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A Suboptimal Controller Design Methodology for Input-Output Feedback-Linearizable Systems

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Abstract: This paper addresses suboptimal control of nonlinear systems which can be feedback-linearized from input to output. The case of input-to-state linearizable systems is also covered as a special case. The method is thus applicable to all nonlinear systems which can be partially linearized using the method of output-feedback linearization while having a stable internal (or zero) dynamics. The well-known LQR technique applied to the linearized system does not guarantee the suboptimality of the nonlinear system. This paper uses output feedback linearization technique to partially linearize the system and then designs an output-feedback for the feedback-linearized system in such a way that it ensures suboptimal performance of the original nonlinear system. The proposed method can optimize any arbitrary smooth function of states and input. The proposed controller is, however, suboptimal due to the facts that (1) the form of the controller is a linear static feedback of the linearized state, (2) the search algorithm may fall into a local extremum rather than a global, and (3) the calculated controller depends on the initial conditions. The method is successfully applied to control design of the longitudinal subsystem of a laboratory double-rotor helicopter and the results are discussed and compared with those of the LQR method.

Keywords: Nonlinear system control; Output feedback control; Optimal control theory.

1. INTRODUCTION

Feedback linearization technique is an important technique in the study of nonlinear control systems. Different from the regular concept of Jacobian linearization of a nonlinear system, the purpose of feedback linearization is to transform a given nonlinear system into a linear system via state-feedback. Differential geometric control is a direct synthesis method in which the controller is derived by requesting a desired closed-loop output response in the absence of input constraints. A widely used differential geometric control method is input-output linearization, which cannot be used to operate a process at a nonminimum-phase (NMP) steady state. Efforts to make input-output linearization applicable to processes with a NMP steady state include the use of equivalent outputs for the controller design (Niemiec-1998), coordinated control (McLain-1996), controller design by inverting the minimum-phase part (Doyle-1996), (Kravaris-1990), and approximate input-output linearization (Kanter-2002) and (Panjapornpon-2004).

To enlarge the class of nonlinear systems which can be handled using the differential geometric approach, the dynamic feedback linearization problem was initiated and addressed in (Charlet-1959) by introducing dynamic compensators and searching for the corresponding state and control transformations in the augmented state spaces. Sufficient conditions for dynamic feedback linearization were given in (Charlet-1991) and necessary conditions were established in (Sluis-1993). Partial feedback linearization problem was formulated and studied in (Marino-1986) and (Respondek-1986) by identifying the largest feedback linearizable subsystems, where conditions were given to transform a portion of the nonlinear system into a linear part. When the relative degree of the considered nonlinear system is less than system dimension, feedback linearization based nonlinear control can also render the transformed system consisting of a nonlinear zero dynamics plus a linear controllable system (the so-called normal form) (Isidori-1995). The difference between the normal form and the partial feedback linearizable form is that the nonlinear zero dynamics in the normal form is only driven by the states of the linear controllable system while the nonlinear part in partial feedback linearizable system can contain control inputs. More recently, nonregular feedback linearization problem was defined in (Sun-2003), where the purpose is to transform the nonlinear system into the linear controllable form with reduced control input dimensions.

Feedback linearization technique transforms the original nonlinear system into a linear system. A stable controller for the linearized system will then also stabilizes the original nonlinear system. Performance of the nonlinear system, however, is not directly related to that of the linear system and cannot be inferred based on that. An optimal design such as linear quadratic regulator (LQR) for the linearized system, for instance, does not necessarily correspond to any optimality in the performance of the nonlinear system. This paper is to address an suboptimal state-feedback design for the feedback-linearized system to achieve suboptimal performance of the nonlinear system. A technique is presented which arrives at the solution for any arbitrary cost function which is a smooth function of state variables and input. The method is successfully applied to a physical system and results are discussed and compared with those of the LQR design.

