

Statistical Learning applied to the energy management in a Fuel Cell Electric Vehicle

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Abstract: The paper considers a high efficiency energy management control strategy for a hybrid fuel cell vehicle using neural networks and Statistical Learning theory. Hybrid Electric Vehicles may potentially improve fuel economy, reduce emission gases, and achieve performance similar to conventional cars. The use of different power sources and the presence of different constraints makes the power management problem highly nonlinear. Probabilistic and statistical learning methods are used to design the weights of a neural networks to minimize the fuel consumption during a given path. Numerical results are obtained using the model of a real hybrid car, “*Smile*” developed by FAAM, using a stack of fuel cells as the primary power source in addition to ultracapacitors. The results are satisfactory in terms of fuel consuming and efficiency of ultracapacitors and batteries.

Keywords: Fuel Cell Electric Vehicle; Statistical Learning; Neural Network Control.

1. INTRODUCTION

The history of electric vehicles started with the invention of the battery by A. Volta and the discovery of electromagnetic induction by M. Faraday. This culminated, in 1873, with the invention of the first electric vehicle (Westbrook [2005]). Even though the first vehicles were actually electric, gasoline and diesel cars overtook electric vehicles since the 19th century, thanks to their better energy-weight ratio. In recent years, the increase in the size and weight of passenger cars have made gasoline and diesel vehicles more pollutant and less efficient. In addition, the ever increasing cost of fuel as well as pollution problems are motivating car companies to look for new solutions to minimize fuel consumption and the production of polluting gases (O.Fuji [2002]).

Hybrid Electric Vehicles (HEVs) actually combine the efficiency of electric cars with the high autonomy of conventional vehicles and are considered a potential solution to such problems. The combination of electric motors with various storage elements (i.e. fuel cell, thermal engine, ultracapacitors, etc..) brought about more complex systems, as well as different control strategies to manage the vehicle powertrain (Maggetto and Mierlo [April 2000]).

Hybrid vehicle controllers are based on a supervisor that chooses, in the presence of different constraints, the best power path to satisfy the power demands of the drive line, while minimizing the fuel consumption and the production of the pollution gases. Various solutions were developed in the literature in order to achieve different performances: Dynamic Programming and Quadratic Program-

ming are used to minimize the fuel consumption over all paths (G Rizzoni [December 2003], Sciarretta and Guzzella [April 2007], Koot [2006]). Heuristic controllers, based on Boolean or fuzzy logic rules, are used to minimize the fuel consumption using different vehicular variables such as torque demand or car speed (N. Jalil and Salman [1997], Sciarretta and Guzzella [April 2007]). Artificial neural networks have also been used to achieve various performance objectives during different driving cycles (J. Moreno and Dixon [2006], N. Jalil and Salman [1997]).

An alternative solution to analytical optimization approaches is provided by statistical learning methods (Koltchinskii et al. [2001]). In such an approach, a performance index is minimized empirically while guaranteeing that the difference between the empirical solution and the optimal one is arbitrarily small with high probability.

In this paper, a neural network controller is proposed and Statistical Learning theory is used to choose the networks' weights in order to reduce the fuel consumption during a given path. The controller is applied to a Fuel Cell Electric Vehicle (FCEV) called “*Smile*” and produced by FAAM S.p.A. (Italy). The vehicle has fuel cell stacks, that convert hydrogen to electric power, using hydrogen as primary power source. A buffer of energy in the powertrain is provided by the lead battery pack and ultracapacitors. The performance of the proposed controller is evaluated via numerical simulation. The paper is organized as follows. In Section 2 the powertrain and main power devices are described. The details of the neural network controller are discussed in Section 3 and Statistical Learning theory is presented in Section 4. The results of the numerical

simulations are reported in Section 5, and the paper concludes with comments on the performance of the proposed controller.

