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Half-order modelling of electrical network Application to stability studies

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Abstract: The fractional order systems have been used successfully to model the development of induced currents in synchronous generators. State-space theory can be generalized to give a dynamical representation of fractional order systems. In this paper, modal analysis of an electrical network modeled by fractional order systems is presented. For stability aspects, a trade-off between modal and frequency approaches is performed.

1. INTRODUCTION

For several years, a very close attention has been paid to the study of the electrical power systems in order to improve their sizing, their quality, their safety and their performances. Indeed, the electrical networks become more and more complex, including an increasing number of power electronics converters, which are used as interface or control devices (Barruel, 2005, Wildrick *et al.,* 1993).

An accurate modelling of power systems over a large frequency range is difficult and leads to models including a high number of parameters (Kundur, 1994). There is one main reason for this complexity. At high frequency, power networks are distributed parameter systems described by a set of partial differential equations (PDE). Classical approaches use lumped parameters model as an approximation of the PDE. But the accuracy of this approximation is directly linked to the number of parameters. And over a large frequency range, this number may be very high. Truncated realizations are then performed when design or control synthesis are (Skogestad, 1996).

An alternative approach uses the fractional order derivation (Riu, 2001). It is characterized by its inherent capability to describe systems of infinite dimension. The overall aim of the authors is to prove that all the classical tools used in electrical engineering for assessing dynamical performances can be adapted to the fractional order derivation. Fractional order modelling will then become a powerful tool for the performance analysis of distributed electrical systems.

In this paper, the authors focus on the small signal stability of electrical systems. The considered system includes a synchronous generator connected to a power bus. Because an electrical machine can be considered as a distributed system, the fractional order derivation will be used for the generator model. A trade-off between two methods commonly used for dynamical studies of power systems studies will be then performed. It will prove the interest of fractional order models for key issues of dynamical studies.

2. HALF-ORDER MODELLING OF A SYNCHRONOUS GENERATOR

2.1 *Fractional order systems*

The fractional order derivation has been applied since several years in many scientific fields as rheology, chemistry, signal processing, control or electromagnetic computation (Machado *et al.*, 2007). It is used for an accurate modelling of diffusion, fractal structures (Oldham *et al.,* 1974) or distributed systems which are represented by partial differential equations. It is also the mathematical principle of a particular technique of robust control introduced by (Oustaloup, 1995).

Fractional order derivation can be defined by (Podlubny, 1999), where Γ is the Gamma function:

$$D^{(\alpha)} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(\alpha)}{\Gamma(k).\Gamma(\alpha - k)} f[t - k.h] \quad (1)$$

It clearly shows the inherent capability of fractional order derivation to take into account all the past of the function. In other words, dynamical behaviour of a system can be modelled over a large range of frequency.

2.2 *Non integer order model of synchronous generator*

The impedance of a rectangular damping bar of a synchronous generator (Fig. 1) can be expressed by a half-order system (Retière *et al.*, 1999):

$$\overline{Z}^{1/2}_{resistive} = R_o \sqrt{1 + j\frac{\omega}{\omega_o}} \quad (2)$$

where $R_o = \dfrac{L_b}{\sigma \cdot h \cdot e}$ and $\omega_o = \dfrac{1}{\sigma \cdot \mu \cdot h^2}$

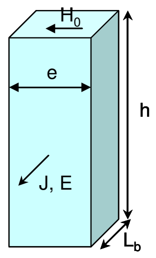

Fig. 1. Damping bar of a synchronous generator

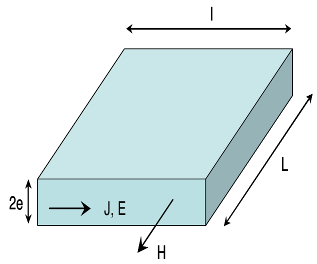

Fig. 2. Representation of a solid ferromagnetic piece.

Fig. 3 proves how relevant is this modelling over a large frequency range. The reference case is given by the analytical expression of the impedance (Alger, 1970):

$$\overline{Z}_{ana} = R_o \cdot \frac{\alpha}{th(\alpha)} \text{ with } \alpha^2 = j\omega\sigma\mu h^2 \qquad (3)$$

In the same way, it has been shown that the impedance characterising a solid rotor ferromagnetic piece (Fig. 2) can be expressed as:

$$\overline{Z}^{\frac{1}{2}}_{inductive} = \frac{j\omega L_o}{\sqrt{1 + j\dfrac{\omega}{\omega_o}}} \qquad (4)$$

where $L_o = \mu \dfrac{2 \cdot e \cdot l}{L} n^2$ and $\omega_o = \dfrac{1}{\sigma \cdot \mu \cdot e^2}$ (n is the number of coil turns surrounding the sheet).