2. BACKGROUND AND PROBLEM STATEMENT

Consider the affine nonlinear system represented by

$$\dot{x} = f(x) + g(x)u, \quad y = h(x), \tag{1}$$

where x is the n-dimensional state vector, f and g are sufficiently smooth vector fields on $D \subset \mathbb{R}^n$ and u is the scalar input signal. The basic approach of input-output linearization is simply to differentiate the output function repeatedly until the input u appears. The differentiated output can then be rewritten using the following expression:

$$\begin{aligned} y = h(x) &= L_f^0 h(x), \\ \frac{dy}{dt} &= L_f^1 h(x), \\ &\vdots \\ \frac{d^{\rho-1}y}{dt^{\rho-1}} &= L_f^{\rho-1} h(x), \\ \frac{d^{\rho}y}{dt^{\rho}} &= L_f^{\rho}(h(x)) + L_g L_f^{\rho-1} h(x) u. \end{aligned} \tag{2}$$

where $L_f^1 h(x) = \frac{\partial h}{\partial x} f(x)$ is called the Lie Derivative of h with respect to f or along f. This is the familiar notion of the derivative of h along the trajectories of the system $\dot{x} = f(x)$ which is more convenient notation when we repeat the calculation of the derivative with respect to the same vector field or a new one.

Definition: The nonlinear system (1) is said to have relative degree ρ, $1 \leq \rho \leq n$, in a region of $D \subset \mathbb{R}^n$ if

$$\begin{aligned} L_g L_f^{i-1} h(x) = 0, \ i = 1, 2, \cdots, \rho - 1, \\ L_g L_f^{\rho-1} h(x) \neq 0. \end{aligned} \tag{3}$$

for all $x \in D$.

Theorem: Consider the system (1), and suppose it has relative degree $\rho \leq n$ in $D \subset \mathbb{R}^n$. If $\rho = n$, then for every $x_0 \in D$, a neighborhood N of x_0 exists such that map

$$T(x) = \begin{pmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{pmatrix} \tag{4}$$

is a diffeomorphism on N. If $\rho < n$, then for every $x_0 \in D$ a neighborhood N of x_0 and smooth functions, $\varphi_1(x), \ldots, \varphi_{n-\rho}(x)$ exist such that

$$\frac{\partial \varphi_i(x)}{\partial x} g(x) = 0, \ i = 1, \cdots, n - \rho, \ \forall x \in N \tag{5}$$

is satisfied for all $x_0 \in N$ and the map $T(x)$

$$z = T(x) = \begin{pmatrix} \varphi_1(x) \\ \vdots \\ \varphi_{n-\rho}(x) \\ --- \\ h(x) \\ \vdots \\ L_f^{\rho-1} h(x) \end{pmatrix} = \begin{pmatrix} \Phi(x) \\ --- \\ \Psi(x) \end{pmatrix} = \begin{pmatrix} \eta \\ - \\ \xi \end{pmatrix} \tag{6}$$

is diffeomorphism on N (Khalil-2002).

The input-output linearization technique is based on applying

$$z = T(x), \quad v = \alpha(x) + \beta(x)u \tag{7}$$

where $z = T(x)$ is an admissible state transformation witch is expressed in (6) and v is the new control input signal. the functions $\alpha(x)$ and $\beta(x)$ are then expressed in terms of $h(x)$ as

$$\alpha(x) = L_f^{\rho} h(x), \quad \beta(x) = L_g L_f^{\rho-1} h(x). \tag{8}$$

Upon using the linearizing transformation T and associated functions α and β, the representation (1) will change to the normal form as

$$\begin{aligned} \dot{\eta} &= f_0(\eta, \xi), \\ \dot{\xi} &= A_c \xi + B_c v, \\ y &= C_c \xi \end{aligned} \tag{9}$$

where

$$A_c = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ \vdots & & & 0 & 1 \\ 0 & \cdots & \cdots & 0 & 0 \end{pmatrix}, B_c = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \tag{10}$$
$$C_c = (\, 1 \ 0 \ \ldots \ 0 \ 0 \,).$$

This form decomposes the system into a linear subsystem described by ξ and an internal nonlinear subsystem described by η and $\dot{\eta} = f_0(\eta, \xi)$. Setting $\xi = 0$ in the internal dynamics results in

$$\dot{\eta} = f_0(\eta, 0). \tag{11}$$

which is called zero dynamics. The system is said to be minimum phase if (11) has an asymptotically stable equilibrium point in the domain of interest.