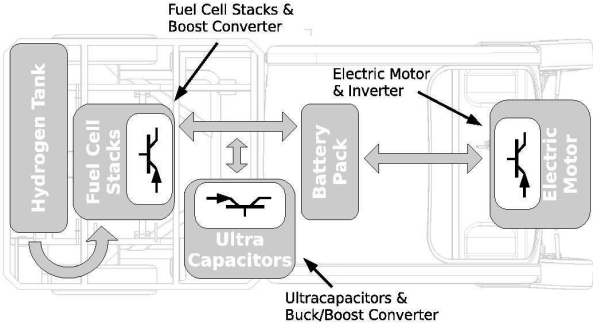


Fig. 1. Powertrain scheme of a fuel cell electric vehicle

2. FUEL CELL ELECTRIC VEHICLE

Fuel cells (FCs) are electrochemical devices that convert the chemical energy of Hydrogen directly into electric energy without combustion products. The combination of fuel cells and electric batteries allows to design clean (zero emission) and high efficiency vehicle. FCEVs are thus considered a potential solution to various pollution problems. As shown in Figure 1, the configuration of the FCEV powertrain consists of a battery pack, a fuel cell stack (PEM), ultracapacitor bank and an inverter that provides power to the electric motor. The fuel cell stack, by a dc/dc converter (Boost), are in parallel with the battery and the supercapacitors bank are connected to the power bus by a Buck/Boost converter. The Boost converter allows to push power from the fuel cell stack to the battery. On the other hand, the Buck/Boost converter works in both directions. The fuel cell stack provide the mean power to the vehicle; the ultracapacitors supply and receive power during the acceleration or the brake conditions, respectively. At the end, the inverter converts the dc voltage into an ac voltage used to drive the motor. Along a given path, the amount of power required by the vehicle is provided by the different power devices as described by the following equation:

$$P_t(t) = P_{fc}(t) + P_{uc}(t) + P_{bat}(t). \quad (1)$$

where $P_t(t)$ is the power required by the inverter at each time instant, and $P_{fc}(t)$, $P_{uc}(t)$ and $P_{bat}(t)$ are the power provided by the fuel cell, the ultracapacitors and the battery pack, respectively. The low-level control architecture of the power devices is shown in Figure (2), where the current provided by the fuel cell stacks is controlled by the *Controller*₁, using the reference signal $I_{ref}^{fc}(T)$. The complete closed-loop system of fuel cell stacks and Boost converter is modeled by the following equations:

$$I_{fc}(kT) = \sum_{j=1}^{n_1} a_j^{fc} I_{fc}((k-j)T) + \sum_{j=1}^{n_2} b_j^{fc} I_{ref}^{uc}((k-j+1)T)$$

$$V_{fc}(kT) = f_1(I_{fc}(kT))$$

$$\Delta h(kT) = f_2(I_{fc}(kT))$$

$$\eta_{bs}(kT) = f_3(I_{fc}(kT))$$

$$I_1(kT) = V_{fc}(kT)I_{fc}(kT)/\eta_{bs}(kT)V_{bat}(kT)$$

$$I_{bs}(kT) = \sum_{j=1}^{n_3} a_j^{bs} I_{fc}((k-j)T) + \sum_{j=1}^{n_4} b_j^{bs} I_1((k-j+1)T) \quad (2)$$

where T is the sampling time, $I_{fc}(kT)$ and $V_{fc}(kT)$ are the current and voltage provided by the fuel cell, $I_{ref}^{fc}(kT)$ is the reference signal for *Controller*₁ and represents the current required to the fuel cell stack, $I_{bs}(kT)$ is the current output of the Boost converter and $V_{bat}(kT)$ is the battery voltage at time kT . The hydrogen consumption is $\Delta h(kT)$, while $\eta(kT)$ is the efficiency curve of the Boost converter and a_j^{bs} , b_j^{bs} , a_j^{fc} and b_j^{fc} are the parameters of the dynamics of the fuel cell and of the Boost converter. The Boost steady-state current is $I_1(kT)$, the nonlinear functions f_1 is the current-voltage characteristic of the fuel cell and f_2 is the nonlinear function that relate the request of the current to the instantaneous fuel consumption of the fuel cell. The nonlinear function f_3 is the efficiency function of the Boost related to the fuel cell current $I_{fc}(kT)$. The power provided by the fuel cell is given by

