Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Higher Order Digital Delta-Sigma Modulator with Small Fluctuation: Sliding Mode Operation Approach

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Abstract: A new design approach of a digital delta-sigma modulator with small output fluctuation is proposed. The digital delta-sigma modulators are used in the digital-analog converters, the fractional-N phase lock loop, etc. and its output fluctuation is one of the important performances although a higher order modulator often produces large fluctuations. In this work, the noise shaping filter of the modulator is operated so that the operating point may maintain close to a sliding plane by selecting the output value to reduce the fluctuation of the output signal. By employing the sliding plane, the degrees of the freedom of the output signal is enhanced.

1. INTRODUCTION

The Delta-Sigma Modulators (DSMs) are used in the Analog-to-Digital Converters (ADCs) and the Digital-to-Analog Converters (DACs) to realize fine resolutions. The DSMs can be classified to two types: one is the analog DSM which is used in the ADCs and its noise shaping filter is implemented with analog circuits. The other type is the digital DSM which is used in the DACs and its noise-shaping filter is implemented as a digital filter.

The digital DSM becomes used in many applications, recently. The fully digital audio amplifier is one of the applications (Tan *et a*l. 2000, Yoneya and Watanabe 2005). The source signals with Pulse Coded Modulation (PCM) are converted to the Pulse Width Modulation (PWM) signals with the digital DSMs and the pulse width modulators and the PWM signals control the switching stages to drive the loads such as laud speakers. In this case, the fluctuation of the digital DSM output signal causes depression of the amplifier performance.

One of the features of the digital DSM is that a multi-level quantizer is used in the modulator since the quantization operation can be performed ideally in digital processing, whereas an analog DSM uses a bi-level quantizer because of the linearity problem. Another feature is that the filter operation is implemented in digital manner. Therefore arbitral accuracies of the operations can be realized by preparing demanded word lengths.

Despite the facts that the digital DSM has a great potential, its design method has not established well yet. Some approaches have been proposed but trials and errors are required to obtain a digital DSM with a desired performance. The main issue of the design is the trade off between the rejection of the quantization error component in a lower frequency region and the variance of the modulation error. Generally, the higher the rejection ratio of the quantization error is high in a lower frequency, the larger the modulation error variance. So it is required to design the digital DSM with a desired spectrum of the modulation error keeping the variance of the modulation error below a certain level by trials and errors.

The main purpose of this work is to propose a design method of a digital DSM with small output signal fluctuations. The idea is that the noise shaping filter is realized in a state-space form and fed back from the DSM output signal so that the operating point of the filter may keep close to a sliding plane in the state space. By minimizing the distance between the operating point and the sliding plane, the amplification of the output signal fluctuation may be prevented.

The use of the sliding mode operation brings a significant flexibility to the design of the DSM. It means various types of the modulations by the output signal of the DSM can be considered. For example, a PWM the pulse position is also modulated can be treated and the modulated signal with desired low-frequency components may be obtained. The nonlinear sliding mode DSM is described for a general case and a numerical example is presented in this paper.

This paper is organized as follows: after this introduction, the digital DSM is explained briefly showing the structure of the ordinary DSM in section 2. Section 3 describes the proposed DSMs for both the linear and the nonlinear cases. In 3.1, the design scheme of the sliding mode DSM is described, and in 3.2, a new structure of the sliding mode DSM for nonlinear case is presented. Some numerical examples including the ordinary, the linear sliding mode and the nonlinear sliding mode DSM cases are shown in section 4.

2. Ordinary Delta-Sigma Modulator

In this section, the ordinary digital DSM is explained briefly. The purpose of a digital DSM is to produce a coarse digital signal whose spectrum in a lower frequency region is close to that of the input signal. A typical block diagram is shown in Fig. 1. Although other types are proposed, this type will be considered throughout this paper. The input signal $u[k]$ is a

PCM signal with a fine resolution and the output signal $y[k]$ is a PCM signal with a coarse resolution.

