputs subset. The coupling and decoupling properties of the partial model allows us to detect occurrence of only selected unknown inputs or faults and to ignore the rest of them. This scheme is used to produce structured fault indicators to detect and isolate occurring failures based on continuous-time model. The proposed decomposition methodology is based on a particular algebraic formalism, named *algebra of functions* introduced by Shumsky [1991], Zhirabok and Shumsky [1993] and revisited in Berdjag et al. [2006]. This algebraic formalism remains valid for all types of deterministic models. Therefore, classical fault detection and isolation scheme, described in Patton [1994], may be extended, and specific models like sequential machines may be used for fault detection.

The present paper presents an extension of the decomposition algorithm to propose a constrained decomposition of sequential machines using *pair algebra* Hartmanis and Stearns [1966]. Output data is used to loosen decomposition constraints (output injection technique). Output injection technique is well known and was used to the decoupling continuous-time models.

Fault detection and isolation using discrete event systems was addressed by Sampath et al. [1995] and derived papers. Using output injection improves fault isolation rate. To the best of our knowledge, it was not employed in the past to decouple discrete-event models. This particular point is the major contribution of this paper.

The paper is organized as follows : In section 2, constrained decomposition problem is formulated using set theory framework. In section 3, the decomposition constraints and conditions are detailed. Section 3 contains basic reminders about *partitions* and *pair algebra* operators. In section 4, constrained decomposition of discrete-event models is presented along with output injection for discrete-event models. An example is given section 5 to illustrate the decomposition and the benefits of output injection. Conclusions and perspectives on future works close the paper.

2. PROBLEM FORMULATION

As a general principle, it is possible to represent deterministic behavioral models, noted Σ, using the following quintuple

$$(\mathcal{X}, \mathcal{U}, \mathcal{Y}, \mathcal{F}, \mathcal{H}) \tag{1}$$

where $\mathcal{X}$ is the state set, $\mathcal{U}$ is input set and $\mathcal{Y}$ is the output set. $\mathcal{F}$ and $\mathcal{H}$ are functions defined by :

$$\mathcal{F} \; : \; \mathcal{X} \times \mathcal{U} \longrightarrow \mathcal{X} \text{ and } \mathcal{H} \; : \; \mathcal{X} \times \mathcal{U} \longrightarrow \mathcal{Y} \tag{2}$$

The function $\mathcal{F}$ is the state function and the function $\mathcal{H}$ is the output function.

This representation allows to describe continuous-time and discrete-event deterministic models using the same formalism. Indeed, if Σ is a continuous-time model, then the sets $\mathcal{X}, \mathcal{U}, \mathcal{Y}$ are infinite sets of dimensions n, l, m respectively, i.e. $\mathcal{X} \subseteq \mathbb{R}^n$, $\mathcal{U} \subseteq \mathbb{R}^l$, $\mathcal{Y} \subseteq \mathbb{R}^m$, and the state and output functions are defined by :

$$\mathcal{F} \; : \; \mathbb{R}^n \times \mathbb{R}^l \longrightarrow \mathbb{R}^n \text{ and } \mathcal{H} \; : \; \mathbb{R}^n \times \mathbb{R}^l \longrightarrow \mathbb{R}^m \tag{3}$$

If the model Σ is a discrete-event model with a finite number of states, then the sets are $\mathcal{X}, \mathcal{U}, \mathcal{Y}$ are finite sets of cardinalities n', l', m' :

$$\mathcal{X} = \{x_1, \cdots, x_{n'}\}, \; \mathcal{U} = \{u_1, \cdots, u_{l'}\}, \; \mathcal{Y} = \{y_1, \cdots, y_{m'}\}$$

If the model Σ is excited by multiple inputs, we assume that Σ contains multiple dynamics. Every single dynamic is affected by a group of inputs, and remains decoupled from the rest. These dynamics are represented by *partial models*.

Definition 1. Let $\Sigma(\mathcal{X}, \mathcal{U}, \mathcal{Y}, \mathcal{F}, \mathcal{H})$ be a model. The model $\Sigma_*(\mathcal{X}_*, \mathcal{U}_*, \mathcal{Y}_*, \mathcal{F}_*, \mathcal{H}_*)$ is a partial model of Σ, if for a given relation $\Theta_{\mathcal{U}} : \mathcal{U} \to \mathcal{U}_*$, there is two relations $\Theta_{\mathcal{X}} : \mathcal{X} \to \mathcal{X}_*$ and $\Theta_{\mathcal{Y}} : \mathcal{Y} \to \mathcal{Y}_*$ satisfying

$$\forall u \in \mathcal{U} \ , \ \forall x \in \mathcal{X} \ , \ \forall x_* \in \mathcal{X}_* \ :$$

$$x_* = \Theta_{\mathcal{X}}(x) \Leftrightarrow \begin{cases} \mathcal{F}_*(x_*, \Theta_{\mathcal{U}}(u)) = \Theta_{\mathcal{X}}(\mathcal{F}(x,u)) \\ \mathcal{H}_*(x_*, \Theta_{\mathcal{U}}(u)) = \Theta_{\mathcal{Y}}(\mathcal{H}(x,u)) \end{cases} \quad (4)$$

$(\Theta_{\mathcal{X}}, \Theta_{\mathcal{U}}, \Theta_{\mathcal{Y}})$ is a homomorphism. The partial model Σ_* is coupled with respect to inputs $u_* \in \mathcal{U}_*$ and decoupled from inputs $u' \in \mathcal{U}$ if $u' \notin \mathcal{U}_*$.