$$P_{fc}(kT) = I_{bs}(kT)V_{bat}(kT). \quad (3)$$

In the proposed low level control architecture shown in Figure (2), the model of the power provided to the vehicle by the ultracapacitors is obtained considering the closed-loop system of the ultracapacitors with the Buck/Boost converter, where the reference signal $I_{ref}^{uc}(kT)$ is the current required to the ultracapacitors bank. The current provided by the ultracapacitors bank is controlled by *Controller*₂ and the corresponding closed-loop system (ultracapacitors bank and Buck/Boost converter) is given by:

$$I_{uc}(kT) = \sum_{j=1}^{n_5} a_j^{uc} I_{uc}((k-j)T) + \sum_{j=1}^{n_6} b_j^{uc} I_{ref}^{uc}((k-j+1)T)$$

$$V_{uc}(kT) = \frac{1}{C} \sum_{j=0}^k I_{uc}(jT)T - R_{uc}I_{uc}(kT)$$

$$\eta_{bb}(kT) = f_4(I_{uc}(kT))$$

$$I_2(kT) = V_{uc}(kT)I_{uc}(kT)/\eta_{bb}(kT)V_{bat}(kT)$$

$$I_{bb}(kT) = \sum_{j=1}^{n_7} a_j^{bb} I_{fc}((k-j)T) + \sum_{j=1}^{n_8} b_j^{bb} I_2((k-j+1)T) \quad (4)$$

where $I_{uc}(kT)$ and $V_{uc}(kT)$ are the current and voltage of the ultracapacitors, $I_{ref}^{uc}(kT)$ is the amount of current required by the ultracapacitors. It is also the reference signal for *Controller*₂ while $I_{bb}(kT)$ is the current output of the Buck/Boost. $I_1(kT)$ is the Buck/Boost steady-state current and $V_{bat}(kT)$ is the battery voltage and a_j^{bb} , b_j^{bb} , a_j^{uc} and b_j^{uc} are the model parameters of the dynamics of the ultracapacitors bank and the Buck/Boost. Moreover C and R_{int} are the capacity and internal resistance of the ultracapacitors, respectively. The nonlinear function f_4 relates the amount of the output current $I_{uc}(kT)$ to the efficiency of the Buck/Boost. The power by the ultracapacitors is given by

$$P_{uc}(kT) = I_{bb}(kT)V_{bat}(kT). \quad (5)$$

The battery pack are modelled by the following equations:

$$\begin{aligned}
 V_{bat}(kT) &= f_5(I_{dyn}(kT)) - R_{bat}I_{bat}(kT) \\
 Q_{dyn}(kT) &= \sum_{j=1}^{n_9} a_j^{bat} Q_{dyn}((k-j)T) + \\
 &\quad + \sum_{j=1}^{n_{10}} b_j^{bat} Q_{ref}^{int}((k-j+1)T) \\
 Q_{int}(kT) &= \sum_{j=0}^{kT} I_{bat}(jT)T
 \end{aligned} \quad (6)$$

where at time kT , $I_{bat}(kT)$ and $V_{bat}(kT)$ are the current and the voltage of the battery packs. $Q_{dyn}(kT)$ takes into account the dynamical aspects of the battery and $Q_{int}(kT)$ is used to model the charge of the battery. Moreover, a_j^{bat} and b_j^{bat} are the model parameters, while R_{bat} is the internal resistance of the battery pack. The power provided by the battery pack is given by

$$P_{bat}(kT) = I_{bat}(kT)V_{bat}(kT). \quad (7)$$

A complete analysis of the considered model is provided in (Cavalletti [2007]).

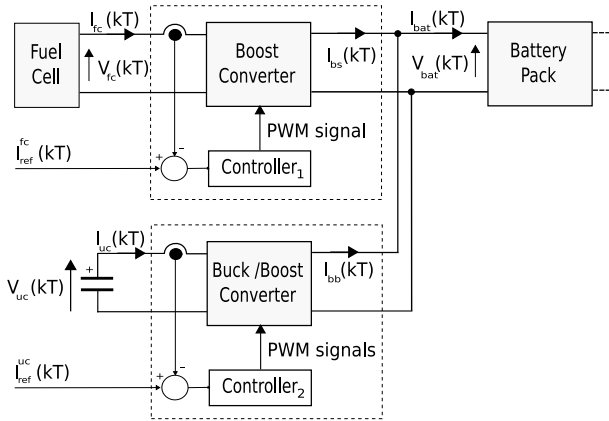


Fig. 2. Low level control architecture of the FCs current $I_{fc}(T)$ and ultracapacitors current $I_{uc}(T)$ in a FCEV