Assume that the system (1) has relative degree ρ and is minimum phase; hence, having transformed (1) into (9), the stabilization problem can easily now be addressed by choosing

$$v = -K\xi, \tag{12}$$

where K is an $1 \times \rho$ constant vector such that all eigenvalues of $(A - BK)$ lie on the negative left-half of the complex plane (Khalil-2002). This selection of control input ensures *stability* of the original system (1). However, desired *performance* of the system (1) cannot be inferred from desired performance of the system (9). As an illustration, if the relative degree ρ is n, according to (6), $T(x) = \xi$ and the system (9) is transformed to

$$\begin{aligned} \dot{\xi} &= A_c \xi + B_c v, \\ y &= C_c \xi. \end{aligned} \tag{13}$$

A Linear Quadratic Regulator (LQR) can then be designed for system (13) by properly selecting the vector K but it may or may not result an optimal performance for the original system (1). The LQR optimal controller for the linear system (13) is the one which minimizes the following cost function

$$J_\xi = \int_0^\infty (v^2 + \xi^T \bar{Q} \xi) dt, \tag{14}$$

where Q is a positive-definite $n \times n$ matrix and T stands for matrix transposition. The solution can easily be obtained using *lqr* command in Matlab. We used the subscript ξ to emphasize that this cost function is defined on ξ-space not on the original x-space. The solution to this problem does not necessarily minimize

$$J_x = \int_0^\infty (u^2 + x^T Q x) dt, \tag{15}$$

which is the associated cost function in the x-space. In other words, optimality of the linearized system does not result in optimality of the nonlinear system.

Formulating the general solutions to (15) is challenging due to the nonlinearities involved. That is why this problem has not been carefully addressed in the literature. This paper is to address this problem and to formulate a solution to (15) for the special case where the control input is of the form (12). In other words, we use the input-output linearization technique to partially linearize the system but then we design the controller coefficients K to ensure the suboptimality of nonlinear system (quantified by J_x) rather than the linearized system (quantified by J_z). The presented suboptimal controller is numerically examined on a real example and the results are discussed. The results show that the proposed controller can perform ways better than the LQR controller.

Problem Statement. *For the nonlinear affine system (1) with the original state vector x and the transformed system of (9) with the state vector of z, determine the suboptimal K in (12) which minimizes J_x of (15). We assume that the zero dynamics (11) is globally asymptotically stable.*

The solution presented in this paper is not limited to the quadratic-type cost functions such as J_x and the proposed solution is formulated for any smooth form of a cost function.

3. PROPOSED METHOD

Using $v = -K\xi$, the system (9) can be represented as

$$\begin{aligned} \dot{\eta} &= f_0(\eta, \xi), \\ \dot{\xi} &= (A_c - B_c K)\xi, \\ y &= C_c \xi. \end{aligned} \tag{16}$$

In the x-space, it will be

$$\begin{aligned} \dot{x} &= F(x, K), \\ y &= h(x). \end{aligned} \tag{17}$$

where $K = [k_1, k_2, \cdots, k_\rho]^T$ is the controller coefficients vector to be determined. Let us assume a general form for the cost function J as

$$J = \int_0^{T_f} \Gamma(x, K) dt. \tag{18}$$

where Γ is a function from $\mathbb{R}^{n+\rho}$ to $\mathbb{R}$. Initial condition $x_0 = x(t)|_{t=0}$ and final time T_f are assumed to be known. The objective is to reach at a constant, suboptimal vector K that minimizes J.