$\mathcal{U}_*$ is a subset of $\mathcal{U}$. If the output set $\mathcal{Y}_*$ is not empty, then Σ_* provides outputs that are *bisimilar* with Σ outputs. Two outputs are considered as bisimilar, if all of their possible values are taken from the same set, and these values are consistent if the two models are excited by the same input sequence. Thus, if Σ_* and Σ are excited by the same inputs from $\mathcal{U}$, then their common outputs will be consistent. Discrepancy appears when the two models are excited by different inputs. Therefore, discrepancy can be used to detect unexpected events in input sequence. Moreover, if the occurring unexpected event belongs to $\mathcal{U} - \mathcal{U}_*$, no output discrepancy is observed, since Σ_* is decoupled from $\mathcal{U} - \mathcal{U}_*$.

Concretely, decomposition objective is to obtain a decoupled partial model which allows detection of selected faults and ignore the rest of them. The input set $\mathcal{U}$ is divided in three disjoint subsets :

$$\mathcal{U} = \mathcal{U}_c \cup \mathcal{U}_\rho \cup \mathcal{U}_\gamma \quad (5)$$

where $\mathcal{U}_\rho$ contains inputs to be coupled to, $\mathcal{U}_\gamma$ contains inputs to be decoupled from and $\mathcal{U}_c$ regroups the rest of inputs. $\mathcal{U}_\rho$ and $\mathcal{U}_\gamma$ forms unknown inputs set. For example, on a real modeled system, $\mathcal{U}_\gamma$ is the set failure events that must be detected, $\mathcal{U}_\rho$ es the set of perturbations or failures that must be ignored and $\mathcal{U}_c$ is the set of command events. The decoupled partial model Σ_* is described by the quintuple $(\mathcal{X}_*, \mathcal{U}_*, \mathcal{Y}_*, \mathcal{F}_*, \mathcal{H}_*)$ which satisfies :

- $\mathcal{U}_* \subseteq \mathcal{U}$
- $\mathcal{F}(\mathcal{X}, \mathcal{U}_*) \subseteq \mathcal{X}_*$
- $\mathcal{X}_* \cap \mathcal{F}(\mathcal{X}, \mathcal{U}_\gamma) = \emptyset$
- $\mathcal{Y}_* = \mathcal{H}(\mathcal{X}_*, \mathcal{U})$

The following section details the method to obtain Σ_*, i.e. to determine $\mathcal{X}_*, \mathcal{Y}_*, \mathcal{F}_*, \mathcal{H}_*$.

3. DECOMPOSITION OF GENERIC BEHAVIORAL MODELS

3.1 Decomposition objective

Consider a partial model Σ_* obtained by decomposition of Σ (1). In order to ensure that Σ_* is decoupled from $\mathcal{U}_\gamma$ and coupled with respect to $\mathcal{U}_\rho$, decomposition procedure is constrained to *coupling with respect to* $\mathcal{U}_\gamma$ and *decoupling from* $\mathcal{U}_\rho$ properties of the state set $\mathcal{X}_*$. Since $\mathcal{X}_*$ is decoupled, the descriptions

$$(\mathcal{X}_*, \mathcal{U}_*, \mathcal{Y}_*, \mathcal{F}_*, \mathcal{H}_*) \text{ and } (\mathcal{X}_*, \mathcal{U}, \mathcal{Y}_*, \mathcal{F}_*, \mathcal{H}_*)$$

are equivalent for Σ_*.

The functions $\mathcal{F}_*$ and $\mathcal{H}_*$ are restrictions of the functions $\mathcal{F}$ and $\mathcal{H}$ on the sets $(\mathcal{X}_*, \mathcal{U}, \mathcal{Y}_*)$:

$$\begin{aligned} \mathcal{F}_* : \ & \mathcal{X}_* \times \mathcal{U} \to \mathcal{X}_* \\ & (x_*, u) \mapsto x_*^+ = \mathcal{F}(x_*, u) \end{aligned} \quad (6)$$

$$\begin{aligned} \mathcal{H}_* : \ & \mathcal{X}_* \times \mathcal{U} \to \mathcal{Y}_* \\ & (x_*, u) \mapsto y_* = \mathcal{H}(x_*, u) \end{aligned} \quad (7)$$