To analyze the DSM, the quantizer can be considered as an adder of the quantization noise $e[k]$ as shown in Fig. 2. Therefore the block diagram can be reduced as shown in Fig. 3 and the z-transform $Y[z]$ of the output can be expressed as follows:

$$Y[z] = U[z] + N[z]E[z], \tag{1}$$

$$N[z] = \frac{1}{1 + F[z]}, \tag{2}$$

where $U[z]$ and $E[z]$ are the z-transform of $u[k]$ and $e[k]$, respectively and $F[z]$ is the transfer function of the noise shaping filter. The term $N[z]E[z]$ is the modulation noise. With a glance, it looks easy to set the transfer function $N[z]$ from the quantization noise to the output signal arbitral by setting $F[z]$ appropriately but there are implicit constraints on $F[z]$ to work the modulator.

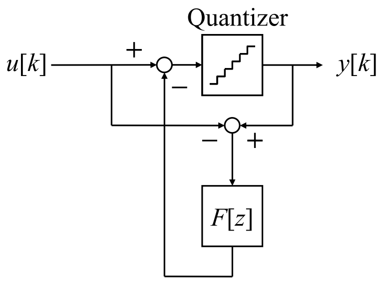

Fig. 1. An example of the ordinary DSM structure.

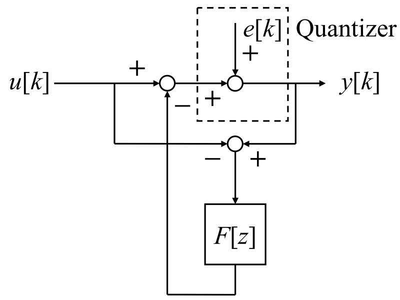

Fig. 2. An equivalent structure of the ordinary DSM.

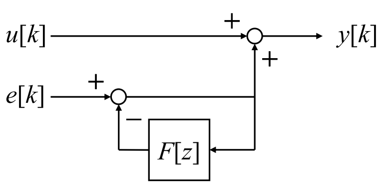

Fig. 3. An reduced structure of the ordinary DSM.

When the modulator works well, the quantization noise $e[k]$ can be considered a white one. Although this is an assumption for the analysis, many of the experiments prove this fact. In other words, $F[z]$ needs to be set so that the quantization noise may be white. In this work, first, $N[z]$ is set to satisfy the specifications, and then, $F[z]$ is calculated and implemented. Usually, $N[z]$ is set as a high-pass filter. Since $F[z]$ needs to be strictly proper, the following condition needs to be satisfied:

$$\lim_{z \to \infty} N[z] = 1 \tag{3}$$

The design method of a filter has been established and the practitioner can design the filter readily. Usually the automatically calculated filter does not satisfy this condition since the gain at a higher frequency may be set unity, and the filter needs to be normalized with the value of $\lim_{z \to \infty} N[z]$.

Unfortunately condition (3) tends to make the gain of $N[z]$ in a higher frequency region larger. This fact means that the fluctuation in the output signal becomes large. Generally, the higher the high-pass filter performance is, the larger the gain of $N[z]$ in a higher frequency region is. Therefore there are trade-off between the low-cut performance and the high-emphasis characteristics of the quantization noise to the output signal.

3. Sliding Mode Delta-Sigma Modulator

The DSM operating in sliding mode is introduced in this section. The noise shaping filter is operated in a kind of sliding mode to reduce the fluctuation of the output signal. First, the linear case is treated, and then the nonlinear case is considered.

3.1 Sliding Mode DSM in Linear Case

In this subsection, the case the lower frequency components of the DSM output signal should be close to the DSM input signal, namely linear case.