3. NEURAL NETWORK CONTROL

As described earlier, a major aim of this paper is to develop a control system that integrates with the low-level control architecture in order to reduce the fuel consumption along a given path. A Neural Network is proposed to generate the two control inputs $I_{ref}^{uc}(kT)$ and $I_{ref}^{fc}(kT)$ used as reference signals in the low level architecture. The corresponding closed-loop system is shown in Figure 3, where $P_t(kT)$ is the requested power by the vehicle along the desired path, and $P_t^y(kT)$ is the generated power by the three power devices. The outputs of the radial basis function (RBF) neural network may be written as:

$$I_{ref}^{fc}(kT) = \theta_{fc}^T \phi(\kappa(kT)) \quad (8)$$

$$I_{ref}^{uc}(kT) = \theta_{uc}^T \phi(\kappa(kT)) \quad (9)$$

where θ^{fc} and θ^{uc} are the weight vectors of the RBF network, the vector $\phi(\kappa(kT)) \in \mathbb{R}^n$ is Gaussian and defined as

$$\phi_i(\kappa(kT)) = \exp\left(-\frac{\|\kappa(kT) - c_i\|^2}{\sigma_i^2}\right), \quad i = 1, 2, \dots, n \quad (10)$$

where n is the number of nodes, $c_i \in \mathbb{R}^n$ are the centers of the basis functions and σ_i are scaling or “width” parameters (Chen et al. [1991]). The considered input vector $\kappa(kT)$ is defined in this work as:

$$\kappa(kT) = [SoC_{bat}(kT) \ SoC_{uc}(kT) \ P_{fc}(kT) \ P_t(kT)]^T \quad (11)$$

where $SoC_{bat}(kT)$ and $SoC_{uc}(kT)$ are two numbers ranging between 0 and 1 and are proportional to the State of Charge of the battery and the ultracapacitors, respectively (0 when the device is empty, 1 when the device is full charged). $P_t(kT)$ and $P_{fc}(kT)$ are the power functions defined in (1) and (3), respectively. In the proposed approach, the weight vectors θ^{fc} and θ^{uc} are designed using statistical learning theory.

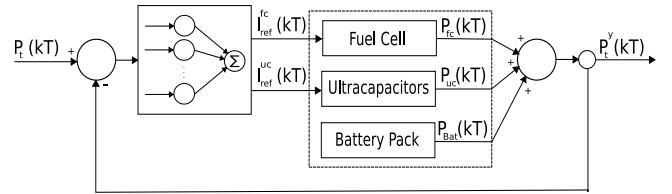


Fig. 3. Closed-loop scheme for the whole powertrain system

4. STATISTICAL LEARNING THEORY

The general supervised learning problem is considered (Vapnik [2000]). Assume there is a system producing input/output pairs (x, y) . Moreover, assume that each input is distributed according to a probability measure $F(x)$ (fixed but unknown), and that y is returned according to a conditional distribution $F(y|x)$ (also fixed but unknown). Consider a “learning machine” capable of implementing a set of functions $f_k(x) \in \mathcal{F}$, and that this learning machine is given a training set of N independent and identically distributed (i.i.d.) samples $(\mathbf{x}, \mathbf{y}) = (x_1, y_1), \dots, (x_N, y_N)$ distributed according to $F(x, y)$. Then, given a function $L(y, f_k(x))$, that measures the loss or discrepancy between the real system response y and the function $f_k(x)$, the problem is to use the information contained in (\mathbf{x}, \mathbf{y}) to choose f_k such that the risk functional

$$R(f_k(x)) = \int L(y, f_k(x)) dF(x, y) \quad (12)$$

can be minimized, trying to reproduce the behavior of the real system with the learning machine $f_k(x)$. This problem is very difficult to solve directly. First, there is the already mentioned lack of knowledge of $F(x)$ and $F(y|x)$. Moreover it may be very difficult to come up with the actual form of $f_k(x)$ such that the response of the system can be exactly reproduced. Instead, an approximation of the real $f_k(x)$ as closely as possible is estimated (et al. [2000]). This optimization problem is then reformulated as follows. Given a desired accuracy $\epsilon > 0$ and confidence parameter $\delta \in (0, 1)$, find an estimate $\hat{f}_k(x)$ of $f_k(x)$ such that