Define a new variable

$$x_{n+1}(t) = \int_0^t \Gamma(x(\tau), K) d\tau.$$

It is clearly observed that $x_{n+1}(0) = 0$ and $x_{n+1}(T_f)$ is equal to J in (18) which is to be minimized. Moreover, for all $0 < t < T_f$

$$\dot{x}_{n+1}(t) = \Gamma(x(t), K). \tag{19}$$

Augmenting (17) and (19) yields

$$\dot{X} = H(X, K), \tag{20}$$

where X is the augmented (n+1)-dimensional state vector defined by

$$X(t) = [x(t), x_{n+1}(t)]^T, \tag{21}$$

and $H(X, K)$ is a function from $\mathbb{R}^{(n+1+\rho)}$ to $\mathbb{R}^{n+1}$ given by

$$H(X, K) = \begin{pmatrix} F(x(t), K) \\ \Gamma(x(t), K) \end{pmatrix}. \tag{22}$$

The initial condition for (20) is $X_0 = [x_0, 0]^T$ and the final time is T_f, both are assumed to be known. Thus, the objective will now be to find a constant vector K to minimize $x_{n+1}(T_f) = J$.

To arrive at a solution, define

$$W = \frac{\partial X}{\partial K} \tag{23}$$

which implies that $W \in \mathbb{R}^{(n+1)\times\rho}$ is in the form of

$$W = \begin{pmatrix} \dfrac{\partial x_1}{\partial k_1} & \cdots & \dfrac{\partial x_1}{\partial k_\rho} \\ \vdots & & \vdots \\ \dfrac{\partial x_{n+1}}{\partial k_1} & \cdots & \dfrac{\partial x_{n+1}}{\partial k_\rho} \end{pmatrix}. \tag{24}$$

Taking the time derivative of W in (24) and using the chain rule results in

$$\dot{W} = \frac{\partial H}{\partial X} \times \frac{\partial X}{\partial K} + \frac{\partial H}{\partial K} = \frac{\partial H}{\partial X} W + \frac{\partial H}{\partial K}. \tag{25}$$

Notice that $W(0) = 0$ because X at $t = 0$ is independent from choice of K. It is also interesting to note that the last row of W is the gradient of J with respect to K which shows variational behavior of J with respect to changes in K.

Based on the above observations, it is now possible to propose an iterative algorithm to obtain the suboptimal K as follows.

- *Step 1.* Choose an initial value for K. A proper initial value can, for example, be obtained by solving the LQR problem in the ξ-space.
- *Step 2.* Jointly solve (20) and (25) with initial conditions $X(0) = [x(0), 0]^T$ and $W(0) = 0$. This involves a set of $(n+1) + \rho(n+1) = (n+1)(\rho+1)$ ordinary differential equations.
- *Step 3.* Update K using the information at time T_f. It can simply be done using the gradient descent rule as below

$$K_{i+1} = K_i - \mu W_i^{n+1}, \tag{26}$$

where W_i^{n+1} is the last row of matrix W at stage i and μ is a positive definite matrix which controls the convergence rate of the algorithm.

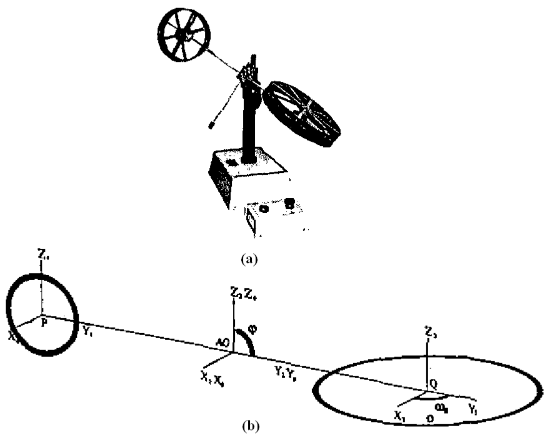

Fig. 1. (a) Double-Rotor Laboratory Helicopter, (b) Longitudinal Subsystem (Lopez-Martinez-2003,L).

A sequence of K_i yields a sequence of J_i. Since J is a positive cost function, it will have a lower bound and selecting an appropriate μ ensures that J_i is a decreasing sequence and converges at least to a local minimum.

4. NUMERICAL RESULTS

This section studies feasibility of the proposed algorithm to design an suboptimal state-feedback controller for a realistic control system as described and discussed below.