The parameters R_o, L_o and ω_o depend on the geometrical and physical data (Riu, 2001).

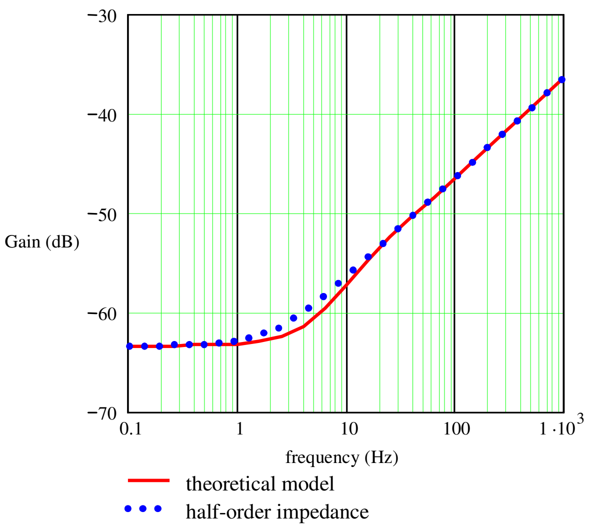

Fig. 3. Frequency response of a damping bar.

The variations of the impedances are due to the development of induced currents when frequency increases. Induction is a distributed phenomenon represented by partial differential equations and is well described by fractional order models.

Of course, these variations should be included in the synchronous generator dynamical model. In Park reference frame, the new fractional order model is established following the rules below:

- no induced current flows in the armature windings for frequencies lower than 1 kHz. It is then modelled by a resistance (r_s) and an inductance (l_s);.
- the inductance l_{ad}, corresponding to the stored energy in the air gap, is also supposed to be constant versus frequency;
- due to the development of induced currents in the solid ferromagnetic parts of the rotor, an "inductive" half-order impedance is added in parallel to l_{ad}: $\overline{Z}^{\frac{1}{2}}_{1d/q}(\omega) = \dfrac{j\omega L_{1d/q}}{\sqrt{1 + j\dfrac{\omega}{\omega_{1d/q}}}}$
- eddy currents are neglected in the excitation winding, which is therefore modelled by a resistance r_f and an inductance l_f, both constant versus the frequency;
- a constant inductance l_{12d} is associated to the mutual field between the excitation and the damper windings;
- the damping windings are modelled by a "resistive" half-order impedance: $\overline{Z}^{\frac{1}{2}}_{2d/q}(\omega) = R_{2d/q}\sqrt{1 + j\dfrac{\omega}{\omega_{2d/q}}}$
- the effect of speed variation on stator voltage are given by $\omega\lambda_q$ in d axis and $\omega\lambda_d$ in q axis.

The resulting "half-order" equivalent circuits of a synchronous machine in the d- and q-axes are shown in (Fig. 4) and (Fig. 5) for steady-state conditions.

It should be noted that each parameter can be linked directly to geometrical dimensions and physical characteristics of the generator.

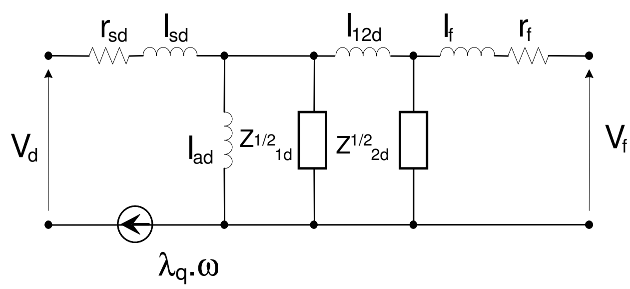

Fig. 4. Direct axis equivalent half-order circuit.

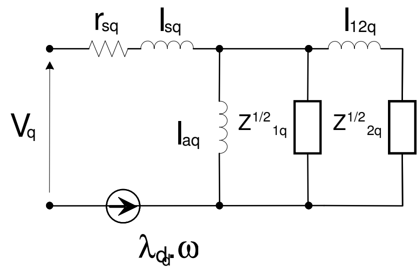

Fig. 5. Quadrature axis equivalent half-order circuit

These equivalent circuits have been identified and compared to classical integer models (Riu, 2001). Their principal properties relate to their compactness (a fractional-order model includes twice less parameters than a traditional integer model), their accuracy over a large frequency range (until approximately 1000 Hz) and their physical sense.