The partial model Σ_* is obtained using the *decomposition function* φ defined by :

$$\begin{aligned} \varphi : \Psi & \to \Psi \\ \Sigma & \mapsto \Sigma_* \\ (\mathcal{X}, \mathcal{U}, \mathcal{Y}, \mathcal{F}, \mathcal{H}) & \mapsto (\mathcal{X}_*, \mathcal{U}, \mathcal{Y}_*, \mathcal{F}_*, \mathcal{H}_*) \end{aligned} \quad (8)$$

where Ψ is the set of all the possible quintuples. The function φ determines all the elements of the quintuple that describes Σ_*. Fortunately, the knowledge of the subset $\mathcal{X}_*$, if not empty, is sufficient to determine the other remaining elements. Let ϕ be a function that determines $\mathcal{X}_*$:

$$\begin{aligned} \phi : \mathcal{X} & \to \mathcal{X}_* \in \Psi_{\mathcal{X}} \\ x & \mapsto x_* = \phi(x) \end{aligned} \quad (9)$$

where $\Psi_{\mathcal{X}}$ is the set of all the possible subsets generated by elements of $\mathcal{X}$. $\mathcal{Y}_*$ is determined by the relation $\mathcal{Y}_* = \mathcal{H}(\mathcal{X}_*)$. $\mathcal{U}_*$ and $\mathcal{U}$ are known. If restrictions of $\mathcal{F}$ and $\mathcal{H}$ are possible on $\mathcal{X}_* \times \mathcal{U}$, then the decoupled partial model of Σ is completely defined.

Hence, the existence of the partial model Σ_* is linked to the existence of a restriction of the state function $\mathcal{F}$ on $\mathcal{X}_* \times \mathcal{U}$. This condition is called *invariance condition*. Output consistency between Σ_* and Σ is linked to the existence of a restriction of the output function $\mathcal{Y}$ on $\mathcal{U}_* \times \mathcal{X}_*$. This condition is called *output condition*.

In the following, the conditions and the constraints are detailed.

3.2 Decomposition constraints

Decoupling constraint Σ_* is decoupled for $\mathcal{U}_\gamma$ if and only if $\mathcal{U}_\gamma \cap \mathcal{U}_* = \emptyset$. It means that $\mathcal{X}_*$ do not intersect the state set coupled with respect to the subset $\mathcal{U}_\gamma$.

$$\mathcal{X}_* \cap \mathcal{X}_\gamma = \emptyset \quad (10)$$

$\mathcal{X}_*$ and $\mathcal{X}_\gamma$ are given by

$$\mathcal{X}_* = \mathcal{F}(\mathcal{X}, \mathcal{U}_*) \text{ and } \mathcal{X}_\gamma = \mathcal{F}(\mathcal{X}, \mathcal{U}_\gamma) \quad (11)$$

Coupling constraint Σ_* is coupled with respect to $\mathcal{U}_\rho$ if $\mathcal{U}_\rho \cap \mathcal{U}_* \neq \emptyset$. It means that $\mathcal{X}_*$ intersects the state subset coupled with respect to the subset $\mathcal{U}_\rho$.

$$\mathcal{X}_* \cap \mathcal{X}_\rho \neq \emptyset \quad (12)$$

$\mathcal{X}_\rho$ is given by $\mathcal{X}_\rho = \mathcal{F}(\mathcal{X}, \mathcal{U}_\rho)$

3.3 Decomposition conditions

Invariance condition Let $\mathcal{F}_*$ be the restriction of $\mathcal{F}$ such as

$$\begin{aligned} \mathcal{F}_* : \mathcal{X}_* \times \mathcal{U} & \to \mathcal{X}_* \\ (x_*, u) & \mapsto x_*^+ = \mathcal{F}(x_*, u) \end{aligned} \quad (13)$$

Since Σ_* replicates a part of Σ, the response of the two systems must be equivalent for the same inputs

$$\forall u \in \mathcal{U} \ , \ \forall x \in \mathcal{X} : \ x_* = \phi(x) \Leftrightarrow \mathcal{F}_*(x_*, u) = \phi\left(\mathcal{F}(x,u)\right) \quad (14)$$

Which implies

$$\mathcal{F}_*(\phi(x), u) = \phi\left(\mathcal{F}(x,u)\right) \quad (15)$$

If this relation is satisfied then the decomposition function ϕ is *invariant* under $\mathcal{F}$. By extending the relation (15) to the whole state set, we obtain the invariance condition

$$\mathcal{F}(\phi(\mathcal{X}),\mathcal{U}) \subseteq \phi(\mathcal{X}) \quad (16)$$

Output condition If there is a link between outputs of Σ_* and outputs of Σ for all the states $x_* \in \mathcal{X}_*$ and $x \in \mathcal{X}$, then it is possible to check discrepancy between outputs of Σ_* and Σ and the *output condition* is fulfilled. This condition make sense only if the invariance condition is already fulfilled.