$$\sup_{F(x, y)} Pr\{R(\hat{f}_k(x)) \geq \inf_{\mathcal{F}} R(f_k(x)) + \epsilon\} \leq \delta. \quad (13)$$

or in other words, $R(f_k(x))$ is within ϵ (small) of $\inf_{f_k} R(f_k(x))$ with probability $1 - \delta$ (high). To formalize this concept the following definition is considered:

Definition 1. (Approximate Near Minimum). Given $R(f) = R(f_k(x))$, $\epsilon > 0$ and $\delta \in (0, 1)$, a number $R_0 \in R$ is said to be an approximate near minimum of $R(f)$ to accuracy ϵ and confidence $1 - \delta$ if

$$Pr\{|R_0 - \inf_{\mathcal{F}} R(f)| \leq \epsilon\} \geq 1 - \delta \quad (14)$$

Another important concept in optimization via randomized algorithms is the so called “level” (Vidyasagar [2001], Vidyasagar [1998]). Loosely speaking, the “level” describes a set of potential solutions that may not be represented in the sample taken for optimization. So if the size of this set is large, the optimization may not be valid, since the sample is not representative of the family of possible solution. On the other hand if this set can be guaranteed to be small, then there will be a small probability of finding another solution that provides considerably better performance than those found during the sampling. Combining the level with the confidence a new type of minimum is defined, where the objective of high accuracy (ϵ) is replaced by that of low probability of not finding the best solution (α).

4.1 Statistical Learning theory applied to the energy management in a FCEV

Statistical Learning theory may be used to solve the optimization problem when it is difficult to find an analytic solution. In this work, Statistical Learning theory is used to solve the optimization problem to reduce the fuel consumption in a FCEV. In the following the problem is reformulated as a statistical learning one. Consider the performance index $J(\cdot)$ related to the fuel consumption

$$\begin{aligned} J(\theta^{fc}, \theta^{uc}, KT) = & \alpha_1 \sum_{j=0}^K \Delta h(\theta^{fc}, \theta^{uc}, jT)T \\ & + \alpha_2 |SoC_{bat}(t_0) - SoC_{bat}(t_1)| \\ & + \alpha_3 |SoC_{uc}(t_0) - SoC_{uc}(t_1)| \quad (15) \end{aligned}$$

where K is the number of the samples required, $\Delta h(\cdot)$ is the fuel consumption function defined in (2) which depends on $I_{ref}^{fc}(kT)$ and $I_{ref}^{uc}(kT)$ and by (8) and (9) it is a function of θ^{fc} and θ^{uc} . The terms $|SoC_{bat}(t_0) - SoC_{bat}(t_1)|$ and $|SoC_{uc}(t_0) - SoC_{uc}(t_1)|$ are used to obtain at the end of the path the same initial amount of energy stored in the power devices, α_1 , α_2 and α_3 are three design parameters. The minimization of the performance index $J(\cdot)$ is given by the design of the optimal weight vectors θ^{fc} and θ^{uc} according with Statistical Learning theory. Note that instead of looking for a solution $J^*(\cdot)$ that guarantees that the cost function (15) achieves its exact minimum, an approximation $J_0(\cdot)$ is calculated. The approximation value that evaluates the cost function (15) will be arbitrarily close to its exact minimum with probability almost equal to one.

The condition on the number of samples needed to guarantee that the solution is sufficiently close to the optimal solution are based on Lemma 1 on (et al. [2007]). The

minimum performance value for the system over the weight vector space is

$$J^* = \min_{\Theta \in \mathbb{R}^{2n}} J(\Theta) = J(\Theta^*), \quad (16)$$

the optimal solution for the system. Denote by $\{\hat{\Theta}\}$ the set of the weight samples $\{\hat{\Theta}_1, \dots, \hat{\Theta}_N\}$, with $\hat{\Theta}_i = (\theta_i^{fc}, \theta_i^{uc})$, let

$$J_0 = \min_{1 \leq i \leq N} J(\hat{\Theta}_i) = J(\hat{\Theta}_0), \quad (17)$$

be the minimum performance value for the system over the set of vectors $\{\hat{\Theta}\}$. We then have the following result.