With the proposed method, the noise shaping filter is realized in a state-space form:

$$\begin{aligned} x[k+1] &= \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{b}\big(u[k] - y[k]\big) \\ v[k] &= \boldsymbol{c}\boldsymbol{x}[k] \end{aligned}. \tag{4}$$

The output signal $y[k]$ is determined so that the state variable $x[k]$ may maintain small. This is the same approach with the ordinary DSMs. Since the state variable is a vector, a measure of the magnitude of the state variable is needed. Then a sliding plane is prepared and the measure is set as the distance $v[k]$ between the state variable and the sliding plane. If the state variable keeps close to the plane, the state variable tends to move to the origin with the dynamics determined by the normal vector $\boldsymbol{c}$ of the sliding plane, although the state variable is exited persistently. Therefore the normal vector should be set appropriately as well as the system matrix of the noise shaping filter.

The output signal $y[k]$ is determined as follows:

$$y[k] = \arg\min_{y[k]} \boldsymbol{c}\left\{ \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{b}\big(u[k] - y[k]\big)\right\}. \tag{5}$$

Since the contribution of the output signal $y[k]$ to the state variable is linear, (5) can be stated with a linear expression and a quantizer. Although such an expression is not treated here, this fact means the sliding mode DSM has the same structure with the ordinary DSM. It is interpreted that the sliding model DSM has a restriction on the noise shaping filter to have small fluctuation in the output signal. The restriction is automatically introduced by using (5). Therefore the sliding mode DSM in a linear case is a kind of the design procedure of the noise shaping filter.

Note that the sliding mode DSM does not guarantee the smallest fluctuation of the output signal. The fluctuations are minimized in each step but the dynamics of the noise shaping filter are not considered. Practically, the sliding mode DSM has small output fluctuation.

The block diagram of the sliding mode DSM for a linear case is illustrated in Fig. 4. The one-step-ahead prediction is applied to determine the output signal to follow to the input signal. This prediction is automatically introduced by utilizing (5). The variable $v[k]$ does not appear directly in the diagram.

3.2 Sliding Mode DSM in Nonlinear Case

In an actual use, the output signal of a digital DSM is often used for the PWM. In such a case, the relationships between the output signal of the DSM and the PWM signal is not linear in the strict sense (Gwee *et al.* 2003, Pascual *et al.* 2003): a PWM signal has two or three states, and a PWM can not be a linear operation. When a nonlinear component is

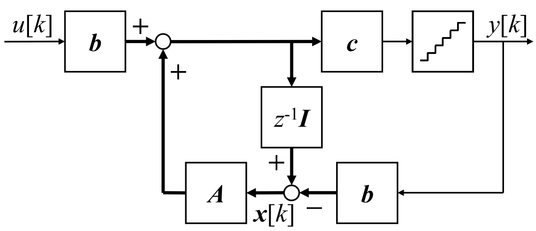

Fig. 4. The structure of the linear sliding mode DSM.

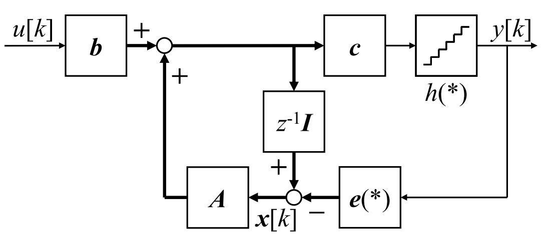

Fig. 5. The structure of the nonlinear sliding mode DSM.

connected to the DSM output, the nonlinearity of the component should be considered in the DSM. In the case of the PWM, the nonlinearity of the PWM can be treated by considering a continuous-time noise shaping filter, whereas the implemented filter is discrete-time one (Yoneya 2005, 2006). In this paper, a general case is dealt with.

The noise shaping filter is realized in the state-space form as follows:

$$\begin{aligned} x[k+1] &= \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{b}u[k] - \boldsymbol{e}\big(y[k]\big) \\ v[k] &= \boldsymbol{c}\boldsymbol{x}[k] \end{aligned}. \tag{6}$$

The difference from the linear case is the feedback term from the output signal $y[k]$ is expressed as a nonlinear vector function $\boldsymbol{e}(y[k])$. This nonlinear function vector depends on the nonlinearities of the component and can be calculated readily, while other terms are same as the linear case.