If all the outputs of the two models are bisimilar then following relation is satisfied

$$\forall x \in \mathcal{X}\ ,\ \forall u \in \mathcal{U}\ ,\ \exists \zeta, \xi\ :\ \xi(\mathcal{H}_*(\phi(x),u)) = \zeta(\mathcal{H}(x,u)) \quad (17)$$

where ζ, ξ are some functions. If the function $\mathcal{H}_*$ is a restriction of $\mathcal{H}$, the relation (17) is always true. This is the perfect case. Practically, only one single consistent output is required to check consistency of Σ and Σ_*. Relation (17) becomes

$$\exists\, \zeta, \xi\ :\ \xi(\mathcal{H}_*(\mathcal{X}_*,\mathcal{U})) \cap \zeta(\mathcal{H}(\mathcal{X},\mathcal{U})) \neq \emptyset \quad (18)$$

Given that $\mathcal{H}_*$ is a restriction of $\mathcal{H}$, relation (18) can be simplified, giving the final form of output condition :

$$\mathcal{H}(\phi(\mathcal{X}),\mathcal{U})) \cap \mathcal{H}(\mathcal{X},\mathcal{U}) \neq \emptyset \Leftrightarrow \mathcal{Y}_* \neq \emptyset \quad (19)$$

3.4 Set delimiters and extension to the finite set case

In order to implement the decomposition conditions and constraints, *set delimiters* are defined. In the case of infinite sets, set delimiters are defined using functions Shumsky [1991]. In the case of finite sets, set delimiters are defined using partitions Hartmanis and Stearns [1966].

The set of all delimiters with the corresponding operations forms a mathematical structure called *algebra*. If definition sets of a model are finite, *pair algebra* is involved Hartmanis and Stearns [1966]. The infinite set case is addressed using *algebra of functions* Shumsky [1991], Zhirabok and Shumsky [1993]. Recently, algebra of functions were used in several topics of model-based monitoring Berdjag et al. [2006]. Unfortunately, pair algebra was not addressed recently.

In the next sections, partitions and pair algebra Hartmanis and Stearns [1966] are presented.

4. PARTITIONS AND PAIR ALGEBRA

Some notions on partitions are given, and definitions for the main algebraic operators of pair algebra are detailed.

4.1 Mathematical background

Partition Consider some finite set S. A partition π on S is a collection of disjoint subsets of S whose set union is S. These subsets are called blocks and noted B_α where α is an element of S.

$$\pi = \{B_\alpha\} \text{ such that } \begin{cases} B_\alpha \cap B_\beta = \emptyset \text{ for } \alpha \neq \beta \\ \bigcup\{B_\alpha\} = S \end{cases} \quad (20)$$

Consider a block B from π, and two elements s and t from S. If s and t are contained in the same block B of π, then we note $s \equiv t(\pi)$.

Operations on partitions Let S be a set and let π_1 and π_2 be two partitions on S. s and t are two elements from S. The following operation and relationship are possible on partitions :

- $\pi_1.\pi_2$ is the partition on S such that :
  $$s \equiv t(\pi_1.\pi_2) \text{ iff } s \equiv t(\pi_1) \text{ and } s \equiv t(\pi_2)$$
- $\pi_1 + \pi_2$ is the partition on S such that :
  $$s \equiv t(\pi_1 + \pi_2) \text{ iff there is a sequence in } S$$
  $$s = s_0, s_1, \ldots, s_n = t$$
  for which either
  $$s_i \equiv s_{i+1}(\pi_1) \text{ or } s_i \equiv s_{i+1}(\pi_2)\ ,\ 0 \leq i \leq n-1$$
- $\pi_1 \leq \pi_2$ if and only if $\pi_1.\pi_2 = \pi_1$ and $\pi_1 + \pi_2 = \pi_2$. π_2 is said larger than or equal to π_1.
- $\pi_1 \cong \pi_2$ if and only if $\pi_1 \leq \pi_2$ and $\pi_2 \leq \pi_1$. Partitions π_1 and π_2 are equivalent.

The set of all the possible partitions on S is ordered by the order relation $\leq$. The smallest partition is noted $\mathbb{O}$ and the greatest partition is noted $\mathbb{I}$. For example, let $S = \{1, 2, 3\}$. The smallest partition is given by $\mathbb{O} = \{\{1\}, \{2\}, \{3\}\}$ and the greatest is given by $\mathbb{I} = \{\{1, 2, 3\}\}$

4.2 Substitution property and pair algebra

Let S, I two sets and δ a function defined by

$$\delta : S \times I \longrightarrow S$$

Let π be a partition on S. The partition π is said to have the *substitution property* with respect to the function δ if and only if

$$s \equiv t(\pi) \Rightarrow \delta(s,a) \equiv \delta(t,a) \forall a \in I \quad (21)$$

If π has the substitution property then the function δ_π, defined by $\delta_\pi : \pi \times I \longrightarrow \pi$ such as

$$\delta_\pi(B_\pi, i) = B'_\pi \Leftrightarrow \delta_\pi(B_\pi, i) \subseteq B'_\pi$$

with $i \in I$, is the image of δ by π. δ_π is a restriction of δ on π.