Theorem 1. (Minimum number of input samples). The minimum number of samples N that guarantee that J_0 is a probable near minimum to level α and confidence δ of J^* is

$$N \geq \frac{\ln(1/\delta)}{\ln(1/(1-\alpha))}. \quad (18)$$

5. NUMERICAL RESULTS

Numerical tests of the proposed controller have been performed on a Fuel Cell Electric Vehicle (FCEV) called “Smile” developed by FAAM of Monterubbiano (Italy). “Smile” is a commercial vehicle that requires a main power of 5 Kw and which has a maximum velocity of 50 Km/h. The vehicle uses hydrogen (that converts to electric power by a fuel cell stack) as a primary source and ultracapacitors as an energy buffer as shown in figure (1). The fuel cell stack and the Boost converter (embedded on the fuel cell module) are produced by “HydrogenicsTM”; the module provides 12 Kw of maximum power, with current ranging between 0 to 300 A and the operating voltage ranging between 40 to 55 V. The mathematical model is given by (2) and the parameters are estimated using data acquired during different tests. A complete description of the modeling phase is reported in (Cavalletti [2007]).

Ultracapacitors The ultracapacitors are produced by the *MaxwellTechnologiesTM* and the module has 165 F of capacity and the voltage ranging between 24.3 to 48.6 V. In order to avoid damage of this power device, *Controlloer₂* works under the constraint that the capacitor voltage is in the required range. The ultracapcitors and Buck/Boost model is given in (4). The State of Charge (*SoC*) is a variable that represents the charged state of the device, and given as a number between 0 to 1. The Battery pack is composed of 6 batteries of 12 V and 105 Ah each one. The mathematical model is given in (6). Different constraints are considered for the battery pack. The State of charge (*SoC*) of the battery ranges between 0 to 1. In fact, the battery may not be completely discharged or over charged. The maximum current provided by the battery is limited to 200 A and the maximum current provided to the battery is limited to -70 A.

The total power amount $P_t(kT)$ is obtained from the request of power during a given drive test. The drive test is chosen to be representative of a typical drive condition. For this reason different road conditions (uphill, downhill, and flat), different velocities, and different drive conditions (speedup, brake) are considered.

Two different drive tests are used here: the first is used

to design the weight vectors, while the second one was chosen to test the proposed controller. In figure (4), the vehicle velocity (top) and the request of power (bottom) are shown as a function of the time during the training phase. Figure (5) show the power path used to verify the obtained performance of the proposed control.

The Neural Network controller was designed off line using Statistical Learning theory as described in the previous paragraph. The size of the RBFN has been chosen to cover all the input space spanned by the input vector $\kappa(kT)$ while maintaining as small a size as possible. Therefore, the hidden layer is chosen to have 27 placed to cover the whole input space spanned by the input vector $\kappa(kT)$. The two linear vectors θ^{fc} and θ^{uc} are designed to generate the two control inputs $I_{ref}^{fc}(kT)$ and $I_{ref}^{uc}(kT)$ using statistical learning theory. The values of α and δ are chosen to be 0.01 and 10^{-3} , respectively. This leads to the number of samples $N = 688$. Figure 6 shows the control inputs generated by the neural network controller using the second path described in figure (5). Note that $I_{ref}^{fc}(t)$ takes on only positive values only because the fuel cell stack is a power generator, while the reference signal for the ultracapacitors $I_{ref}^{uc}(t)$ can be positive and negative. In figures 7, 8 and 9 the behaviors of the voltage (top) and the current (bottom) during the drive path are reported. The figures show that all the defined constraints are satisfied. In figure 10, the states of charge of the battery pack and of the ultracapacitors are shown. The figures clearly show that the two power devices are used as power buffers, and that the final amount of energy is almost the same of the starting one. For this reason, all the energy required during the drive path is provided by the fuel cell. At the end of the drive path the efficiency of the whole system is 41.20%.