When the PWM is used, the nonlinear vector function $\boldsymbol{e}(y[k])$ is calculated as follows. Let us assume the noise shaping filter (6) is the discretized one of the following filter,

$$\begin{aligned} \dot{x}_c(t) &= \boldsymbol{A}_c \boldsymbol{x}_c(t) + \boldsymbol{b}_c\big(u_c(t) - w(t)\big) \\ v_c(t) &= \boldsymbol{c}\boldsymbol{x}_c(t) \end{aligned}, \tag{7}$$

where $w(t)$ is the PWM output signal and the suffix c means a continuous time variable or a parameter for the continuous time system. The output vector $\boldsymbol{c}$ is same for both cases. Discretizing (7) applying a zeroth hold for $u_c(t)$ and a sampler for $v_c(t)$, the parameters in (6) are obtained as:

$$\boldsymbol{A} = \exp\big(\boldsymbol{A}_c T\big), \tag{8}$$

$$\boldsymbol{b} = \int_0^T \exp\big(\boldsymbol{A}_c \tau\big)\boldsymbol{b}_c d\tau, \tag{9}$$

$$\boldsymbol{e}\big(y[k]\big) = \int_0^T \exp\big(\boldsymbol{A}_c \tau\big)\boldsymbol{b}_c w\Big(\overline{k+1}T - \tau\Big)d\tau, \tag{10}$$

where T is the sampling period. Since the PWM signal $w(t)$ is determined by the DSM output signal $y[k]$ during the period of $kT < t < (k+1)T$, the evaluated value of (10) is a function of $y[k]$.

Then the output signal $y[k]$ is calculated as follows:

$$y[k] = \arg\min_{y[k]} \boldsymbol{c}\left\{ \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{b}u[k] - \boldsymbol{e}\big(y[k]\big)\right\}. \tag{11}$$

Since $\boldsymbol{e}(*)$ is a nonlinear function vector of $y[k]$, the output signal $y[k]$ is a complicated function of $\boldsymbol{x}[k]$ and $u[k]$. Because it is not realistic to solve the optimization problem (11) in real time, the DSM should have the map from $\boldsymbol{x}[k]$ and $u[k]$ to $y[k]$ and its relationships are assumed to be calculated previously. The relationship can be expressed as follows:

$$y[k] = h\Big(\boldsymbol{c}\big(\boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{b}u[k]\big)\Big), \tag{12}$$

where $h(*)$ is a nonlinear function and has discrete values. Note that $h(*)$ is a map from a scholar variable to a scholar one, and the map can be implemented with a table.

The block diagram of the sliding mode DSM for the nonlinear case is shown in Fig. 5. This block diagram differs from the one for linear case with two points. One is that the feedback component from the output signal $y[k]$ is nonlinear and the other is that the quantizer to calculate the output signal $y[k]$ has a complicated relationship between its input and output and is implemented with a table. From the viewpoint of the accuracy, the table should be realized as an associated type one. But from the viewpoint of the hardware resource, the table should be a direct map from the discretized input value to the output value. The resolution of the table affects the magnitude of the output signal fluctuation but the resolution is not so sever in many cases.

4. Numerical Example

4.1 Linear Case

First, some numerical examples for the linear cases are presented.

The application is assumed a fully digital audio amplifier, and the sampling period is set 705.6 kHz. The output signal $y[k]$ has 61 levels in [-30, 30] which corresponds to the input signal range [-30/32, 30/32].