3. SMALL SIGNAL STABILITY ANALYSIS OF FRACTIONAL ORDER SYSTEM

Synchronous generator stability refers to voltage stability, speed (or frequency) stability and rotor angle stability (Kundur, 1994). Disturbances may be small or large, leading to small-signal or large-signal analysis. In this article, the authors focus on small-signal voltage stability of an electrical system.

Sources of voltage instability are related to interactions between devices connected to the network (electrical motors, generators, power converters, filters). A special attention has to be paid to the power converters. Indeed, their control mode is so that they behave as constant power loads. From a small-signal point of view, it means that a power converter can be represented by a negative resistance, which is typical of an unstable load.

In this paper, a synchronous generator connected to a power bus is analyzed. The generator is supposed driven by a constant speed turbine. For simplicity purpose, the generator voltage is not controlled. This simplification does not change the relevancy of the methodology given here after. Indeed, the aim of this paper is first to prove the efficiency of fractional order models for key issues of dynamical studies.

3.1 Frequential analysis (Middlebrook criterion and Nyquist theorem)

Stability analysis of an electrical system can be performed by using impedance method (Sudhoff *et al.*, 2000). The most straightforward application of the impedance method to power systems is called "Middlebrook criterion" (Middlebrook, 1979). According to this criterion, a system is stable if the gain of Z_{source} is "much lower" than the gain of Z_{load}, where Z_{source} is the generator impedance and Z_{load} is the constant power load impedance (Fig. 6). A second Middlebrook criterion consists in verifying that the contour of $|Z_{source}/Z_{load}|$ stays within the unit circle.

These criteria are a conservative interpretation of Nyquist theorem. Nonetheless, they are nowadays widely used to assess stability of power converters and to design input filter with stability margin.

In this article, the authors have preferred to guarantee the stability of the system by using the Nyquist theorem itself. For that purpose, the ratio Z_{source}/Z_{load} will be plotted in Nyquist diagram.

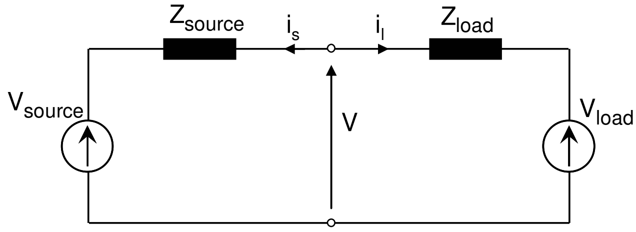

Fig. 6. Thevenin equivalent circuit of the studying system.

Both impedances are analytically calculated using Thevenin theorem. Of course, we need to apply the Nyquist theorem in both direct and quadrature axes of the Park model of the generator. The transmission line is included in the charge impedance. As an instance, the source impedance in q-axis is given in equation (5).

$$Z_{source,q} = r_{sq} + s \cdot l_{sq} + \frac{\dfrac{\left(r_{2q} + s \cdot r_{2q}\right) \cdot \dfrac{s \cdot L_{1q}}{1 + \left(\dfrac{s}{\omega_{1q}}\right)^{1/2}}}{\left(r_{2q} + s \cdot r_{2q}\right) + \dfrac{s \cdot L_{1q}}{1 + \left(\dfrac{s}{\omega_{1q}}\right)^{1/2}}} \cdot s \cdot l_{aq}}{\dfrac{\left(r_{2q} + s \cdot r_{2q}\right) \cdot \dfrac{s \cdot L_{1q}}{1 + \left(\dfrac{s}{\omega_{1q}}\right)^{1/2}}}{\left(r_{2q} + s \cdot r_{2q}\right) + \dfrac{s \cdot L_{1q}}{1 + \left(\dfrac{s}{\omega_{1q}}\right)^{1/2}}} + s \cdot l_{aq}} \qquad (5)$$

It should be noticed that expressions of half-order impedances are simplified as:

$$Z_{2d/q}^{1/2} \approx R_{2d/q}\left(1 + \sqrt{j\frac{\omega}{\omega_{2d/q}}}\right) \text{ and } Z_{1d/q}^{1/2} \approx \frac{jL_{1d/q}\omega}{1 + \sqrt{j\dfrac{\omega}{\omega_{1d/q}}}}.$$

Indeed, such an approximation is available for considered low cut-off frequencies of electrical machines (Riu, 2001) and allows to determine the generalized state-space system in section 3.2.