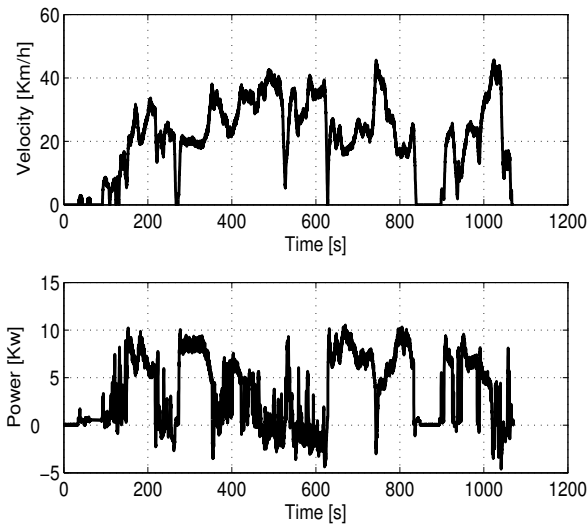


Fig. 4. Velocity and Power used for the training phase.

6. CONCLUSIONS

In this paper, the energy management problem applied to a fuel cell electric vehicle is analyzed and solved using statistical learning theory. Statistical learning theory is a powerful tool to resolve optimization problems when analytic solutions are difficult to find. This theory was used

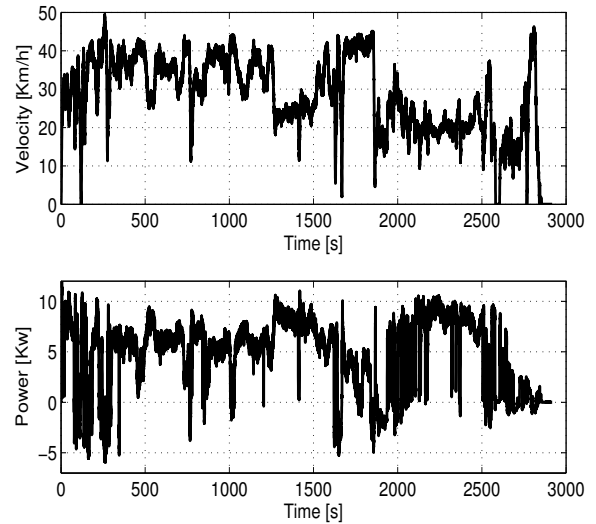


Fig. 5. Velocity and Power used to evaluate the performance of the proposed controller.

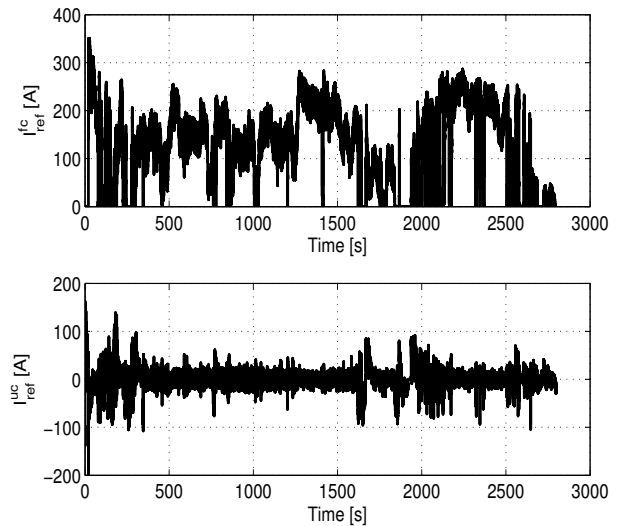


Fig. 6. Control input $I_{ref}^{fc}(t)$ and $I_{ref}^{uc}(t)$ generated by the neural network controller.

to design the weight vectors of a neural network controller with the aim to reduce the fuel consumption during a given path. Numerical results show that statistical learning may be a good tool to solve such problems. Future work is oriented towards using this approach along different paths with the aim to generalize the results. Moreover, in order to allow a comparison with other techniques, the use of standard drive paths will be consider in the future works.