The partition pair is an extension of the substitution property to two partitions. A partition pair (π, π') is an ordered pair of partitions on S such as

$$s \equiv t(\pi) \Rightarrow \delta(s,a) \equiv \delta(t,a)(\pi') \quad (22)$$

The partition pair is anti-symmetric. If (π, π') and (π', π) are partition pairs then $\pi \cong \pi'$. By the way, if π has the substitution property then π satisfies the relation (22), and (π, π) is a partition pair.

The partition pair concept is useful to describe the minimal and the maximal operators m and M.

Definition 2. Let π be a partition on S. $m(\pi)$ is the minimal partition that forms a partition pair on the left with π, i.e. $(m(\pi), \pi)$ is a partition pair and if (π', π) is a partition pair then $m(\pi) \leq \pi'$. The result $m(\pi)$ is also given by the following relation

$$m(\pi) = \prod\{\pi_i | (\pi, \pi_i) \text{ is a partition pair }\} \quad (23)$$

Definition 3. Let π be a partition on S. $M(\pi)$ is the maximal partition that forms a partition pair on the right with π, i.e. $(\pi, M(\pi))$ is a partition pair and if (π', π) is a partition pair then $m(\pi) \leq \pi'$. The result $M(\pi)$ is also given by the following relation

$$M(\pi) = \sum\{\pi_i | (\pi_i, \pi) \text{ is a partition pair }\} \quad (24)$$

Consider know some set of partitions L ordered by the ordering relation $\leq$, and a function δ. The subset $\Delta \subseteq L \times L$ of all the partitions pairs with respect to δ, with the partition operations "." and "+" forms an algebra called pair algebra. If the pair (π_1, π_2) is a partition pair, then we note $(\pi_1, \pi_2) \in \Delta_\delta$.

5. DECOMPOSITION OF SEQUENTIAL MACHINES

In this section, a constrained decomposition methodology is proposed for sequential machines, which are a common type of deterministic discrete-event models. Sequential machines are noted $(S, I, O, \delta, \lambda)$ to make distinction with the general case. S,I,O are respectively the state set, the input set and the output set of the model. δ is the state function and λ is the output function.

The decomposition problem is formulated as follows : Consider a sequential machine $\Sigma(S, I, O, \delta, \lambda)$ whose input set is given by

$$I = I_c \cup I_\gamma \cup I_\rho$$

A partial sequential machine Σ_* decoupled from I_γ and coupled with respect to I_ρ is searched. The machine Σ_* is defined by the quintuple $(S_*, I_*, O_*, \delta_*, \lambda_*)$ with

- $S_* = \pi$, where π is a partition of S.
- $I_* = \pi_I$, π_I is a partition of I or of $I \times O$.
- $O_* = \pi_O$, where π_O is a partition of O.
- $\delta_* : \pi \times I_* \rightarrow \pi$ is a restriction of δ.
- $\lambda_* : \pi \times I_* \rightarrow \pi_O$ is a restriction of λ

5.1 Decomposition constraints

In order to express coupling and decoupling constrains using partitions, a neutral element i_0 is added to I.

$$\forall s \in S : \delta(s, i_0) = s \tag{25}$$

Hence, Σ is decoupled from the element i_0 by definition. Consider now a block of the partition π_I that contains i_0. The corresponding partial model of Σ will be also decoupled from all the elements of this block.

Let $I_\gamma = \{a_1, a_2, \ldots\}$ and $I_\rho = \{b_1, b_2, \ldots\}$.

Decoupling constraint Consider the following partition

$$\pi_\gamma = \{\{i_0, a_1, a_2, \ldots\}, \{i_1\}, \ldots, \{i_l\}, \{b_1\}, \{b_2\}, \ldots\} \tag{26}$$

where i_j, with $j = 1, \ldots, l$, are elements of I_c. The partition π_γ decouples I_γ. Using the operator m, the corresponding state set partition is determined.

$$\pi_*^0 = m_\delta(\pi_\gamma) \tag{27}$$

π_*^0 is the state set partition that is decoupled from I_γ. If the machine Σ_* is decoupled from I_γ then its state set is a partition of π_*^0, i.e.

$$\pi_*^0 \leq \pi_* \tag{28}$$

Coupling constraint Consider the partition π_ρ that decouples I_ρ

$$\pi_\rho = \{\{i_0, b_1, b_2, \ldots\}, \{i_1\}, \cdots, \{i_l\}, \{a_1\}, \{a_2\}, \ldots\} \tag{29}$$

and the corresponding state set partition

$$\bar{\pi}_*^0 = m_\delta(\pi_\rho) \tag{30}$$

We saw previously that if the machine Σ_* is decoupled from I_ρ then its state set is a partition of $\bar{\pi}_*^0$. Accordingly, if Σ_* is coupled to I_ρ then

$$\bar{\pi}_*^0 \not\leq \pi_* \tag{31}$$

5.2 Decomposition conditions

Invariance condition Consider the sequential machine Σ and a partition π which has the substitution property with respect to δ. It means that the restriction of δ on π exists and the quintuple $(\pi, I, \pi_O, \delta_*, \lambda_*)$ describes a partial model of Σ, for some partition π_O and function λ_*.