3.2 Modal analysis

From the electrical equations given by the equivalent half-order circuits (Fig. 4 and Fig. 5), one can build a generalised state-space system including the fractional order derivation of state-space variables. This system equation can be written as follows (Oustaloup, 1995):

$$\left[\underline{x}\right]^{(n)} = [A]\cdot\left[\underline{x}\right] + [B]\cdot\left[\underline{u}\right] \qquad (6)$$

where n is a real number, $[A]$ is the state matrix, $[x]$ is the state vector, $[B]$ is the input matrix and $[u]$ is the input vector.

If n is equal to unity, (6) gives the well-known expression of a classical state-space system.

From this relation, it is possible to calculate the eigenvalues λ_l and their associated vectors (right and left vectors) resolving the following equation:

$$s^n - \lambda_l = 0 \qquad (7)$$

where s is the Laplace operator. Then, poles are given by relation (8) where m_k is an integer (Matignon, 1996):

$$p_k = \left(\left|\lambda_l\right|\right)^{1/n} e^{i\cdot\left(\frac{1}{n}arg(\lambda_l)+\frac{2m_k\pi}{n}\right)} \qquad (8)$$

For a system with a fractional order between 0 and 1, the small signal stability condition is then given by the following relation (Matignon, 1996):

$$arg(\lambda_i) \geq \frac{n\cdot\pi}{2} \qquad (9)$$

This result is well-known for n=1.

The generalized state including the fractional order derivation of state-space variables is obtained from the set of differential equations describing the system (eq. (9)).

$$\begin{cases} V_f = -r_f\cdot i_f - l_f\cdot s\cdot i_f - l_{ad}\cdot s\cdot\left(i_d + i_f + i_{1d} + i_{2d}\right) \\ \quad - l_{12d}\cdot s\cdot\left(i_f + i_{2d}\right) \\ 0 = -R_{2d}\cdot i_{2d} - \frac{R_{2d}}{\sqrt{\omega_{2d}}}s^{1/2}\cdot i_{2d} - l_{ad}\cdot s\cdot \\ \quad \cdot\left(i_d + i_f + i_{1d} + i_{2d}\right) - l_{12d}\cdot s\cdot\left(i_f + i_{2d}\right) \\ 0 = -L_{1d}\cdot s\cdot i_{1d} - l_{ad}\cdot s\cdot\left(i_d + i_f + i_{1d} + i_{2d}\right) - \\ \quad -\frac{l_{ad}}{\sqrt{\omega_{1d}}}\cdot s^{3/2}\cdot\left(i_d + i_f + i_{1d} + i_{2d}\right) \\ V_d = -r\cdot i_d - l_d\cdot s\cdot i_d - l_{ad}\cdot s\cdot\left(i_d + i_f + i_{1d} + i_{2d}\right) \\ V_q = -r\cdot i_q - l_q\cdot s\cdot i_q - l_{aq}\cdot s\cdot\left(i_q + i_{1q} + i_{2q}\right) \\ 0 = -r_{2q}\cdot i_{2q} - l_{2q}\cdot s\cdot i_{2q} - l_{aq}\cdot s\cdot\left(i_q + i_{1q} + i_{2q}\right) \\ 0 = -L_{1q}\cdot s\cdot i_{1q} - l_{aq}\cdot s\cdot\left(i_q + i_{1q} + i_{2q}\right) - \\ \quad -\frac{l_{aq}}{\sqrt{\omega_{1q}}}\cdot s^{3/2}\cdot\left(i_q + i_{1q} + i_{2q}\right) \end{cases} \qquad (10)$$

It can then be written as :