For the ordinary DSM, the noise shaping filter $F[z]$ is set as follows:

$$F[z] = \frac{1.9715z^4 - 6.0354z^3 + 7.2881z^2 - 4.0511z + 0.8678}{z^5 + 4.9665z^4 - 9.8997z^3 + 9.8997z^2 - 4.9665z + 1}. \tag{13}$$

This noise shaping filter is designed so that the noise spectra in the frequency region below 20 kHz may be small.

For the sliding mode DSM, the combination $(\boldsymbol{c}, \boldsymbol{A}, \boldsymbol{b})$ is set as the realization of $F[z]$ with the observable canonical form.

The spectra of the output signal $y[k]$ for a two-tone test are shown in Fig. 6(a) with the ordinary DSM and in Fig. 6(b) with the sliding mode DSM. The input signal frequencies are 4.13 and 5.51 kHz and the maximum amplitude is 0.92.

In both cases, the noise spectra in the frequency region below 20 kHz are suppressed well, but the noise spectrum shapes are slightly different. The difference comes from the difference of the feedback scheme from the noise shaping filter. One of the purposes to use the sliding mode DSM is to reduce the output signal fluctuation, which corresponds to the noise spectrum in a higher frequency region. From the figures, it is observed that employing the sliding mode DSM makes the noise spectrum in a high frequency region smaller. Since the both DSM have equivalent structures for the linear case, the noise spectrum in the lower frequency region becomes larger in the sliding mode case by making the high frequency noise spectrum lower. Generally, it is a hard work to find out a noise shaping filter with a high performance of the noise

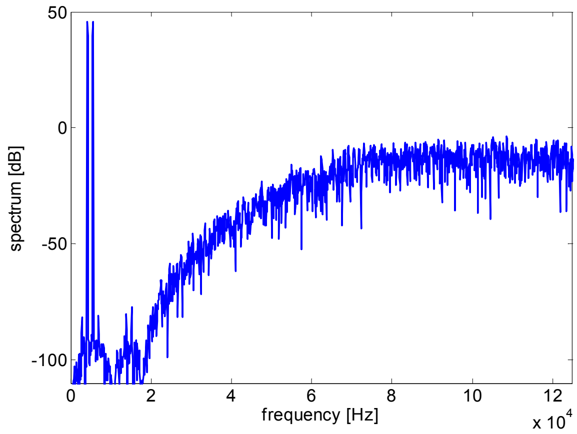

Fig. 6(a). Spectrum of the output PCM signal with the ordinary DSM.

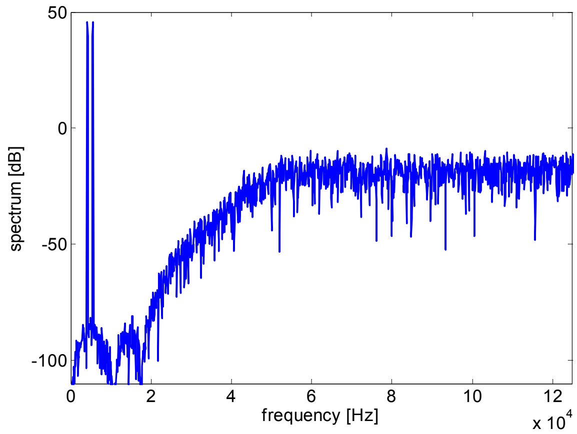

Fig. 6(b). Spectrum of the output PCM signal with the sliding mode DSM.

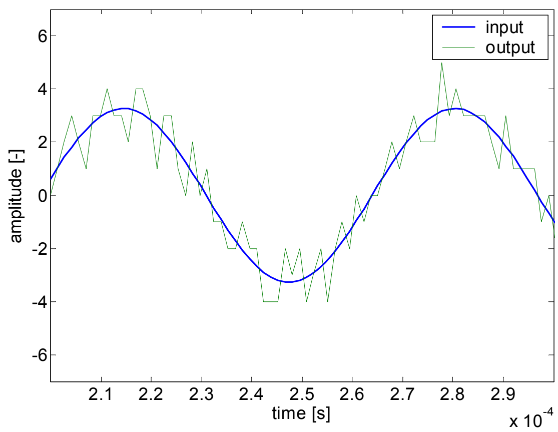

Fig. 7(a). Input and output signals of the ordinary DSM.

suppression in the lower frequency region and small output signal fluctuations for the ordinary DSM. With the sliding mode one, the condition of small fluctuation may be satisfied automatically.