It means that if π has the substitution property, i.e. $(\pi, \pi) \in \Delta_\delta$, then the discrete-event model described by $(\pi, I, \pi_O, \delta_*, \lambda_*)$ is a partial model of Σ.

From the definitions 2 and 3, if $(\pi, \pi) \in \Delta_\delta$, then the following relations are satisfied

$$\pi \leq M_\delta(\pi) \text{ and } \pi \geq m_\delta(\pi) \tag{32}$$

Given that for the sequential machine case, the relation $\pi \leq M(\pi)$ implies $\pi \geq m(\pi)$ and *vice versa*. Thus, either relation of (32) describes the invariance condition.

5.3 Output condition

Consider a partition π of the state set S. By analogy with the invariance condition, if π has the substitution property, then there is a restriction of λ on $S_* = \pi$, $I_* = I$ and a partition π_O of O defined by

$$\begin{aligned} \lambda_\pi : & S_* \times I_* \longrightarrow \pi_O \\ & (s_*, i_*) \mapsto \lambda(s_*, i_*) \end{aligned}$$

Let $\pi_\lambda = M_\lambda(\mathbb{O})$ be a partition induced by the output function λ and the output set O on the state set. If $\pi \geq \pi_\lambda$ is verified, (π, π) is a partition pair, all outputs of Σ and outputs of the partial model determined by π are bisimilar, which obviously is the best case.
Practically, to fulfill the output condition, it is sufficient to have one single bisimilar output. It means that partitions π and π' must share the same block and $\pi' + \pi \neq \mathbb{I}$. The output condition is given by

$$\pi + M_\lambda(\mathbb{O}) \neq \mathbb{I} \tag{33}$$

5.4 Output injection for discrete-event models

In some cases, the constraints of the decomposition are too strong resulting in unsatisfied decomposition conditions. A special technique called output injection may be used to relax the invariance condition. Output injection is a well known technique for continuous-time model decoupling. The main idea is to replace the information loss caused due to the decreased state set S_* by extending the input set of Σ_* with selected outputs of Σ.

Consider the invariance condition, i.e. $(\pi, \pi) \in \Delta_\delta$. An injection of the outputs O is equivalent to rectifying the state set partition π with a partition π_{inj} which is determined according to the injected outputs such that

$$\pi_*^0 . M_\lambda(\pi_{inj}) = \mathbb{O} \tag{34}$$

The extended invariance condition becomes

$$(\pi . \pi_{inj}, \pi) \in \Delta_\delta \tag{35}$$

The relation (35) is satisfied if the following statements are true.

$$M(\pi) \geq (\pi . \pi_{inj}) \text{ and } \pi \geq m(\pi . \pi_{inj}) \tag{36}$$

If the relations (36) are satisfied, then the decomposition partition π determines a partial sequential machine with an extended input set :

$$\Sigma_*(\pi, I \times \pi_{inj}, \pi_O, \delta_*, \lambda_*)$$

from some partition π_O and function λ_*.

5.5 Decomposition algorithm

Partial model determination problem is solved as a constrained optimization problem. The principle is to determine an initial set of partitions satisfying the decoupling constraint, and to determine S_* and π using an iterative loop, based on a scheme proposed in Shumsky [1991]. When the extended invariance condition is fulfilled, the loop ends and the output condition and coupling constraint are checked. Finally, the quintuple describing Σ_* is determined (algorithm 5.5).

Algorithm 1. Decomposition algorithm for discrete-event models
Require: $\Sigma(S, I, O, \delta, \lambda)$ {Complete system}
Require: π_γ, π_ρ { Decomposition constraints}
  $\pi_*^0 = m_\delta(\pi_\gamma)$ { Decoupled state set partition }
  $\bar{\pi}_*^0 = m_\delta(\pi_\rho)$ { Coupled state set partition }
  $\pi_\lambda = M_\lambda(\mathbb{O})$ { State set partition induced by O }
  Determine π_{inj} such that $\pi_*^0.M_\lambda(\pi_{inj}) = \mathbb{O}$
  $\xi^0 = \pi_*^0$, $i = 1$ {Initialization of the iterative loop}
  while $\xi^i \ncong \xi^{i-1}$ **do**
    $\xi^{i+1} = m(\xi^i.\pi_{inj}) + \xi^i$
    Increment i
  end while
  $\pi = \xi^i$
  if $\pi = \mathbb{I}$ **then**
    return Decoupling impossible
  else
    if $\pi + \pi_\lambda = \mathbb{I}$ **then**
      Output condition not satisfied by π
    else
      Output condition satisfied by π
    end if
    if $\pi \geq \bar{\pi}_*^0$ **then**
      Coupling constraint not satisfied by π
    else
      Coupling constraint satisfied by π
    end if
    return $\Sigma_*(S_* = \pi, I_* = (I_c \times \pi_{inj}, O_* = \pi_O, \delta_*, \lambda_*)$
  end if