$$\begin{cases} x_1^{(1/2)} = x_2; \\ x_2^{(1/2)} = a_{2,1}\cdot x_1 + a_{2,5}\cdot x_5 + a_{2,6}\cdot x_6 + a_{2,7}\cdot x_7 + \alpha\cdot V_d; \\ x_3^{(1/2)} = x_4; \\ x_4^{(1/2)} = a_{4,3}\cdot x_3 + a_{4,5}\cdot x_5 + a_{4,6}\cdot x_6 + \beta\cdot V_f; \\ x_5^{(1/2)} = x_6; \\ x_6^{(1/2)} = a_{6,1}\cdot x_1 + a_{6,3}\cdot x_3 + a_{6,5}\cdot x_5 + a_{6,6}\cdot x_6 + a_{6,7}\cdot x_7 + \chi\cdot V_d + \delta\cdot V_f; \\ x_7^{(1/2)} = a_{7,1}\cdot x_1 + a_{7,2}\cdot x_2 + a_{7,3}\cdot x_3 + a_{7,5}\cdot x_5 + a_{7,6}\cdot x_6 + a_{7,7}\cdot x_7 \\ \quad + \varepsilon\cdot V_d + \phi\cdot V_f + \varphi\cdot V_d^{(1/2)}; \\ x_8^{(1/2)} = x_9; \\ x_9^{(1/2)} = a_{9,8}\cdot x_8 + a_{9,10}\cdot x_{10} + a_{9,12}\cdot x_{12} + \gamma\cdot V_q; \\ x_{10}^{(1/2)} = x_{11}; \\ x_{11}^{(1/2)} = a_{11,8}\cdot x_8 + a_{11,10}\cdot x_{10} + a_{11,12}\cdot x_{12} + \eta\cdot V_q; \\ x_{12}^{(1/2)} = a_{12,8}\cdot x_8 + a_{12,9}\cdot x_9 + a_{12,10}\cdot x_{10} + a_{12,11}\cdot x_{11} + a_{12,12}\cdot x_{12} + \lambda\cdot V_q; \end{cases} \qquad (11)$$

which allows constructing the state-space system described by (6).

3.3 *Identification of the fractional order model*

The studied system is composed by a (1101 MVA, 22 kV, 60 Hz) synchronous generator and a constant power load of 650 MW. The parameters of fractional order equivalent circuits are identified by the SSFR procedure (IEEE, 1995). It is based on the identification of operational inductances L_d and L_q in both direct and quadrature axis. These inductances are obtained from source impedances as:

$$L_{d/q}(s) = \frac{Z_{source,d/q}(s) - r_{sd/q}}{s} \qquad (12)$$

The measures were performed by SSFR tests (Canay, 1993). Parameters of fractional order equivalent models are identified using least-square classical tools. Figures 7.a and 7.b present equivalent integer order models obtained in (Riu, 2001). Fig. 8 shows the frequency variation of the d-axis operational inductances for this classical model, the fractional order presented in section 2.2 and measurements.

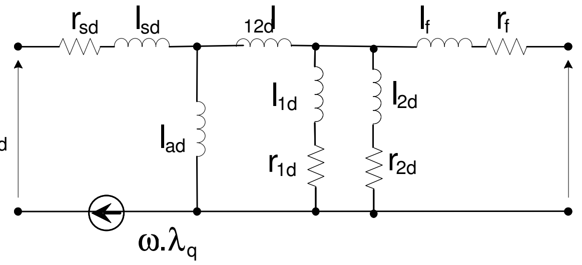

Fig. 7.a Direct axis equivalent integer order circuit

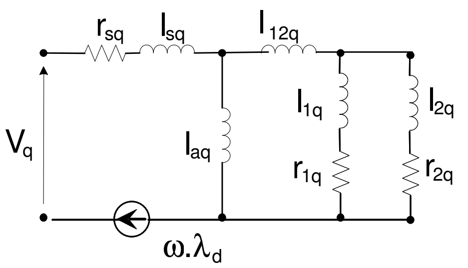

Fig. 7.b Quadrature axis equivalent integer order circuit

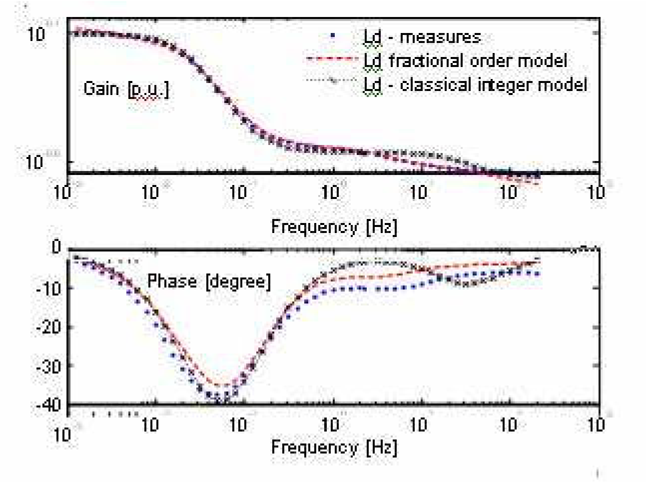

Fig. 8. Direct operational inductance comparison

It was already shown in (Riu, 2001), fractional order model are clearly more accurate than classical integer one, especially if the phase is considered in the medium to high frequency range. The number of parameters is lower than for the classical model, but the main interest concerns the physical signification of each parameter. Indeed, fractional order models are more suitable to study design and robust control systems.