Examples of the input and output signals of the DSMs in time domain is shown in Figs. 7(a) and 7(b) for the ordinary DSM and the sliding mode one, respectively. The input signal is a sinusoidal one with the frequency of 15.2 kHz and the amplitude of 1.02. It can be shown that the fluctuation of the output signal with the sliding mode DSM is smaller than that with the ordinary DSM.

Another merit of the sliding mode DSM is the small overshoot of the transient response of the output signal. Figs. 8(a) and 8(b) show the transient responses of the output signals for the both DSM with the input signals of rectangular waveforms. There is a large possibility to have large overshoot in the output signals with the ordinary DSM, whereas any overshoot is not preferable in many applications. With the sliding mode DSM, any overshoot is not observed. Although a rectangular waveform can not be applied in the fully digital audio amplifier, a truncated waveform may be applied, and such an overshoot issue is important in the application.

4.2 Nonlinear Case

For an example of the nonlinear case, an application of the fully digital audio amplifier is considered. The output signal $y[k]$ of the DSM is fed to a complementary pulse width modulator to produce the driving signal of a switching amplifier. The calculation of the nonlinear vector function $\boldsymbol{e}(y[k])$ (Yoneya 2005, 2006, 2007) is omitted in this paper.

One of the merits of the sliding mode DSM is that various nonlinearity of $\boldsymbol{e}(y[k])$ can be treated readily. Here, the pulse position of the PWM signal is shifted in time as a modulation as well as the pulse width. Since the PWM is a nonlinear operation, the time shift of the pulse affects the nonlinear function $\boldsymbol{e}(y[k])$. The pulse shift modulation brings an effect as a kind of the Pulse Density Modulation (PDM). Let us call this modulation a Pulse Width and Shift Modulation (PWSM).

An example of the PWSM signal spectrum generated with the nonlinear sliding mode DSM is shown in Fig. 9: this is not the spectrum of a PCM signal. The conditions are same as Fig. 6. By shifting the pulse position, the effective resolution of the PWSM becomes finer than that of the PWM, and the noise spectrum becomes smaller by about 6dB.

The resolution of the PWSM depends on the signal level. A spectrum of the PWSM signal with a small input signal is shown in Fig. 10. The sliding mode DSM is artificially excited so that the output signal level remains in the region with a high resolution. It can be recognized that the noise spectrum is reduced remarkably because of a high resolution.

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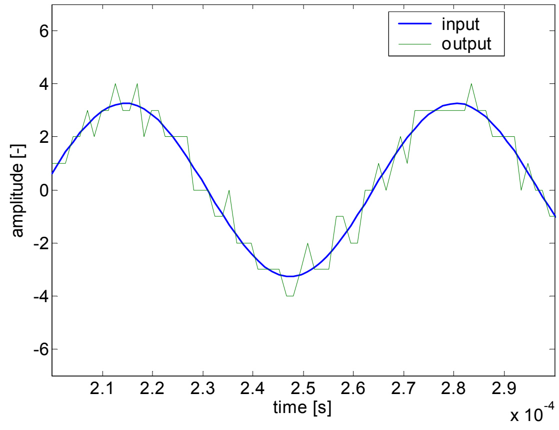

Fig. 7(b). Input and output signals of the sliding mode DSM.