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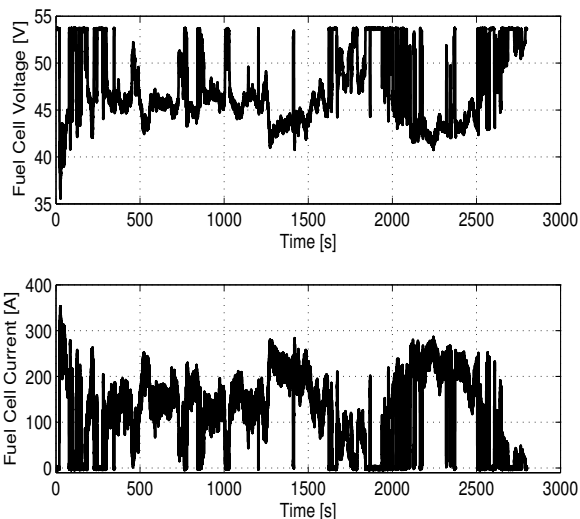


Fig. 7. Voltage and Current provide by the Fuel Cell Stack during the test.

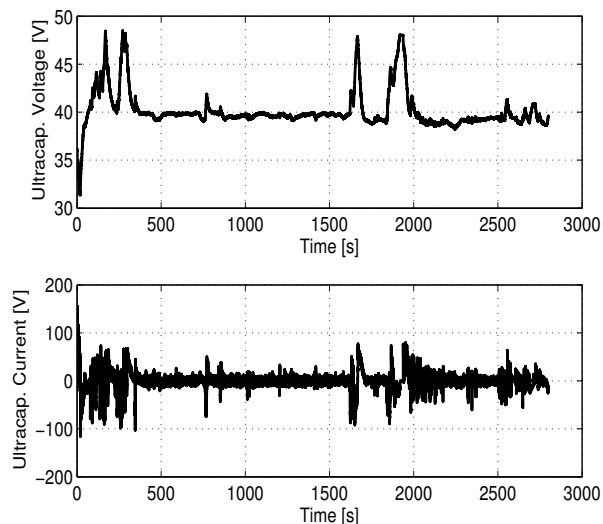


Fig. 9. Voltage and Current provide by the Ultracapacitors Bank during the test.

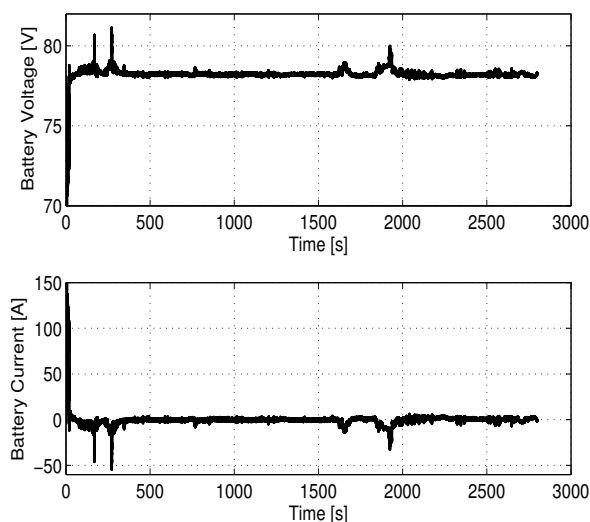


Fig. 8. Voltage and Current provide by the Battety during the test.

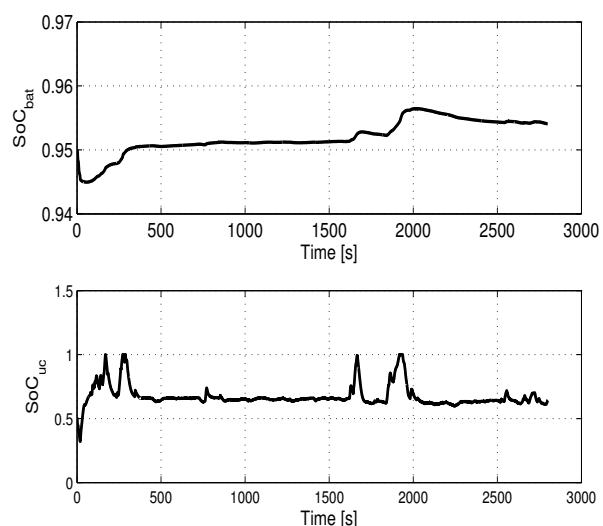


Fig. 10. State of Charge (SoC) of the Ultracapacitors and the Battery, respectively.

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