4.1 Case Study

The laboratory helicopter consists of a 2 degrees of freedom (DOF) mechanism thrusted by two rotors resembling a helicopter, Fig. 4.1. The degrees of freedom are the yaw and the pitch angles. In this analysis, the orientation angle is fixed (θ=constant) , and the angular velocity of the tail rotor is null (ω_t=0). The pitch angle will be controlled by the main rotor.

The equations of the longitudinal dynamics are as follows:

$$\begin{aligned} I_\varphi \ddot{\varphi} + G_s \sin(\varphi) + G_c \cos(\varphi) + K_\varphi \dot{\varphi} = \overline{L}_g |\omega_g| \omega_g \\ I_g \dot{\omega} = P_m - (B_g + \overline{D}_g |\omega_g|)\omega_g . \end{aligned} \quad (27)$$

The output measurement is

$$y_m = \varphi - \varphi_{eq}, \quad (28)$$

where

- φ: Pitch angle measured from the horizontal plane.
- I_φ: Inertia of the longitudinal system with respect to its rotation axis.
- ω_g: Angular velocity of the main rotor.
- I_g: Inertia of the propeller with respect to its rotation axis.
- $\overline{L}_g|\omega_g|\omega_g$: Torque due to the aerodynamic force of thrust in main rotor.
- $K_\varphi\dot{\varphi}$: Friction torque.
- $G_s \sin(\varphi)$: Gravity Torque 1.
- $G_c \cos(\varphi)$: Gravity Torque 2.
- P_m: Engine torque.
- B_g: Friction constant of the engine.
- $\overline{D}$: Drag constant of the propeller.

It can be seen that there is only an engine P_m and 2 DOF, the pitch angle φ and the angular velocity of the rotor ω_g. Therefore it is an underactuated system in the sense that it has less control inputs than degrees of freedom (see (Fantoni J. and Lozano-2002) for details).

With respect to the linearization loop, it was seen in (Lopez-Martinez-2003) that such a law was not suitable next to the static equilibrium point of the system. In order to control the system in a region around this point, the system model is modified (see (Hauser, J.. Sashy, S. and Kokotovic-1992)) and a new approximate law is obtained, which is suitable only in this region. As it was shown in (Lopez-Martinez-2004), the longitudinal system is controllable via output-feedback linearization if the rotor velocity is not next to zero; in fact, a switching control based on the two laws is studied and applied depending on the working point to control the system. While the rotor velocity is near the static equilibrium point of the system, a simplified model of the aerodynamic forces applied to the system is assumed (Lopez-Martinez-2004). The simplification consists of linearizing the aerodynamic force in a region that contains $\omega_g = 0$, that is, linearizing the force when the angular velocity is next to zero. Therefore, this approximate model is valid only for small forces.

The equations of the longitudinal dynamics are now changed by the following ones

$$\begin{aligned} I_\varphi \ddot{\varphi} + G_s \sin(\varphi) + G_c \cos(\varphi) + K_\varphi \dot{\varphi} = L_g \omega_g \\ I_g \dot{\omega} = P_m - (B_g + D_g)\omega_g \end{aligned} \quad (29)$$

where constant L_g and D could be determined from those of the quadratic forces $\overline{L_g}$ and $\overline{D_g}$ to ensure a soft switching between both laws in two step control (Lopez-Martinez-2004). The state vector is defined as

$$x = \begin{pmatrix} \varphi - \varphi_{eq} \\ \dot{\varphi} \\ \omega_g \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (30)$$

and the system state-space equations will be given by $\dot{x} = f(x) + g(x)u$ where

$$f(x) = \begin{pmatrix} x_2 \\ f_2 \\ -kx_3 \end{pmatrix}$$

$$\begin{aligned} f_2 = \\ \frac{-G_s \sin(x_1 + \varphi_{eq}) - G_c \cos(x_1 + \varphi_{eq}) - K_\varphi x_2 + L_g x_3}{I_\varphi}, \\ g = \left(0 \ 0 \ k \right)^T , \\ y = h(x) = x_1 . \end{aligned} \quad (31)$$