6. ILLUSTRATION

Consider a sequential machine Σ described by the table 1. Σ is a five state model, with two known inputs, two unknown inputs f and g and three outputs $\{O, Q, N\}$. g represents the fault we want to detect and occurrence of the event f must be ignored. Therefore, Σ is going to be decomposed in order to obtain the partial model decoupled from $I_\gamma = \{f\}$ and coupled to $I_\rho = \{g\}$.

	a	b	f	g	o
1	2	4	5	1	O
2	2	4	2	2	O
3	3	5	3	3	Q
4	3	4	4	3	Q
5	3	1	5	5	N

Table 1. Transition table of the model Σ

The input set partition decoupled from I_γ is given by

$$\pi_\gamma = \{\{i_0, f\}, \{a\}, \{b\}, \{g\}\}$$

The corresponding state set partition is given by

$$\pi_*^0 = m_\delta(\pi_\gamma) = \{\{1, 5\}, \{2\}, \{3\}, \{4\}\}$$

The greatest partition π which fulfills the invariance condition is obtained by iteration :

$$\xi^0 = \pi_*^0$$
$$\xi^1 = \xi^0 + m(\xi^0) = \{\{1, 4, 5\}, \{2, 3\}\} \ncong \xi^0$$
$$\xi^2 = \xi^1 + m(\xi^1) = \{\{1, 2, 3, 4, 5\}\} \ncong \xi^1$$
$$\xi^3 = \xi^2 + m(\xi^2) = \{\{1, 2, 3, 4, 5\}\} \cong \xi^2$$

Since $\xi^3 \cong \xi^2$ then $\pi = \{\{1, 2, 3, 4, 5\}\} = \mathbb{I}$. In this case, the decomposition is impossible with the decoupling constraints from π_γ.

A solution may be obtained using output injection. There is three outputs generated by Σ : $\{O, Q, N\}$. We don't need to inject all the outputs. The outputs to be injected are a partition π_{inj} of $O = \{O, Q, N\}$ such that

$$\pi_*^0.M_\lambda(\pi_{inj}) = \mathbb{O}$$

Two partitions of outputs are possible : $\{\{O, Q\}, \{N\}\}$ and $\{\{O\}, \{Q, N\}\}$. We choose the first one $\pi_{inj} = \{\{O, Q\}, \{N\}\}$. Choosing the second partition will lead to a similar result. The decomposition algorithm is resumed at the π determination step. The greatest partition π which fulfills the invariance condition with output injection is obtained by the following iteration :

$$\xi^0 = \pi_*^0$$
$$\xi^1 = \xi^0 + m(\xi^0.\pi_{inj}) = \{\{1, 5\}, \{2\}, \{3\}, \{4\}\} \cong \xi^0$$

Since $\xi^1 \cong \xi^0$ then $\pi = \xi^1$. The decomposition with a decoupling constraint is possible using π .

The verification step consists in the test of the output condition and the coupling to $I_\rho = \{g\}$.

State set partition induced by the output is obtained by

$$\pi_\lambda = \{\{1, 2\}, \{3, 4\}, \{5\}\}$$

The output condition is fulfilled by π since

$$\pi + \pi_\lambda = \{\{1, 2, 5\}, \{3, 4\}\} \neq \mathbb{I}$$

To test the coupling constraint, the partition π_ρ which decouples I_ρ is calculated :

$$\pi_\rho = \{\{i_0, g\}, \{a\}, \{b\}, \{f\}\}$$

The corresponding state set partition is given by

$$\bar{\pi}_*^0 = m_\delta(\pi_\rho) = \{\{1\}, \{2\}, \{3, 4\}, \{5\}\}$$

Coupling constraint is fulfilled because

$$\bar{\pi}_*^0 \nleq \pi$$

Finally, partial sequential machine Σ_g is determined using the decomposition partition π. The input set of the partial machine is given by

$$I' = \{aO = \{a, \{O, Q\}\}, bO = \{b, \{O, Q\}\}, \ldots$$
$$\ldots, aN = \{a, \{N\}\}, bN = \{b, \{N\}\}\}$$

The state set is given by

$$S' = \{1' = \{1, 5\}, 2' = \{2\}, 3' = \{3\}, 4' = \{4\}\}$$

and the output set is

$$O' = \{\{O, Q\}, \{N\}\}$$

The state function δ' and the output function λ' are shown through the transition table 2. Output value O' of Σ_* dis

	aO	aN	bO	bN	g	o'
1'	2'	3'	4'	1'	1'	O'
2'	2'	2'	4'	4'	2'	O'
3'	3'	3'	1'	1'	3'	Q'
4'	3'	3'	4'	4'	3'	Q'

Table 2. Transition table of Σ_g

equivalent to both output values O or N for Σ and output value Q' is equivalent to Q. For example, if the current output of Σ is O or Q and the output of Σ_* is O' then outputs are consistent.