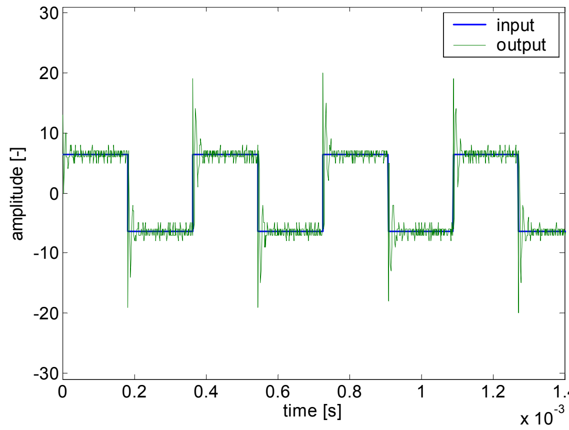

Fig. 8(a). Transient response of the ordinary DSM for a rectangular waveform input.

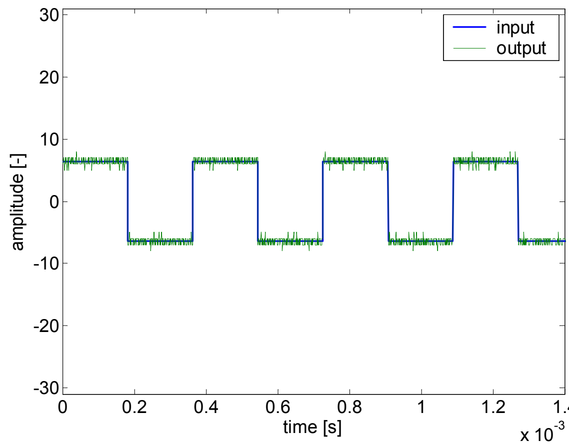

Fig. 8(b). Transient response of the sliding mode DSM for a rectangular waveform input.

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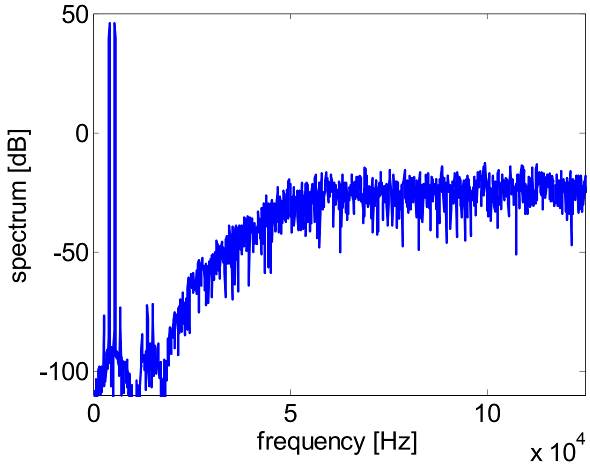

Fig. 9. Spectrum of the PWSM signal with the nonlinear sliding mode DSM.

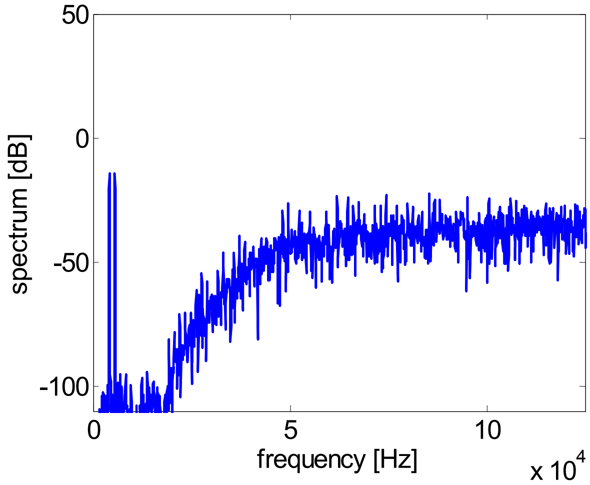

Fig. 10. Spectrum of the PWSM signal with the nonlinear sliding mode DSM for small input signal.