Computing time derivatives of the output gives the following terms

$$y = h(x) = x_1$$
$$\frac{dy}{dt} = L_f h + L_g h(x).u = x_2$$
$$\frac{d^2y}{dt^2} = L_{f^2} h(x) + L_g L_f h(x).u = f_2$$
$$\frac{d^3y}{dt^3} = L_{f^3} h(x) + L_g L_{f^2} h(x).u = L_{f^3} h(x) + \frac{kL_g}{I_\varphi}.u$$

Since $L_g L_{f^2} h(x) = \frac{kL_g}{I_\varphi} \neq 0$ is non-null, the system is said to have the relative degree three, and u can be obtained from

$$u = \frac{v - L_{f^3} h(x)}{L_g L_{f^2} h(x)}, \quad v = -K\xi. \tag{32}$$

where $K = [k_1\ k_2\ k_3]$ and

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \begin{pmatrix} h(x) \\ L_f h(x) \\ L_{f^2} h(x) \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ f_2 \end{pmatrix} \tag{33}$$

In (Lopez-Martinez-2004), K is computed by applying the LQR method to the linearized system in ξ-domain. Let us apply the proposed method of this paper to obtain K and make comparisons with the LQR in sequel.

According to (32) the control signal u is now a function of x and K as $u = u(x, K)$ and the objective is to locate the suboptimal K which minimizes the cost function

$$J(K) = \int_0^{T_f} [u^2(x, K) + x^T Q x] dt \tag{34}$$

for a given positive definite matrix Q. The stability requirement on the eigenvalues of $A - BK$ poses a constraint on the elements of K as

$$k_3 k_2 - k_1 > 0, \quad k_i > 0,\ i = 1, 2, 3. \tag{35}$$

The constraint (35) is derived using the Routh-Hurwitz criterion (on the linearized system) and ensures stability of the closed-loop system. Using the same notations introduced in previous section, we have

$$F(x, K) = f(x) + g(x)u(x, K), \quad \Gamma(x, K) = u(x, K)^2 + \beta \|x\|_2^2, \tag{36}$$

where a selection of $Q = \beta I$ ($\beta > 0$) is made for simplicity.

4.2 General Simulations

Results of computer simulations of the proposed algorithm on the above case study is presented in this section. Numerical values of $\beta = 3$ (for the cost function), $x_0 = [\frac{\pi}{3}\ 0\ 0.8]$ (initial state) and $\mu = 0.01\ I$ (step size of the gradient method) are selected. The physical parameters $I_\varphi, K_\varphi, G_s, G_c, k, \varphi_{eq}, \overline{L}_g$ are estimated as $0.7, 1, 5, 5, 0.5, 0.23, 40$, respectively, from the real physical quantities. The initial value for controller coefficients K is randomly selected as long as the stability requirement (35) is satisfied.

Figure 2 shows evolution of the cost function as well as norm of the gradient vector as iterations go on. Within about 119 iterations, the gradient vector becomes sufficiently small and the algorithm can be stopped. The suboptimal controller is $K_{\text{Opt}} = [20.1601,\ 24.4168,\ 11.6979]$ and the minimum value of the cost function is J_{Opt}=3.84.

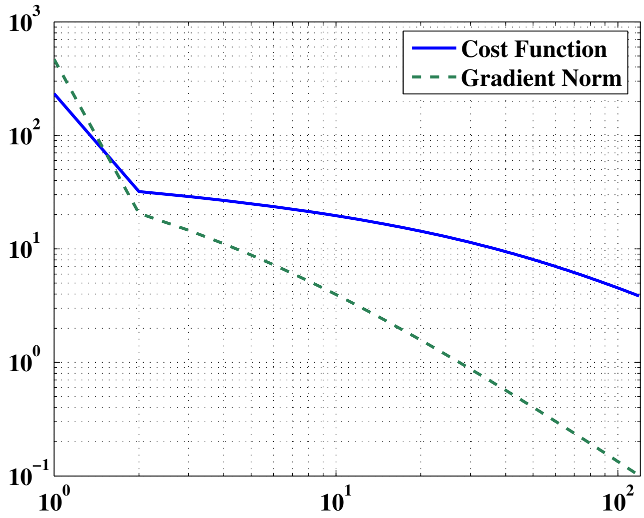

Fig. 2. Evolvement of the cost function J_x and the gradient norm $\|\frac{\partial J_x}{\partial K}\|$ versus the iteration index.