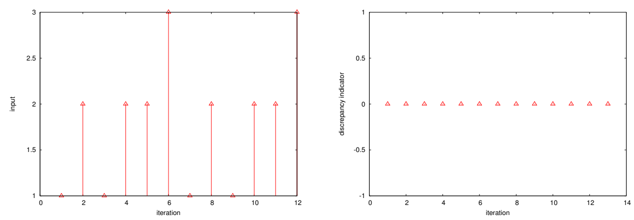
6.1 Simulations

Simulation results are provided here. The model Σ is excited by two sequences of known and unknown inputs, the first one contains several occurrences unknown input f and the second one contains occurrences of the unknown input g. Sequences composed of known inputs (a, b) of seq_1 and seq_2 combined with outputs from Σ are injected into the decoupled partial model Σ_*. Outputs are compared and a discrepancy indicator sequence is computed. The analysis of the discrepancy indicator sequence permits to detect the event g.

Input sequence containing f The first injected sequence is given by

$$seq_1 = [a, b, a, b, b, f, a, b, a, b, b, f]$$

Outputs of Σ and Σ_* are shown figure 1.



(a) Input sequence (b) Discrepancy indicator

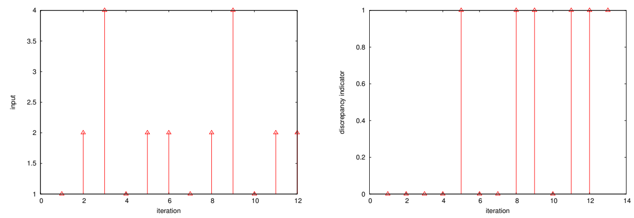
Fig. 1. Simulations for the first sequence

An unknown input f appears at the 6th iteration (Figure 3.a). Outputs of Σ and Σ_* remain coherent after f occurs, so the discrepancy indicator (Figure 3.b) remains nil.

Input sequence containing g The second injected sequence is given by

$$seq_2 = [a, b, g, a, b, b, a, b, g, a, b, b]$$

Outputs of Σ and Σ_* are shown figure 2. The unknown input g appears at the 3rd and the 9th iterations (Figure 4.a). Outputs of Σ and Σ_* remain consistent until the first occurrence of event g after what they become inconsistent. The discrepancy can be seen through the indicator (Figure4.b).



(a) Input sequence (b) Discrepancy indicator

Fig. 2. Simulations for the second sequence

7. CONCLUSION

In this paper, constrained decomposition of sequential machines is addressed. An algebraic formulation of the problem and of the solution is presented, based on previous work on continuous-time model decoupling Berdjag et al. [2006]. It is important to notice that the algebraic formalism used to implement the decomposition remain the same in the case of continuous-time models Berdjag et al. [2006].

While the plain decomposition of sequential machines cannot be considered as a contribution, the introduction of a decoupling constraint is new to the best our knowledge. The resulting decoupled partial model can be used to detect unexpected events in a process using a discrete-event model. Another contribution is the use of the *output injection* technique to extend invariance condition in the decomposition methodology.

Future works of authors addresses the decomposition of mixed dynamic models known as hybrid models.

REFERENCES

D. Berdjag, C. Christophe, V. Cocquempot, and B. Jiang. Nonlinear model decomposition for robust fault detection and isolation using algebraic tools. *International Journal of Innovative Computing, Information and Control*, 2(6):ISSN 1349-4198, December 2006.

J. Hartmanis and R. Stearns. *The algebraic structure theory of sequential machines*. Prentice-Hall, New York, 1966.

R.J. Patton. Robust model-based fault diagnosis : The state of the art. In *Proceedings IFAC Symposium SAFE-PROCESS'94*, volume 1, pages 1–24, Espoo, Finland, June 1994. IFAC.

M. Sampath, R. Sengupta, S. Lafortune, K. Sinnamohideen, and D. Teneketzis. Diagnosability of discrete-event systems. *IEEE Transactions on Automatic Control*, 40:1555–1575, 1995.

A.Ye Shumsky. Fault isolation in nonlinear dynamic systems by functional diagnosis. *Automation and Remote Control*, 12:148–155, 1991.

A.N. Zhirabok and A.Ye. Shumsky. A new mathematical techniques for nonlinear systems research. In *Proceedings 12th World Congress IFAC*, pages 485–488, Sidney, 1993.