3.4 Trade-off between modal and frequential analysis

Fig. 9 shows root locus given by modal analysis for fractional-order model. It exhibits instability. This is due to the constant power load.

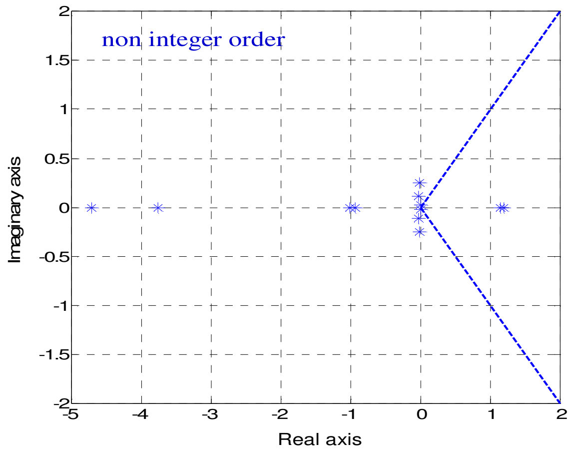

Fig. 9. root locus given by modal analysis.

Of course, using the Nyquist criterion, the same result of instability is found. It should be noted that if the load is a constant impedance one, the system is stable (Fig. 10 and Fig. 11).

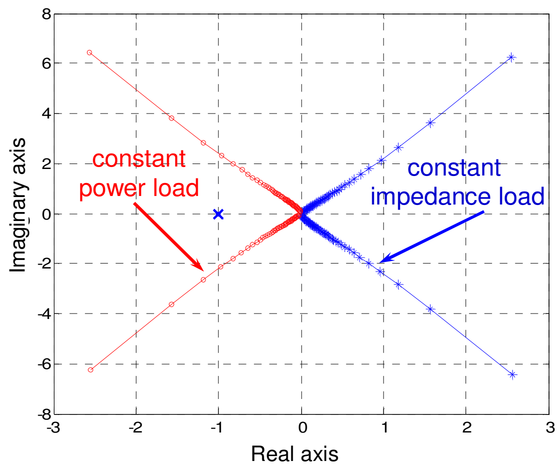

Fig. 10. d-axis Nyquist diagram :

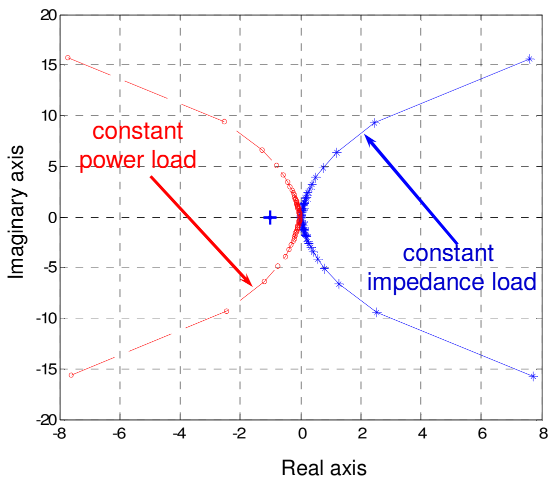

Fig. 11. q-axis Nyquist diagram

As a result, fractional order derivation is very suitable for stability assessment of electrical systems. Same methods as usual can be used (modal analysis, Nyquist criterion). Then, frequential approaches are interesting to focus on fractional systems applied to dynamical analysis. It should be interesting to display this approach to more complicated studies, as MIMO or robust analysis and control as mu-analysis, CRONE and H∞ robust controls (Oustaloup, 1995 - Skogestad, 1990).

4. CONCLUSIONS

In this paper, we have modelled a synchronous generator using fractional order systems. The final modelling has been used for stability assessment of a system connecting a synchronous generator and a constant power load.

Further works are the design of robust control loops by using fractional order controllers. The authors hope increasing the robustness of the system facing to dynamic perturbations (Oustaloup, 1995). The long-term work is to propose a pre-designing tool of electrical power systems using fractional order equivalent circuits. Then, a general methodology would be proposed to analyse a system integrating components modelled with non-integer order systems without any limitation according to this mathematical tool.

The paradigm shift in dynamical modelling introduced by fractional order derivation gives interesting perspective for the analysis of the new electrical systems. It makes stronger the link between physical phenomenon and mathematical representations. Then it contributes to a better understanding of key issues for the design of these critical and complex systems.

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