To compare the results with those of an optimally designed LQR controller, the matlab command lqr is used to obtain the optimal controller $K_{LQR} = [0.7071,\ 1.8419,\ 2.0454]$ with the same β.[1] This controller results in a value of $J_{LQR} = 232.74$ for the same cost function $J(K)$ given in (34). The index function is improved about sixty times by the proposed controller. This large difference is due to the fact the the LQR algorithm addresses the problem in the ξ-space which is only a fictitious space and does not necessarily reflect any optimality. The proposed algorithm, on the other hand, achieves the minimization in the real, physical x-space.

Time responses of the closed-loop control system using both the proposed controller and the LQR controller are obtained and shown in Fig. 3 and Fig. 4. Figure 3, parts (a) and (b) respectively depict the state variables of the system using proposed controller and the LQR controller. The control signals for the proposed controller and the LQR are also respectively shown in Fig. 4 parts (a) and (b). Comparing with the LQR responses, variations of the state variables as well as the control signal are within a much smaller range in the proposed controlled system which confirms suboptimality of the proposed controller.

5. CONCLUSION

The problem of designing an suboptimal output-feedback controller is addressed for a class of nonlinear systems characterized by those which can be partially linearized using the feedback linearization technique while its internal dynamics remain stable. The method is evaluated in the context of a physical system and results confirm that the proposed controller can behave ways better than the conventional LQR designed for the linearized system. The method is presented in a step-by-step algorithm.

[1] We do not deny the possibility of existence of a $\bar{Q}$ in the LQR problem which may generate a lower index J_x but there is no evidence as how to get to such $\bar{Q}$.

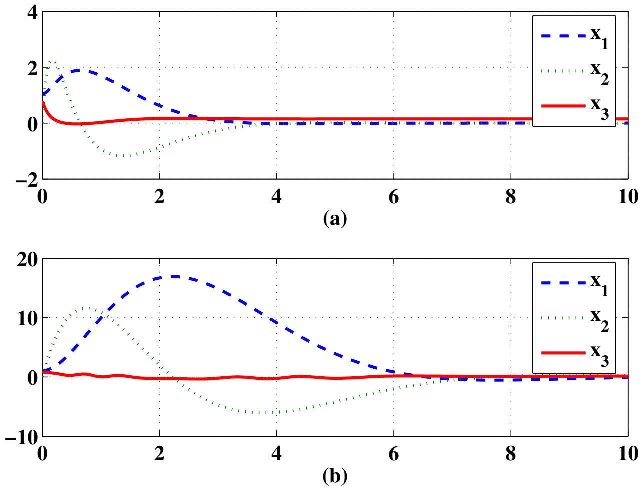

Fig. 3. (a) State variables using the proposed controller, and (b) state variables using the LQR method.

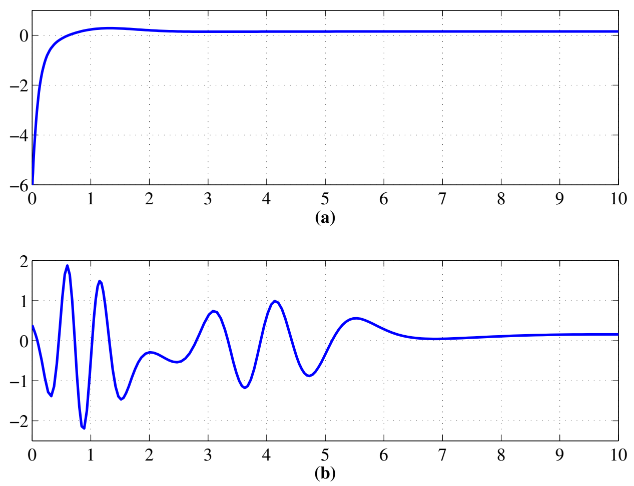

Fig. 4. (a) control signal using the proposed controller, and (b) control signal using the LQR method.

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